Sabin Levi

Counterpoint, Polyphony
Strict (Renaissance) Style

Practical Instruction

In Sacris

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Полифония – строг стил
Практическо ръководство

Сабин Леви

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Foreword by the Editor

The new textbook "Counterpoint – Strict style" completes the cycle of manuals that realize the large-scale spiritual project of Sabin Levi: to provide Bulgarian musicians, students and pupils with abundant material with a detailed instruction to study the polyphonic art. After "Polyphony - Free style...", the author heads to the roots of the high polyphonic tradition. This journey back in time is actually a journey forward into the knowledge.

The variety of polyphonic species, taught in two-voice, three-voice and four-voice textures (with informative examples also in more voices), provides limitless technical possibilities. The author suggests their practical immensity through the calculated probabilistic combinations, which in four-voice polyphony reach well into the thousands. To find a successful path to mastering polyphonic dexterities today requires a unique combination of qualities: compositional skills, performance and research experience, knowledge of the fundamental theoretical works and of the national theoretical and methodological tradition, pedagogical imagination, and resourcefulness. Sabin Levi has it all: a translator of Fux and Jeppesen, an expert on Taneyev and on the Bulgarian polyphonic achievements from the times of Asen Karastoyanov until today, a teacher of polyphony and a composer, a creative figure with unconventional thinking and an original sense of humor.

In this work, Sabin Levi remains faithful to his pedagogical approach: dialogical, provocative, surprising, activating the creative potential and cognitive processes to the limits of what is possible. This interactive educational environment, as modern as it is difficult to achieve, is "managed" by the author of the textbook in "real time", opening visibility to the very creative spirit that shapes the musical matter. Naturally, the rules of style are observed here too, but not as a didactic compulsion, but as a discipline of the musical thinking and as a means of artistry. The rules are therefore "fixed" and "more lenient". The former trace the route, the latter enable it to be traveled each time individually rather than mechanically, personally experienced rather than imitated.

As an artist, Sabin Levi never forgets the genetic link between the art and game. He reminds us of it in this textbook - not just by mentioning historical facts, but also as a way of creating and teaching. The music creator is not afraid of mistakes, but consciously confronts them and elegantly "tames" their opportuneness.

Such a text is a serious challenge for the editor. My advantage is the modest experience I have gained in this adventure of being the first reader, the first "student" who, as if in the conditions of a pedagogical experiment, crosses and approbates, accepts and rejects, is astonished by the totally unexpected surprises appearing - palindromes, puzzles, shifts in space, inversions of the note text. In the end one learns to be part of the game and not to interrupt it, but only to make it as clear as possible. In order to bridge the gap between the previous and the new

textbook, between the free and the fixed style, I will take the liberty of quoting the last words of the old foreword, to the previous textbook, which find here a new and complete confirmation: “I think that even the most experienced theorists and musicologists would find it profoundly meaningful to unfold the pages of this book and walk with the author along the tracks of the staffs and notes. A creative adventure that bestows the knowledge, unleashes the imagination and stirs the inspiration.”

I wish good fortune to all who will embark on the journey to the strict polyphonic style with this textbook!

Prof. Mariyana Bouleva

Introduction - Objective and Teaching Approach

The aim of this textbook is to develop certain aural, practical skills. These skills are very important; they are the foundation of tonal music.

Polyphony as a term can be explained in several ways:

The first definition relates to texture - where more than one linear musical idea is involved.

The second definition concerns a system of rules that regulate and explain the movement of horizontal lines in music, and the relationships between these lines vertically.

The third definition concerns directly or indirectly various polyphonic techniques - it is closely related to composition.

The traditional teaching approach is basically as follows:

Two different historical styles are taught. The first is based on Renaissance music (16th-17th centuries) - in the world of music theory it is called strict style. It is of two types - "scholastic" or "synthetic", and "real".

The second is based on Baroque music (17th - 18th centuries) - and it differs significantly from Renaissance.

The first style is taught here, and is also the first in order in teaching practice. It is conventionally called strict because it has a strict, pragmatic control over dissonances in the music. The free (Baroque) style is not taught in this textbook.

The strict style, also in the old teaching tradition, is taught in three consecutive movements.

The first part contains the so-called Fuchsian polyphonic modes; these were very clearly developed by the Austrian composer and theorist Johann Joseph Fuchs (c. 1660 - 1741). These exercises begin with the so-called Fuchsian cantus firmus, followed by exercises in two, three, four and sometimes more voices. These synthetic, practical exercises have been the basis of the polyphonic course for centuries.

Next comes the section on invertible counterpoint, technical and compositional practices that are not entirely synthetic-they were defined in the Middle Ages, and in some sense are still used today.

The last section deals mainly with the Renaissance canon.

But Fuchs's polyphonic style, and that of the "living" Renaissance differ. The Fuchsian approach is more rigorous. Once it is mastered, certain relaxations follow that make the real Renaissance style possible, and even easy to write.

Like any science devoted to art, polyphony is a dynamic, multifaceted matter. Different composers, teachers, and theorists have different views. These views often differ and even contradict each other. This poses a certain challenge for the teacher as well as the student; we have a somewhat subjective position on the hundreds of details,

axioms, and postulates that make up polyphony. In trying to navigate this matter, we use two main criteria for teaching and practice:

Developing the technical discipline of work. By training our ear, we learn what is possible and what is not, what is the simplest and most direct approach to solving a musical problem.

Developing stylistic skills. What is better suited to the style of a particular era, or even a particular composer.

These two criteria are often not in complete agreement with each other. Sometimes the teaching approach drifts more towards discipline, and sometimes more towards style. The two criteria find themselves in a complex relationship, a love-war. Given the need to develop both, the teacher finds himself or herself in a situation where he or she must balance the two criteria, as fairly and as reasonably as possible. This is no easy task. And this explains the fact that every textbook on polyphony, no matter by which author and from which era, represents a somewhat subjective, personal creative and technical decision of its author.

The same applies to this book. Using the works of Fuchs, Jeppesen, Cherubini, Zdravko Manolov, Dimitar Hristov, and others, I present my personal teaching concept. That is, I also use ideas of other authors, but their interpretation is my own. In order not to mislead the reader that the polyphonic rules are "cut in stone", at times I present the position of one or another authority on a particular issue. This presents a potential subject for discussion - but my personal position is clearly defined. But it also explains my decision to sometimes present technical rules as "hard" and "looser". This also affects the more specific focus of this textbook - it is primarily a practical guide, and to a lesser extent a musicological, descriptive or historical tool.

In order to study polyphony, we must have developed our musical ear. I consider solfege, and musical performance, to be an integral part of the educational process. It is of utmost importance for all of us to hear what is written on the page

In order to study polyphony, we must have developed our musical hearing. I consider solfege and musical performance to be an integral part of the educational process. It is of utmost importance for all of us to hear what is written on the page - that is, our approach in learning this subject should be auditory, not graphic. If we try to study algebra without mastering arithmetic, or classical painting without a knowledge of perspective, we will find that it is about the same.

In addition to a well-developed inner musical ear, it is important to have certain knowledge pertaining to the theory of musical elements. This especially applies to the knowledge and mastery of at least the viola and bass clefs (possibly others), intervals, their types and reversals, and certain musical phenomena - ostinato, sequence.

I recommend that polyphony be studied with a teacher who checks in real time to what technical degree the exercises have been mastered. I also consider it very important that all exercises are played or sung. This component of learning is particularly important if we aim to develop practical skills.

This book contains some excerpts from another textbook of mine, Polyphony, Free Style.

I am obliged to introduce a small caveat here. All the possible technical variants referring to the various polyphonic sections are thousands[1] . Some of them are used relatively often - others not; others exist as theoretical possibilities for which there are almost no precedents in music. If I had attempted to present them all, I would have ended up with material of many thousands of pages - and I am not aware of any author who has even considered undertaking this task. This book aims to teach only the most important principles, the most necessary music theoretical tools - and its focus is distinctly practical.

I would like to thank my music editor, Prof. Dr. Mariana Buleva, Ph.D., Associate Professor Dr. Petya Kirklisiyska, and the English translator Dr. Sabina Yordanova.

Sabin Levi
Sofia, August 2023

[1] All Fuchsian rules are about a hundred. The options for contrastive Fuchsian polyphony are as follows: 36 for two-voice, 216 for three-voice, 1296 for four-voice. All together: 1548. Invertible counterpoint: 4 for two-voice, 6 for three-voice, 24 for four-voice. For combined invertible counterpoint: several hundred. All the possible options and variations for writing and applying canon, including free style canon, I believe number about five or six thousand. Very few are actually used, but they exist.

Table of Note Markings:
Таблица на нотните означения:

Middle C - (do) - До – C
D – (re) - Ре – D
E - (mi) - Ми – E
F - (fa) - Фа – F
G – (sol) - Сол – G
A – (la) - Ла – A
B – (ti, or si) - Си – B

First octave - (Middle octave) - Първа октава – C1, D1, E1, etc.
Second octave – (The one above it) - Втора октава – C2, D2, E2, etc.
Small octave – (The one under the first) - Малка октава – c, d, e, etc.

Cantus Firmus

This is a melody in one voice that serves as the basis for the traditional teaching approach. It is learned through synthetic exercises that are written in a modal context, that is, in the "old modes" - Ionian, Dorian, and so on. Fux lays the foundation for an approach to use these synthetic exercises in a particularly precise and clear way.

Rules for writing a cantus firmus

Fixed rules[1]

I. Only whole notes are written without bar lines (for now), with two final bar lines at the end. It is not necessary to write time signature.

II. The exercise should not contain less than seven tones/notes.

III. No repeated notes (next to each other).

IV. It can be written from any note except B.[2] If we are writing for a hypothetical soprano tessitura, it is preferable that the lowest note be an f or e, and the highest be an A2 or B2.

V. The first note is the first degree of the mode - the same applies to the last note. For example, if the exercise was started from D, it should end on D, but the last note can be an octave below or above the first note. A larger interval between the first note and the last note (two or three octaves) is to be avoided.

VI. No sharps or flats are used. Here the approaches differ drastically between the different teachers. There are occasions of the use of F sharp and B flat in some special cases, and the use of ficta[3]. Sometimes a key signature (tonal approach) is used. The more simplified method is shown here.

VII. The penultimate tone is the seventh or second degree of the mode. That means, if the exercise starts from D, its penultimate tone should be either E (second degree) or C (seventh degree).

---

[1] The rules differ from source to source. For example, the number of notes according to Karastoyanov is 9 -15. There are barlines. He also recommends an odd number of notes. There are also differences in Manolov and Hristov's textbook. In this textbook I use my personal interpretation of the rules.
[2] From B it would mean that the exercise is written in Locrian mode; since this could include a tritone interval between the first and fifth degrees (B-F or F-B), Locrian exercises are not usually used.
[3] These phenomena are sometimes used - this is explained later. Artin Poturlian, in his teaching practice, allowed the use of F sharp and B flat.

VIII. Forbidden leaps between two adjacent tones:

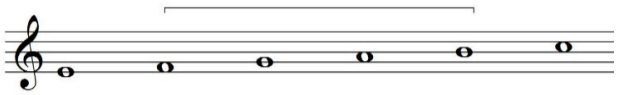
1.

Upwards	**Downwards**
Tritone (F-B or B-F)	Tritone (F-B or B-F)
Major sixth	Sixth (any kind)
Seventh	Seventh
Any interval above the octave	Any interval above the octave

IX. Two consecutive leaps in the same direction are forbidden. Any motion greater than a second is considered a leap. That is, a C-E-G motion in the upward direction would constitute an error. A C-E-G motion in the downward direction would be an error as well - also because the downward sixth leap is forbidden (VI).

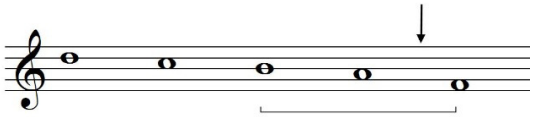
X. Tritone "gap". This phenomenon is sometimes avoided in some textbooks without explanation. Its treatment varies between the different sources. If the notes are placed on a straight line (in the same direction) between F and B or vice versa, there can be no missing notes:

2.

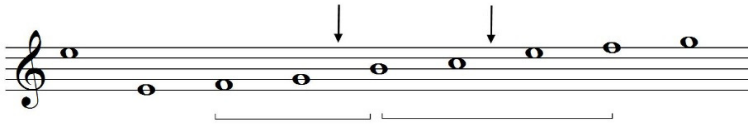


In this example, the notes F and B are placed in a straight ascending line. There are G and A in-between, which means there are no problems here. But if one or two notes are missing, as for example here, then errors will occur:

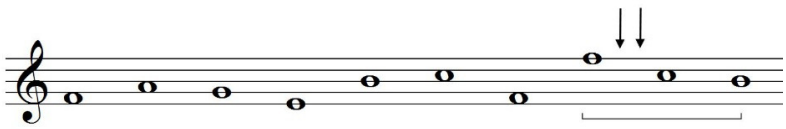
3.



G is missing here



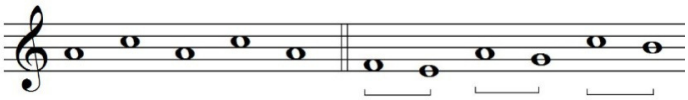
A is missing between F and B, and then D is missing between B and F



There are F and B here, but they are not on the same straight line, except at the end. E and D are missing there

XI. Ostinati and sequences are not allowed:

4.



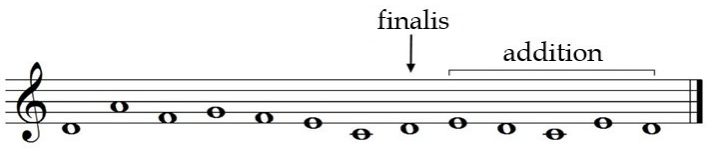
A leap in one direction followed by a leap in the other is allowed. This is the so-called reverse leap, which is considered stylish and appropriate in most cases.

Additional rules

They are not mandatory. Some are recommended, others not so much. The more recommended rules are highlighted:

1. The cantus firmus must have one highest and one lowest tone.
2. **It is recommended that the exercise achieves a smooth melody with the following linear characteristic: there should be neither too many leaps nor too many stepwise moves.**
3. After one or two stepwise moves in one direction one can't leap in the same direction. The reverse is also true - after a leap we can't continue more than one or two stepwise moves in the same direction.
4. A beautiful technique is to leap in one direction and then move stepwise in the opposite direction - leap and fill-in.
5. An interesting phenomenon is the so-called double cadence at the end of the exercise. In such cases, once we have reached the last tone (finalis), we can add a few more notes after it:

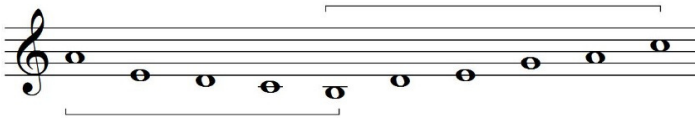
5.



6. A reverse leap is recommended but not mandatory.

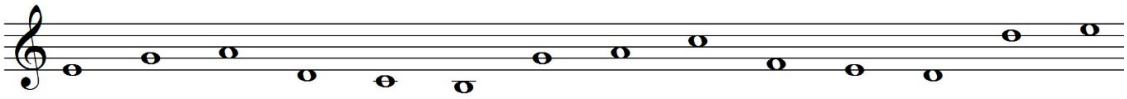
7. Dissonant segments. When we observe the contour of the exercise, we can find the intervals that are obtained between the highest and the lowest tone in a similar motion - i.e., without a change in the direction of motion:

6.



Here, the interval between A (the first tone) and B, achieved in similar motion, is a seventh. Then from the same tone, B, we move in one direction, in similar motion, to the last tone, C. This interval is a ninth. Such intervals are forbidden. Here is another example:

7.



The first segment is between E - the first tone - and A. A is the temporary highest tone. From A the motion changes - we go downwards. The lowest tone is B - the end of the descending segment. Then the motion changes back to ascending and the highest tone is C. From the same tone the motion is descending again and so on.

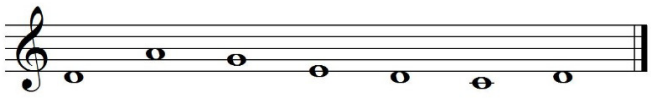
All segments should be checked. The forbidden intervals of the segments are as follows:

A. Tritone
B. Seventh
C. Ninth
D. Tritone over an octave
E. Seventh over an octave

8. Larger sum intervals in similar motion are not allowed.

Here is an example for analysis:

8.



This exercise is subject to all rules, including the additional ones:

Fixed rules

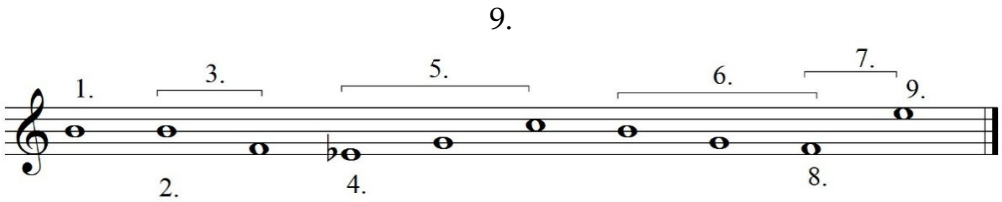
I. There are no bar lines and meter, there are two lines at the end
II. Contains seven tones
III. There are no repeated tones (next to each other)
IV. It doesn't start from B
V. It begins and ends on D
VI. There are no sharps and flats
VII. The penultimate tone is seventh degree of the mode
VIII. There are no forbidden interval leaps
IX. There are no successive leaps in one direction
X. There are no tritone gaps
XI. Lack of ostinati and sequences

Additional rules

1. There is a highest tone (A, second note) and a lowest tone (C, sixth note)
2. There are two leaps and four stepwise motions, it is smooth. The exercise resembles a melody (can be easily sung) and doesn't have a too large singing range
3. After the D-A leap, we are moving in the opposite direction, although we are not moving entirely stepwise - there is a leap and a fill-in
4. There is one leap after a stepwise motion and two stepwise moves after a leap in the same direction
5. There is no reverse leap in this exercise - it is not mandatory
6. There is no double cadence - it is not mandatory
7. Lack of dissonant straight line segments
8. The maximum distance interval in a straight line segment in a similar motion is a sixth

Thus the typical instructional cantus firmus acquires the characteristic of an asymmetrical, distinctive melody.

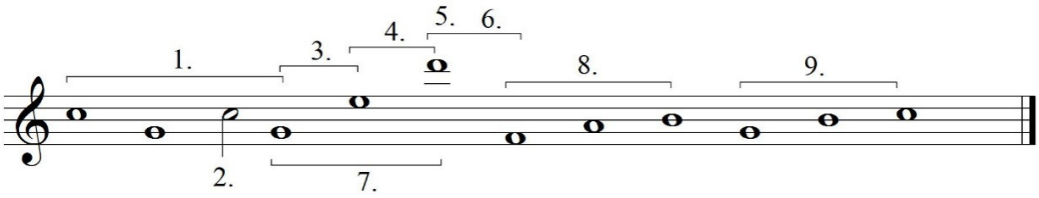
Examples with errors - according to the fixed rules only:



1) It starts from B
2) Repeated tone
3) Descending tritone leap

4) A flat before the tone E
5) Two leaps in one direction
6) Tritone gap (A is missing)
7) Seventh leap
8) The penultimate tone is not the second or seventh degree of the mode (if it is considered that the first tone of the exercise is the first tone of the mode)
9. The last tone is not the same as the first one

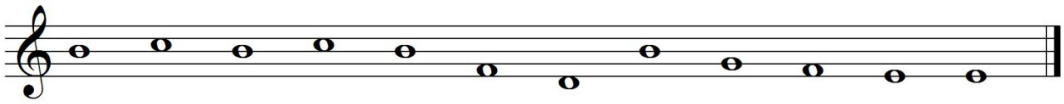
10.



1) Ostinato
2) A half note
3) Major sixth leap upwards
4) Seventh leap
5) Too high a tone for soprano tessitura
6) Descending leap above the octave
7) Two leaps in one direction
8) Tritone gap – G is missing
9. Sequence. Two identical segments - F-A-B, then G-B-C

Following are some examples that can be analyzed by the reader.

11.



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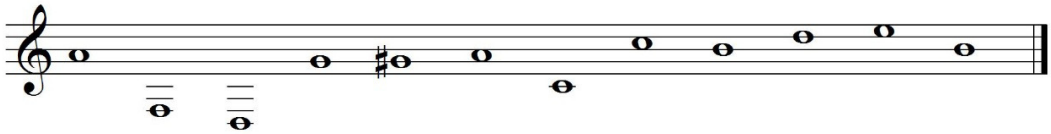

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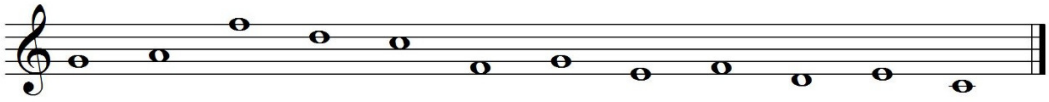

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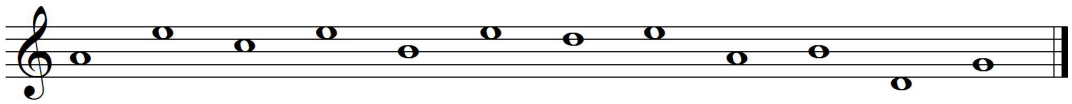

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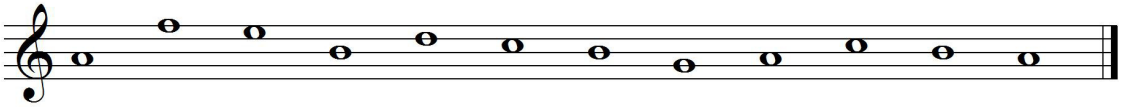

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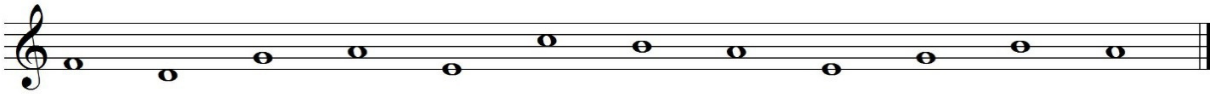

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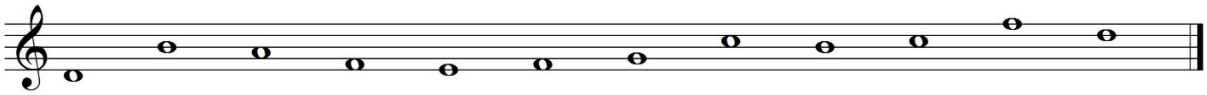

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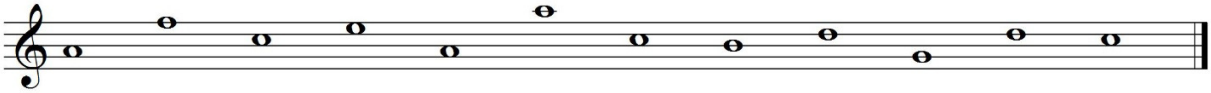

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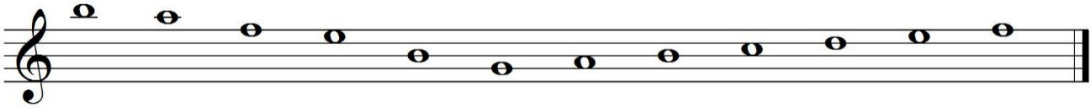

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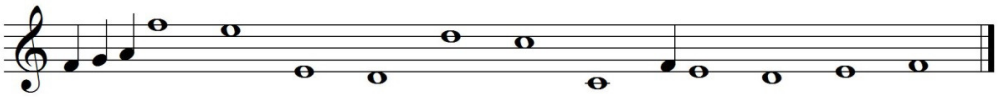

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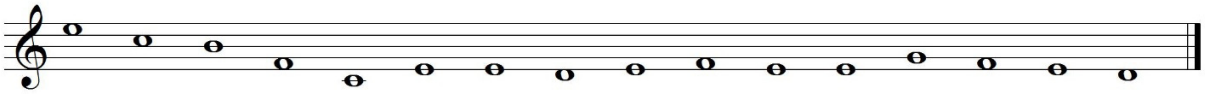

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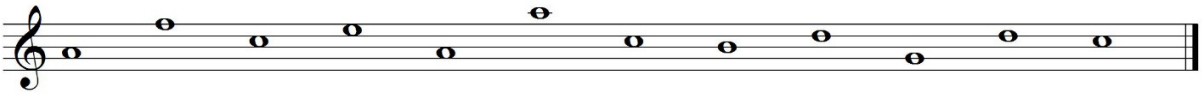

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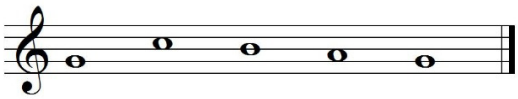

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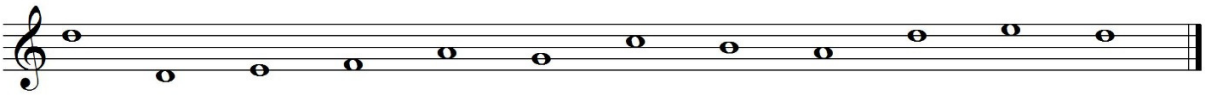

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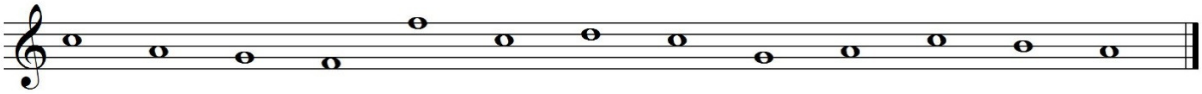


For homework, I recommend writing canti firmi and - especially important – to be played and sung.

First Polyphonic Species – Note-Against-Note

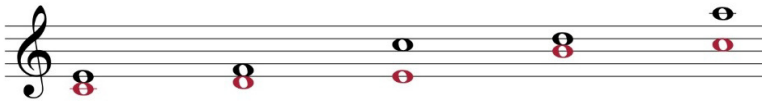
We add a second voice that counterpoints the cantus firmus. Against one note of the cantus firmus there will be one note of counterpoint.

All the rules for the cantus firmus remain in force except one: a repeated tone is allowed here in the counterpoint - this is contrary to Rule III.

Three types of polyphonic motion

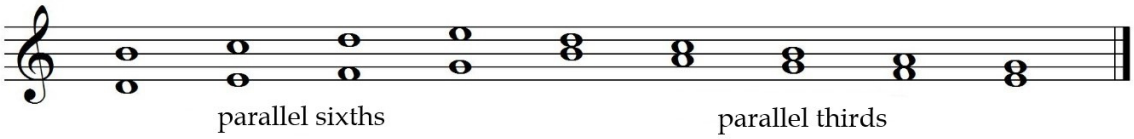
When the two voices move in the same direction, the motion is called similar (straight):

1.



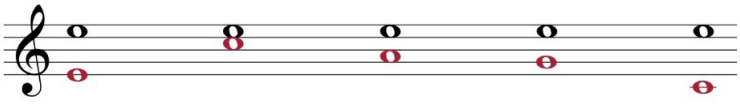
Variant of the similar motion – parallel motion. It has equal intervals on the vertical:

2.



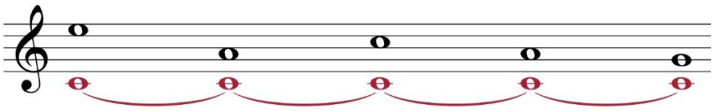
When one of the voices does not move, then the motion is oblique.

3.



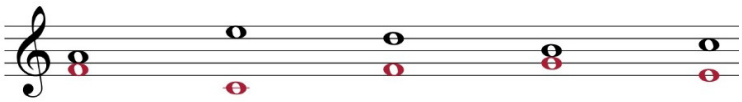
In this example the motionless voice is on the top.
And here the motionless voice is below:

4.



When the voices move in mutually opposite directions,
the motion is called contrary:

5.



In polyphony, the contrary motion is considered the most convenient. It produces the best results and is the easiest to avoid mistakes in.

Fixed rules

I. All fixed cantus firmus rules remain in force.

I. The contrapuntal voice may be below or above the cantus firmus. But crossing of voices is allowed - from the second to the pre-penultimate tone. This means that the first tone of the counterpoint must be in the correct position and the same applies to the penultimate and last tone[1].

II. The maximal distance between the two voices should not exceed two octaves (according to Hristov - a fifth over an octave).

III. When the contrapuntal voice is above, the first tone can only be the first, third or fifth degree of the mode. If the first tone of the cantus firmus is *D*, then the counterpoint can only be *D*, *F*, or *A*. If the contrapuntal voice is below, it can only start from the first degree of the mode[2].

IV. The last tone of the counterpoint can only be the first degree of the mode (the same applies to the cantus firmus) - a whole note.

V. One repeated tone is allowed in the counterpoint, i.e. two identical notes next to each other (according to Hristov - two repeats, three notes). Such a move should be used sparingly and carefully.

VI. If the penultimate tone of the cantus firmus is the second degree, the penultimate contrapuntal tone must be the seventh degree and vice versa.

---

[1] Fux and Jeppesen do not follow this rule. This is a phenomenon that can be observed in the living music. But it is logical to work out some degree of discipline and additional technical skill - therefore I believe that the penultimate and last tone should be in the same position as the first. In a real compositional project this rule can be ignored.

[2] As the contrapuntal voice thus turns out to be a bass voice, we cannot start from another tone to avoid a false impression of starting in the wrong mode. Also, if the lower voice were to start on the fifth degree, a forbidden interval of fourth on the vertical would result.

VII. Forbidden intervals on the vertical - dissonances[3]:

A. Second

B. Fourth

C. Tritone

D. Seventh

Each of these intervals is also forbidden over an octave.

VIII. Perfect unison on the vertical is allowed only on the first and last note of the cantus firmus.

IX. Rules for treating perfect intervals on the vertical:

A. Perfect unison - in this interval one cannot enter or exit in a similar motion[4]

B. Perfect fifth - cannot be entered in a similar motion

C. Perfect octave - cannot be entered in a similar motion

The same rule applies to a fifth over an octave or two octaves.

X. It is not considered to be good taste to have more than three parallel thirds or sixths between the voices.

XI. It is forbidden for both voices to leap simultaneously in one direction - i.e. in a similar motion.

XII. Parallel perfect unisons, fifths and octaves are forbidden.

More lenient rules

1. It is desirable for the contrapuntal voice to be flowing smoothly.

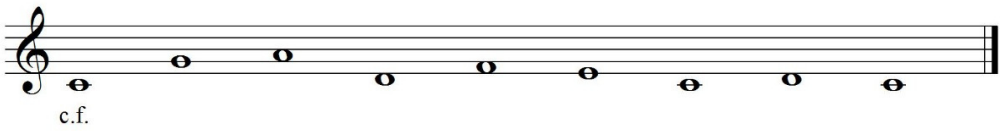
2. It's also not a bad idea for it to have its own highest and lowest tone. Attention is sometimes paid to the contrast between the cantus firmus and the counterpoint - if the two lines have high and low tones in different places, with different contours, this is considered a good achievement - a deliberate contrast between the voices.

3. It is an interesting and important rule to examine what the possibilities are for counterpointing the given cantus firmus in a similar stepwise motion. Sometimes this is possible - sometimes not. As an example, I give the following c.f. ("cantus firmus" - this designation is written in order to be able to distinguish it from the counterpoint, which is sometimes denoted by "c.p."):

[3] Sometimes it happens that we get the interval rules confused whether they affect the horizontal or the vertical: leaping down a sixth is forbidden, as is leaping up a major sixth. But on the vertical both the minor and major sixths are allowed.

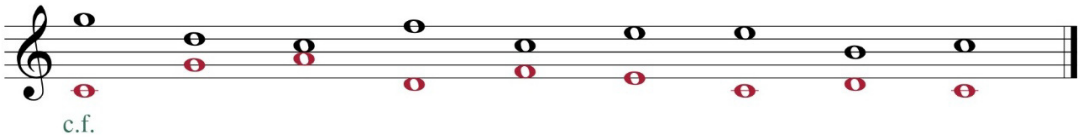
[4] Perfect unison, fifth and octave, in which we have entered with a similar motion, are referred to as "hidden" intervals.

6.



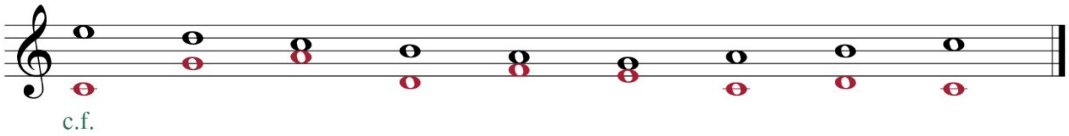
I can write the following counterpoint in the upper voice. This counterpoint follows all the fixed rules [5]:

7.



But the following, more smooth solution is also possible:

8.



Thus I demonstrate that this cantus firmus is susceptible to a much smoother counterpoint. This example is important. If we are able to avoid the leaps, then we have developed a better, more perfect skill to counterpoint. The avoidance of leaps, the smoother and more logical motion, and the economy of means are the most important principles of polyphony.

After we have written an exercise, we have to check it. It is preferable to check the vertical first - the intervals between the voices. Then the horizontal of the contrapuntal voice is to be checked. In this way we demonstrate that we have taken care of every aspect of the exercise - a mandatory goal and task in polyphony[6].

The following is an analysis of exercise 7[7].

[5] Between the first and second note of this example there are antiparallel fifths (battuta) - i.e., fifths that are allowed to occur as a result of contrary motion of the voices. Again, different polyphonists approach this phenomenon differently. Some of them tolerate it, others do not. In my interpretation it is considered a slightly undesirable but not forbidden phenomenon.

[6] It is also advisable to write all polyphonic exercises relatively quickly. For example, writing a cantus firmus rarely takes more than 40-50 seconds, and the learner can try to complete it according to the clock. Then the check is to be done slowly and carefully.

[7] Only the upper voice and the vertical between the voices are analyzed. We consider that the cantus firmus has already been checked.

Vertical

- The upper contrapuntal voice begins on the third degree of the mode (E - Rule III.)
- From the tenth, i.e. the first interval on the vertical, I move to the fifth (G-D - this is the fifth achieved with the contrary motion of the voices. This is allowed to me - IX.
- The next interval is a third - A-C
- Then the interval is a sixth
- Again a third (F-A)
- And again a third
- A sixth (*C-A*)
- Again a sixth. In the cantus firmus there is a D - second degree. In the counterpoint there is a *B* - seventh degree -VI.
- An octave. Both voices are on the first degree of the mode (IV.) I have entered the octave by means of a contrary motion - IX.
- There are no forbidden intervals on the vertical - VII.
- There are no leaps of the two voices in one direction –XI.

Additional observations

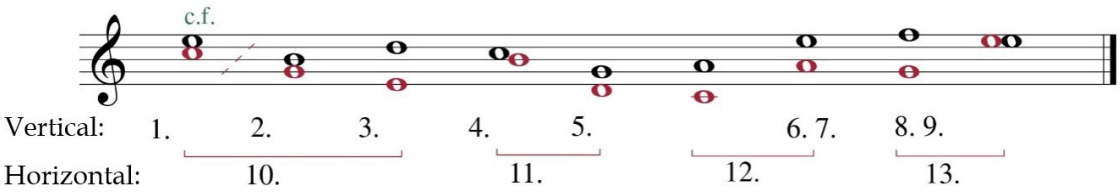
- There are no crossing voices, not obligatory - I.
- No octave has been exceeded on the vertical between the voices - II.
- There are no repeated tones in the counterpoint, not obligatory – V.
- There is no perfect unison anywhere in the exercise –VIII.
- There are no more than three thirds or sixths as well – X.

Horizontal

- Similar stepwise movement. There are no false sums – additional rule 7 of the cantus firmus chapter.
- The counterpoint is as smooth as possible – 1.
- It even has a highest and a lowest tone, and they don't match the highest and lowest tone of the cantus firmus – 2.

The following are examples with mistakes:

9.



In this example, the cantus firmus is in the upper voice and therefore the c.f. marking is placed on the top.

Vertical

1. The first tone of the counterpoint is not the first degree of the mode, as is the first tone of the cantus firmus – IV.
2. The two voices leap simultaneously in one direction, (i.e. in a similar motion) from C-E to G-B – XI.
3. Forbidden interval – seventh - VII.
4. Forbidden interval – second - VII.
5. Forbidden interval – fourth - VII.
6. The two voices leap simultaneously in the same direction, i.e. in a similar motion - from C-A to A-E - XI.
7. The same interval (A-E) is a fifth that is achieved in similar motion – this is a hidden fifth - IX.
8. Forbidden interval – seventh - VII.
9. The penultimate tone of the counterpoint is not the seventh degree of the mode (i.e. not D). It has to be D, because the penultimate tone of the cantus firmus is F – second degree - VI.

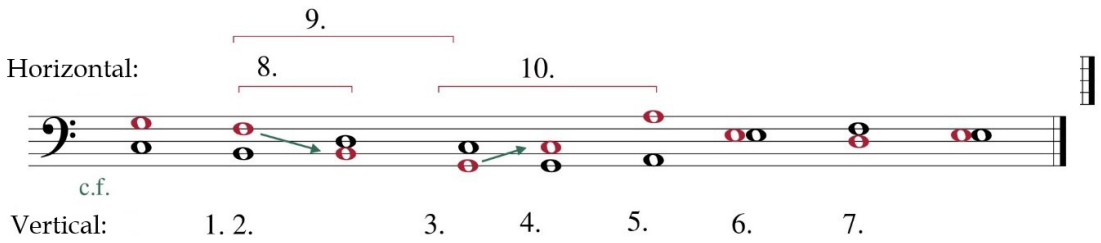
Horizontal (according to the rules for cantus firmus)

10. Two leaps in one direction – *C-G-D* - IX.
11. Descending sixth leap – *B-D* - VIII.
12. Ascending major sixth leap – *C-A* - VIII.
13. Ascending major sixth leap – *G-E* - VIII.

The counterpoint line also contains too many leaps, it is too broken.

The broken diagonal line between *C-E* and *G-B* represents the so-called "ambush"/voice overlapping. This happens when the second tone in the upper voice (B) is lower than the first tone in the lower voice - *C*[8]. The "ambush" is forbidden, but only in two-voice polyphony.

10.



Here the cantus firmus is below. The upper voice at times descends below the lower voice - sometimes this is indicated by an arrow. This is allowed as long as it is not on the first, penultimate and last note.

Vertical

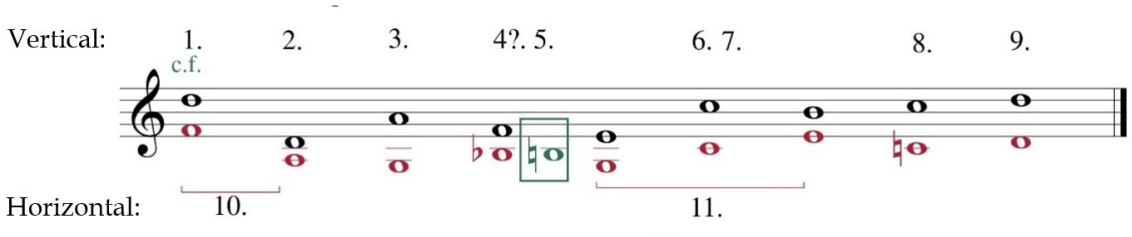
1. Parallel fifths – C-G, then B-F
2. Although the *B-F* interval is a fifth, it is also a tritone – forbidden interval on the vertical
3. Forbidden interval – fourth
4. The same
5. Hidden octave. We have entered this interval by a similar motion of the voices
6. Perfect unison that is not on the first or last tone
7. The penultimate tone of the counterpoint is below that of the cantus firmus – D

Horizontal

8. Tritone leap downwards – F-B
9. Two leaps downwards
10. Two leaps upwards

[8] There is also an "ambush" between the fourth and fifth tones of the exercise.

11.



In this example, where the cantus firmus is on top, I'm trying to place the ficta (B flat), and explain it below.

Vertical

1. The counterpoint does not begin at the first degree – the contrapuntal voice is below
2. Forbidden interval - fourth
3. Forbidden interval – ninth (a second over an octave)
4. There is a B flat here. With it an interval of perfect fifth is obtained. Without it a tritone is obtained – diminished fifth - i.e. mistake 5.[9]
5. Forbidden interval – tritone

It is precisely the added B flat that constitutes a ficta. Ever since the Middle Ages, the practice has existed to alter a tone to avoid unpleasant sounding on the vertical or horizontal. In this textbook I show this phenomenon and explain it, but use it sparingly and rarely.

6. The two voices have leapt simultaneously in a similar motion
7. The octave is reached in a similar motion - this is a hidden octave
8. Since the penultimate tone of the cantus firmus is the seventh degree, the note of the counterpoint should be the second degree (E), but it is not
9. Parallel octaves - C-C, D-D

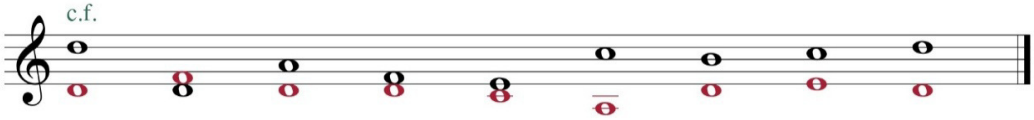
Horizontal

1. A sixth leap downwards – F-A
2. Two leaps in one direction – G-C-E

[9] The addition of B flat (ficta) is considered less important than the presence of a tritone on the vertical.

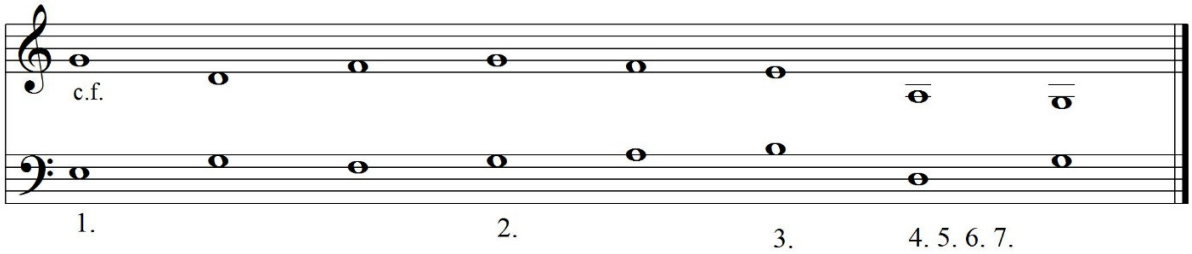
Below I present the correct solution of the above cantus firmus

12.



Here the counterpoint is in the lower voice:

13.

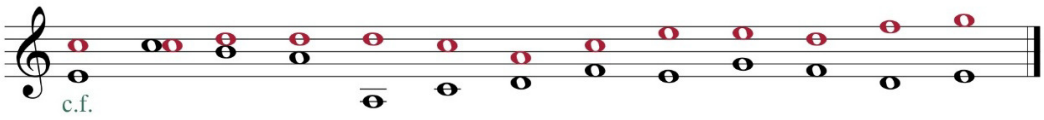


Mistakes

1. The counterpoint did not start from the first degree of the mode
2. Parallel octaves between the third and fourth tones – F-F, G-G
3. Fourth on the vertical
4. Hidden fifth – it is reached with a downward motion in one direction
5. A sixth leap downwards – from the previous interval, B
6. Leap of both voices in descending similar motion and "ambush"
7. The penultimate tone of the counterpoint is not the seventh degree of the mode

Examples for analysis

14.



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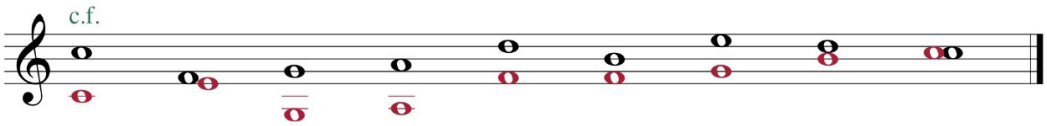

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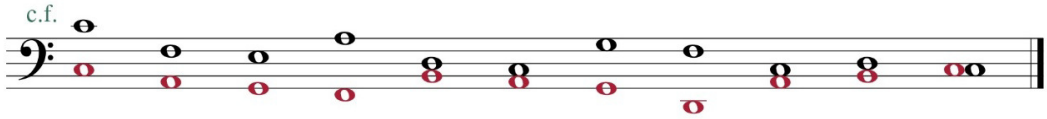

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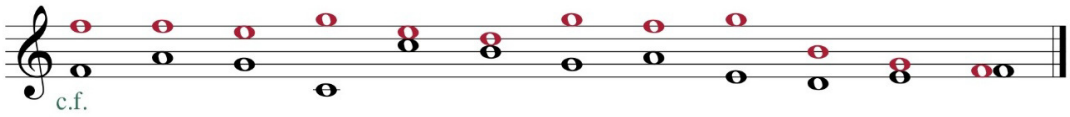

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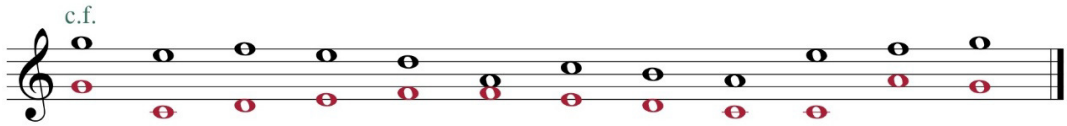

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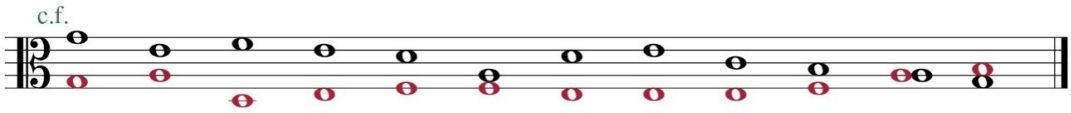

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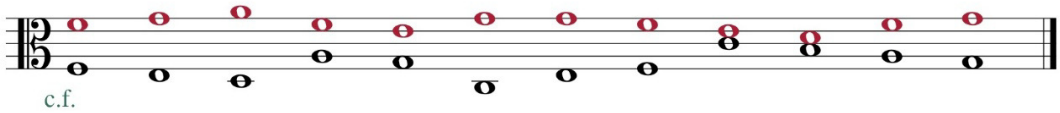
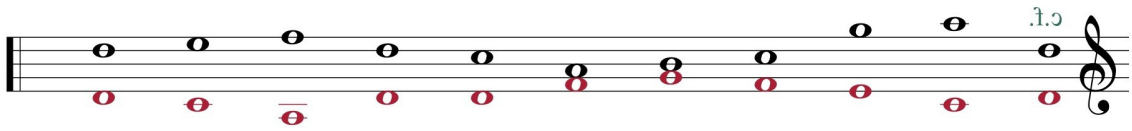


Have you ever tried to read a musical text from right to left? Are the exercises correct?

21.



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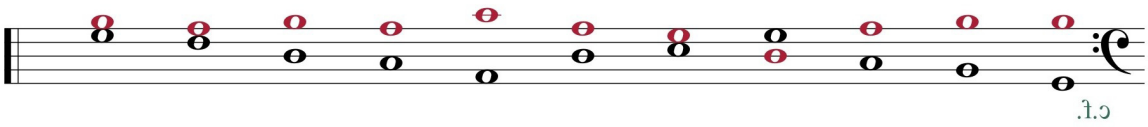
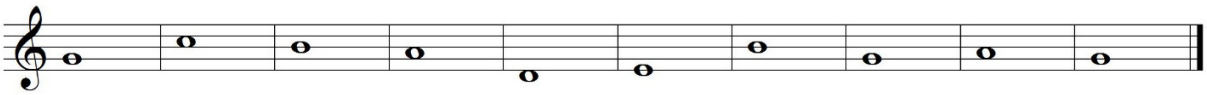


As a homework I recommend to use already checked cantus firmus for contrapuntal exercises. It is a very good practice to play one voice and sing the other, of course at the same time. Singing in a duet is also very beneficial.

Correct c.f. to solve

23.



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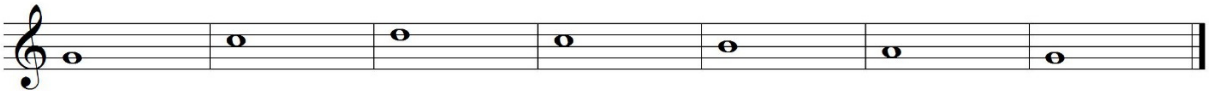

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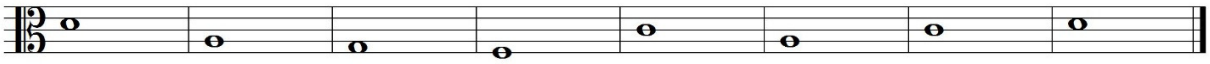

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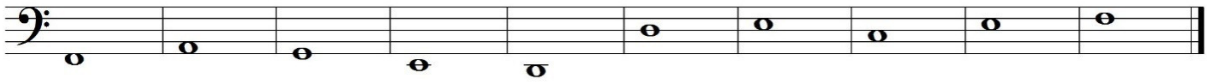


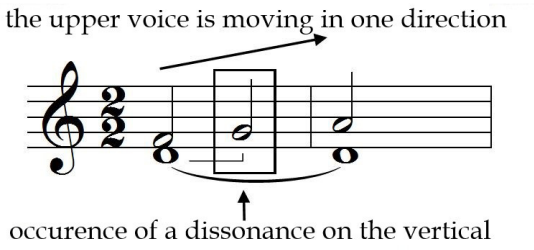
Second Polyphonic Species
Two Notes in the Counterpoint Against One in the Cantus Firmus

We write two half notes of counterpoint against one whole note in cantus firmus. Here again the use of a repeated tone in the counterpoint is forbidden. We introduce a duple meter[1].

Types of dissonances (vertical)

1. Transient dissonance

1.



When there are a minimum of two voices, dissonance can occur between them. This has occurred between the lower voice - D, and the upper voice - G (fourth). It is particularly important that the upper voice moves stepwise, does not stand in the same place, does not change direction and does not leap. As a result, the first interval on the vertical of the strong beat is consonant (third, D-F). Then, as a result of a stepwise upward movement, a fourth interval (D-G) is obtained. The motion does not change, it continues upwards. As a result, a consonance fifth (D-A) is again obtained. Thus the following situation is formed:

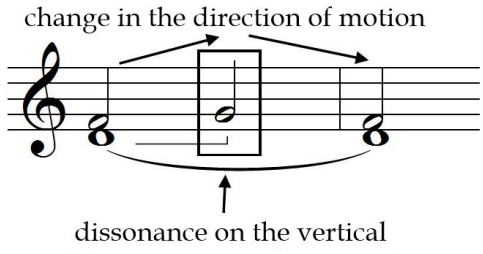
A. The dissonance - the fourth - has turned out to be on a weak beat
B. It occurred in the middle between the two consonants
C. It occurred in a similar stepwise motion in one direction

These three conditions are obligatory for us to be able to call this dissonance transient.

[1] In this textbook, I avoid the use of a triple meter for exercises because there is a distinct contradiction in teaching approaches when it comes to it. Duple meters illustrate the natural state of the matter more clearly and simply.

2. Lateral (transitive) dissonance

2.



Here again we have started with a third in the vertical between the two voices - D-F. Again the upper voice has moved stepwise upwards and a dissonance - a fourth, D-G - has occurred. But then the upper voice changes its direction of motion and goes down. Again the consonance D-F occurs. This lateral dissonance is defined by the following obligatory conditions:

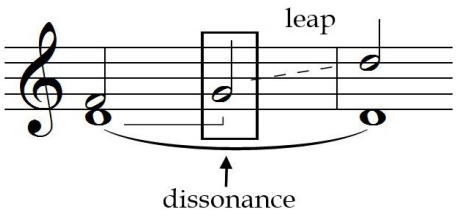
А. It occurred as a result of a stepwise motion - no leap

Б. Its occurrence was followed by a change in the direction of motion

В. After repeated stepwise motion in the opposite direction (no leap), a consonance is again obtained. The dissonance lies between two consonances.

3. Leaping dissonance

3.



The initial conditions are the same. But once dissonance is obtained, the upper voice leaps. This is the condition to call this dissonance leaping.

There is an important learning rule - we can neither leap from nor into a dissonance (the two exceptions to this are explained later in the textbook).

Renaissance music uses all three dissonances, but their use is regulated by strict rules. In this textbook, leaping dissonance is allowed in only two cases. The first case is as an element of the cambiata - this is explained in the chapter on the third polyphonic

species. The second case is a subdivision of a dissonant syncopation - this is explained in the chapter on the fifth polyphonic species.

Fixed rules

I. All fixed rules for cantus firmus remain in force and also apply to the counterpoint (including prohibition of ostinati and sequences).

II. The same applies to the rules for the first polyphonic species with one exception:

III. We are not allowed to repeat notes one after another.

IV. The following rules also take into account the metric beat/time signature in the introduced duple metre:

V. On the downbeat, there must always be a consonance on the vertical.

VI. In the first measure (only) we can write a half pause and start from the second beat.

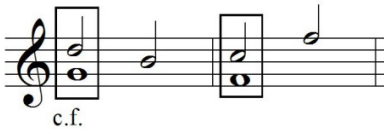
VII. A perfect unison on the vertical on a weak (second) beat is allowed.

VIII. The two half notes of the penultimate measure can be replaced by one whole note.

IX. In the last measure we write a whole note in the counterpoint.

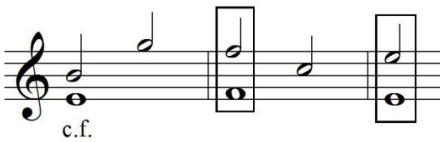
X. Apart from parallel fifths and octaves, which are always forbidden, we cannot allow fifths and octaves that follow in consecutive measures on the downbeat (accented fifths and octaves):

4.



Accented fifths

5.



Accented octaves

Freer Rules

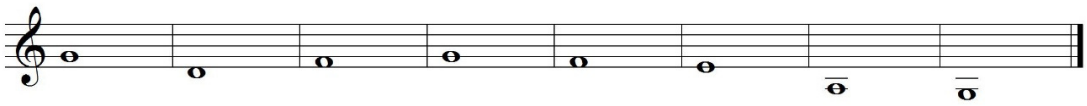
1. This kind of exercise is primarily intended to teach us to use transient dissonance, so stepwise motion is preferred.

2. We can again try to counterpoint using similar stepwise motion as far as possible (illustrated below).

Strategy of solving

On the following cantus firmus a counterpoint should be written in the upper voice:

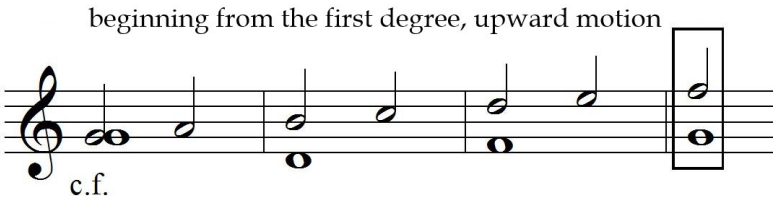
6.



In the first measure I can start without a pause, or with a pause. I also remember that I can start from the first, third or fifth degree of the mode. The stepwise motion is desirable.

Here are a few options to get it started. First I examine the options without a pause in the counterpoint.

7.



I begin in unison - it's fine. *A* forms a second with the G of the cantus firmus, this is a dissonance. In the second measure we get a sixth, consonance, that's good. So the tone A from the first measure turns out to be a transient dissonance, this is allowed to me.

The main reason that A is a transient dissonance is that the counterpoint moves stepwise in one direction - this is good.

In the second measure, the second tone of the counterpoint - C - forms a dissonance with D in the cantus firmus. But the motion continues stepwise upward, and in the third measure we have again a sixth on the vertical - F in the cantus firmus and D in the counterpoint. Once again good. *C* forms a transient dissonance.

The second note of the counterpoint in the third measure, E, forms a dissonance - a seventh - with F in the cantus firmus. In the fourth measure, however, we have a problem - it's enclosed in a rectangle. On a strong beat (the downbeat) we have a seventh interval - dissonance. This is impossible.

Thus the tone E in the third measure can no longer be called a transient dissonance - having moved another degree in the same direction, it does not make consonance with the cantus firmus on the downbeat in the fourth measure. That is, the tone A cannot be used here - the dissonance it forms is not legitimate and not allowed.

Based on my analysis, I can conclude that the solution is correct only up to the downbeat of the third measure. The stepwise motion further upwards, in the same direction, is not possible. I have two options - either to change the direction of motion or to leap to a consonance on the second beat of the measure.

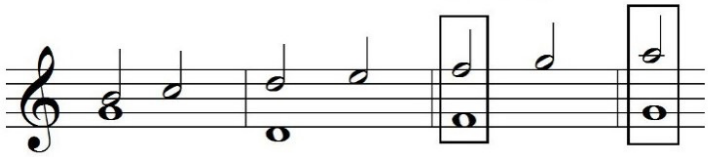
An important conclusion is that there can be only two allowed options on the second beat of the measure:

A. either a transient dissonance (the next tone of the counterpoint must form a consonance with the cantus firmus on the downbeat in the next measure, moving in the same direction), or

B. the second note (on the second, weak beat) must form a consonance with the cantus firmus.

8.

beginning from the third degree, upward motion



I start from B, third degree, C is a transient dissonance.

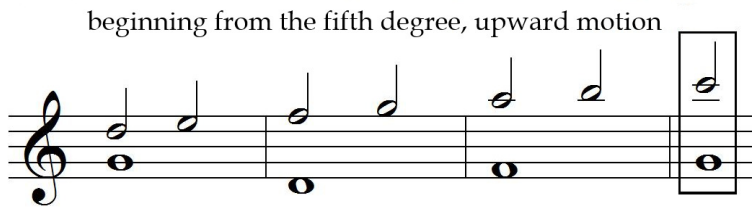
Second measure - octave on the vertical, obtained in contrary motion of the voices (cannot be a similar motion, that would be a hidden octave). All right. E is a transient dissonance.

Third measure - here we have obtained a consonance, an octave. But this octave is hidden. It has been obtained with a similar movement of the voices - both voices have moved upwards. This octave is forbidden to us.

In the fourth measure we have reached a ninth on the vertical. This is a mistake and so it also becomes incorrect to use the tone G in the third measure.

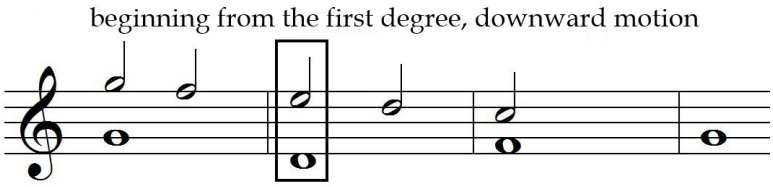
The solution is therefore correct only up to the downbeat of the second measure.

9.



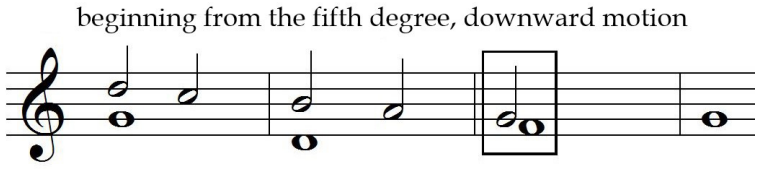
By the same principle everything is correct until the beginning of the fourth measure. E in the first measure is a transient dissonance, the same is valid for G in the second measure. In the fourth measure there is a dissonance - a fourth over an octave on the downbeat. Furthermore, the upper voice is going too high. This solution is correct only up to the downbeat of the third measure.

10.



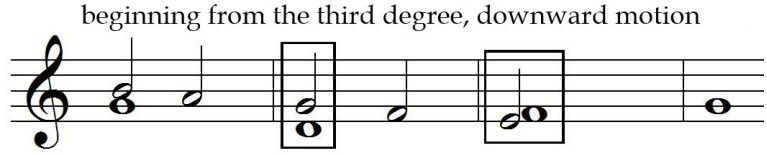
We have a ninth on the downbeat in the second measure. That means the descending stepwise solution from *G*2 is impossible.

11.



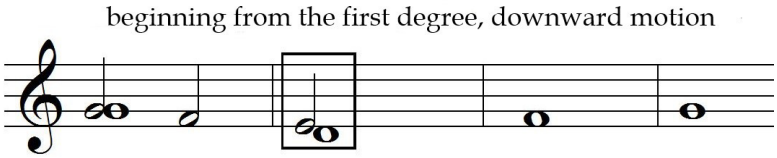
By the same principle, the solution is only possible up to the second note of the second measure. In the third measure there is a second on the vertical.

12.



An unsolvable situation right from the beginning.

13.

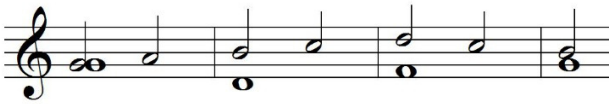


Starting again from *G*, but in the first octave, we find that this solution is also impossible.

My analysis shows that the similar motion in one direction has its limitations. It cannot continue for a long time.

Then I decide to change my strategy a little. After I've made a few stepwise moves, I change the direction of motion. Using example 7 as a model, I move downwards from measure 3:

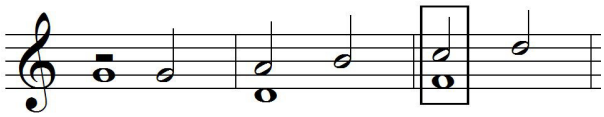
14.



There are no mistakes here.

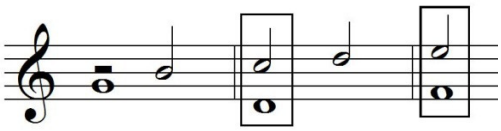
In the same way, I can examine the possibilities for solving by placing a half pause in the first measure. The second note continues to follow the fixed rules. I show a few examples.

15.



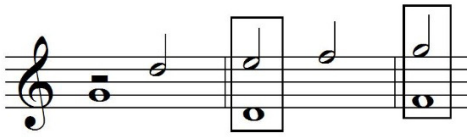
Hidden fifth in measure 3, accented fifths in measures 2-3 (on the downbeat).

16.



Successive dissonances on strong beats.

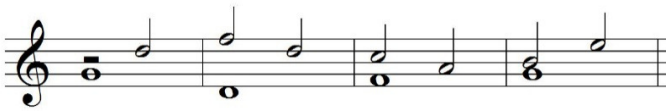
17.



The same.

I have not shown all the possibilities here. Instead, I can use the other fundamentally different principle - to leap into consonance both on the downbeat and the second beat [2]:

18.

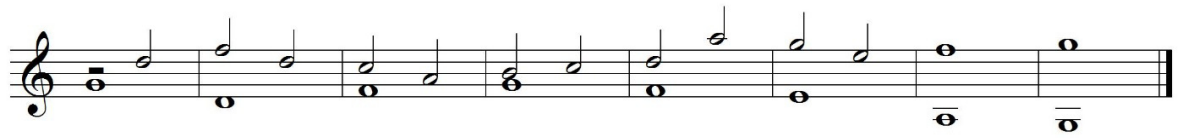


There are no mistakes. Each note of the counterpoint forms a consonance with the cantus firmus. Such a solution is quite possible, although it is a bit "jumpy".

The following is a correct solution, which consists of leaps and stepwise moves:

[2] Both Fux and Jeppesen do not pay much attention to the emphasis on the stepwise motion. This motion is important, but it is not obligatory.

19.



Analysis of the vertical

First measure - fifth, the counterpoint begins at the fifth degree.

Measure 2. – tenth, followed by an octave in a lateral motion on a weak beat.

M. 3. – Fifth obtained in contrary motion, then third in lateral motion.

T. 4. – Third, followed by transient dissonance - fourth.

T. 5. – Sixth, after that - tenth.

T. 6. – Tenth again and then octave, obtained in a lateral motion on a weak beat.

T. 7. – A whole note in the counterpoint, it is allowed. The note is the seventh degree of the mode because the note of the counterpoint is the second degree. Sixth over an octave.

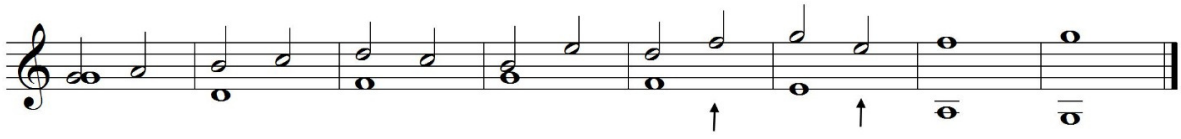
T. 8. – Whole note and first degree in both voices (obligatory). Maximal allowed interval between the two voices - two octaves.

There are no parallel, hidden or accented fifths and octaves here. A transient dissonance is used in m. 4. There are no problems horizontally.

This is the whole strategy of writing this polyphonic species, as well as all the others: we try to counterpoint as smoothly as possible, using stepwise moves. When this proves impossible, we undertake leaps, starting from the smallest.

Below is presented a correct solution that consists of more stepwise moves and fewer leaps:

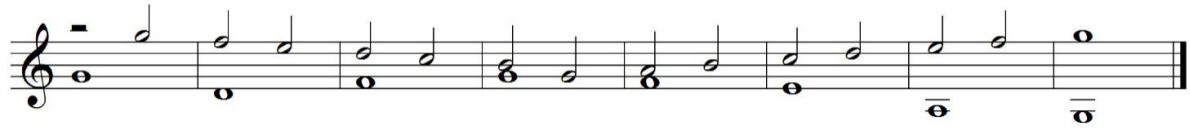
20.



Two consecutive octaves on weak beats are marked with arrows. If this does not occur too often in the exercise, it is considered a minor problem.

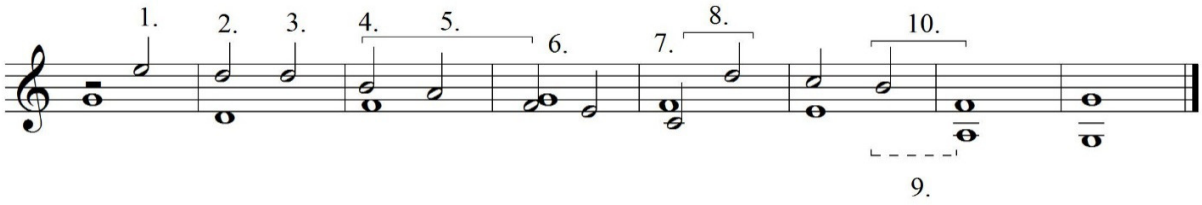
The following is a correct solution in which a similar stepwise motion is used and only one leap (in m. 4):

21.



Incorrect examples:

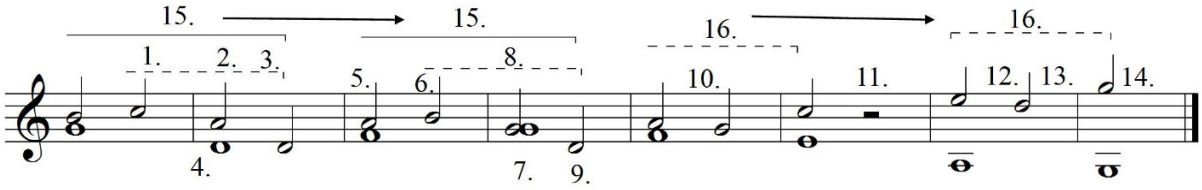
22.



Mistakes on both vertical and horizontal:

1. The counterpoint starts from the sixth degree of the mode
2. Hidden octave
3. Repeated tone
4. Dissonance (tritone) on the downbeat
5. Tritone gap in the horizontal – B-A-F
6. Dissonance (second) on a strong beat
7. Dissonance (fourth) on a strong beat
8. Forbidden horizontal leap - greater than an octave (ninth)
9. The two voices leap simultaneously in one direction
10. Horizontal tritone leap

23.



Mistakes

1. C forms a dissonance with the cantus firmus (fourth). This dissonance is not transitive, it is a leaping one
2. Hidden fifth
3. Two leaps in one direction (horizontal, counterpoint) – C-A-D

4. The two voices leap simultaneously in one direction, C-A in counterpoint, D-A in cantus firmus (between the first and second measures)

5. The two voices leap simultaneously in the same direction, D-A in counterpoint, D-F in cantus firmus (between the second and third measures)

6. *B* forms a dissonance with the cantus firmus (tritone). This dissonance is not transient, it is leaping

7. Unison on a strong beat - only allowed on the first and last measures. An unison on a weak time is always allowed

8. Two leaps in one direction (horizontal, counterpoint) – B-G-D

9. Forbidden interval on the vertical - fourth, between cantus firmus and counterpoint. In this case the counterpoint has leaped into dissonance

10. *G* forms a dissonance with the cantus firmus (second). This dissonance is not transient, it is leaping

11. No pauses are allowed except for a half pause in the first measure.

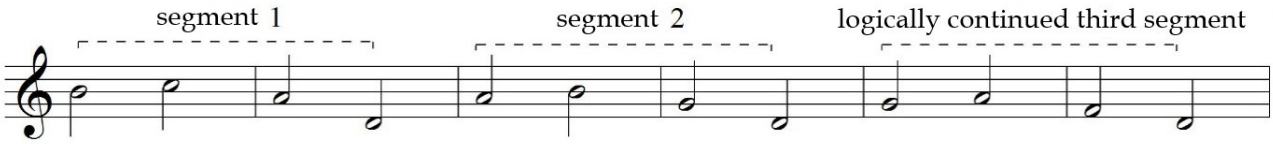
12. D forms a dissonance with the cantus firmus (fifth over an octave). This dissonance is not transient, it is leaping

13. The penultimate tone of the counterpoint is not seventh degree (F)

14. The last note is not a whole note

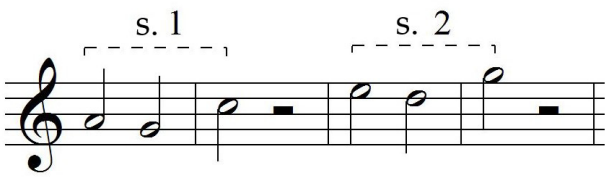
15. Progressive sequence between the segments of m. 1-2 and m. 3-4:

24.



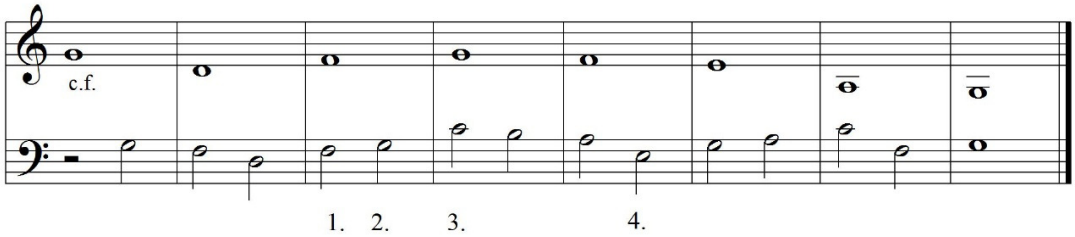
15. Non-progressive (identical) sequence between the segments of m. 5-6 and m. 7-8:

25.



Now I'm writing a contrapuntal voice under the same cantus firmus.

26.

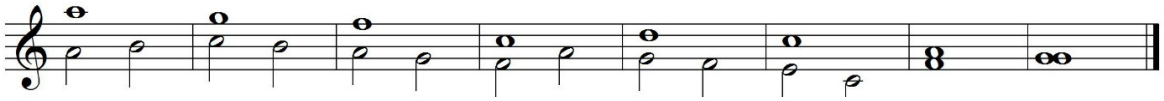


Mistakes

1. Parallel octaves
2. Seventh on the vertical - not transient, it leaps
3. Hidden fifth
4. Leap of ninth on the second beat (*E*)

Examples for analysis

27.



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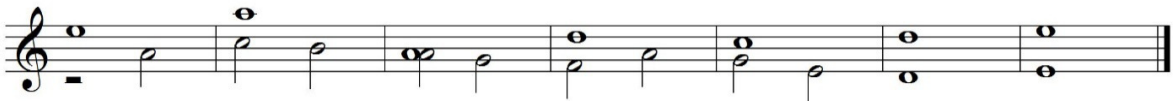

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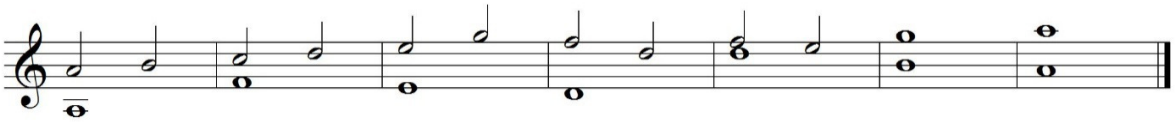

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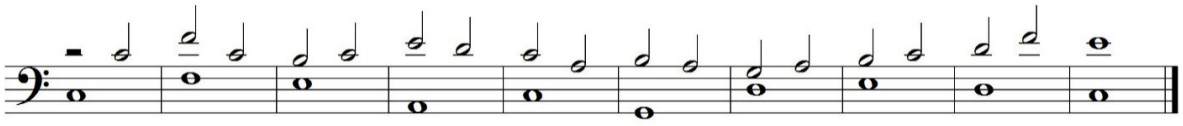

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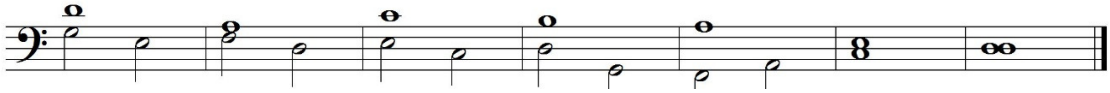

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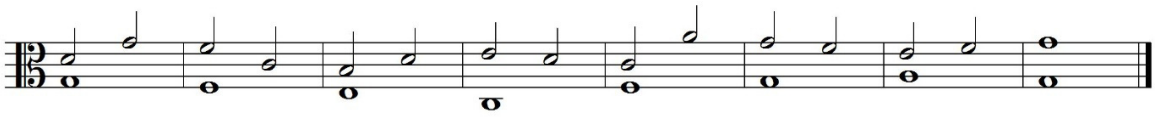

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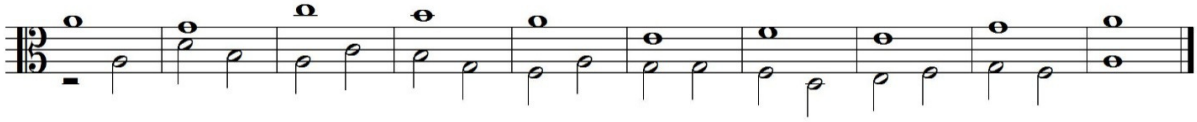

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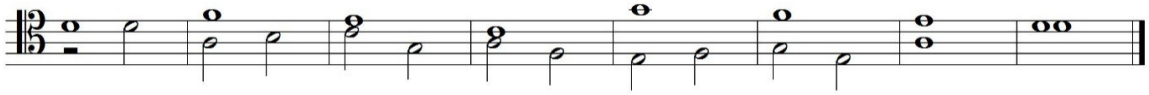
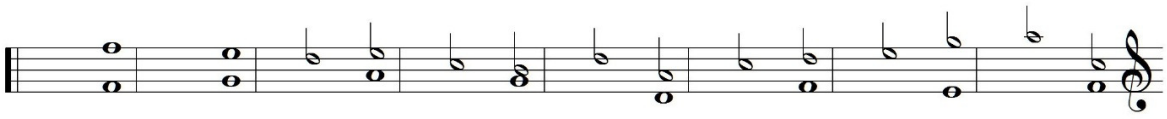


Are the exercises correct?

35.



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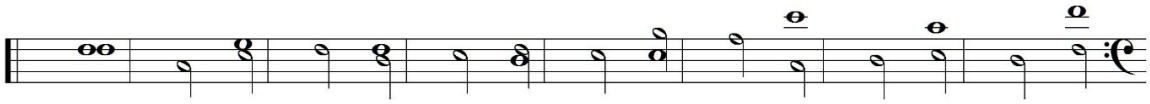

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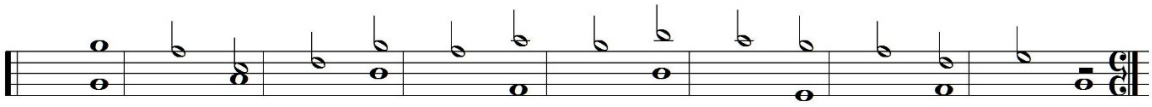
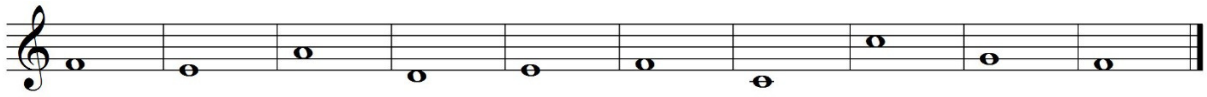


Correct c.f. for solving:

38.



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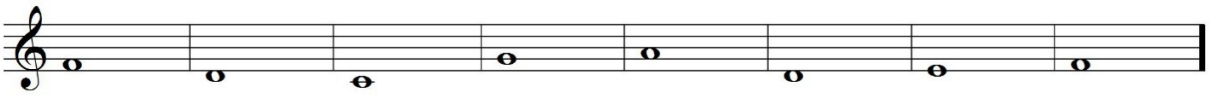

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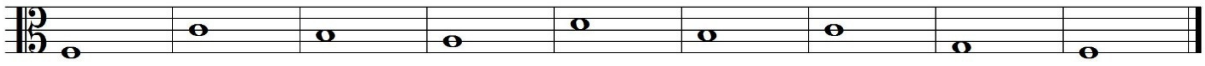

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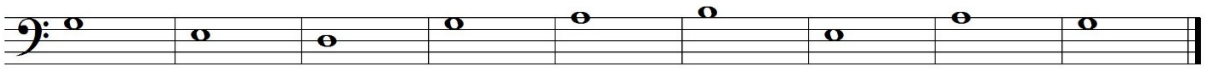


Third Polyphonic Species
Four Notes of Counterpoint Against One of Cantus Firmus

All rules so far remain in force (no repeated notes allowed).

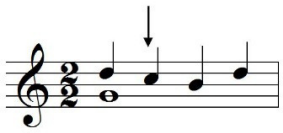
Fixed rules

I. In the first measure in the counterpoint we can begin with a quarter pause, the choice is ours[1].

II. On the downbeat (the first quarter note) the presence of a consonance on the vertical is still obligatory.

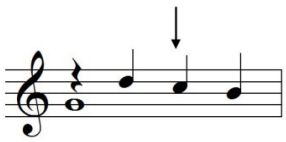
III. On the second, third and fourth quarter note there might be a consonance of course, but there might also be a transient dissonance:

1.



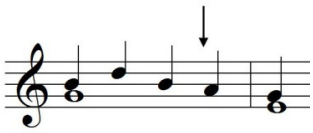
Here it is on the second quarter note

2.



Here it is on the third quarter note, i.e. on the second beat[2].

3.



On the last quarter note in the measure.

IV. It is a good idea to use a cambiata - it is explained below. It incorporates one of the two allowed uses of a leaping dissonance.

V. If we want, we can use two quarter and one half notes in the penultimate measure instead of four quarter notes.

---

[1] No other configuration of notes and pauses is allowed. No pauses in other bars.

[2] There is a Bulgarian term for a transient dissonance on a relatively strong (second) beat - a *hard transition [твърд преход]*. The author of this term is Zdravko Manolov.

A cambiata is a group of five notes[3] in counterpoint that must obey strict rules in the horizontal and vertical. It can be "straight" and "inverted". It can be built from any tone.

Rules for the cambiata on the horizontal

The five notes must be arranged interval-wise as follows:

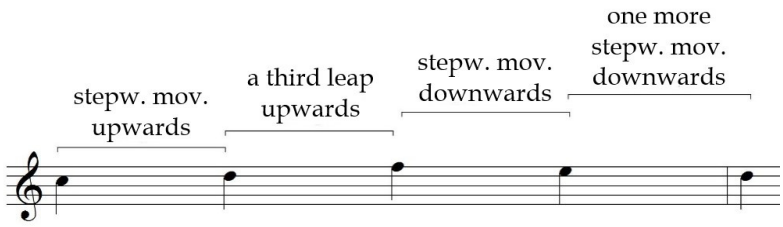
A. Stepwise movement upwards after the first note
B. Leap upwards after the second note
C. Stepwise movement downwards to the third note
D. One more stepwise movement downwards to the last note

The inverted (mirrored) cambiata has the same interval rules but in the opposite direction:

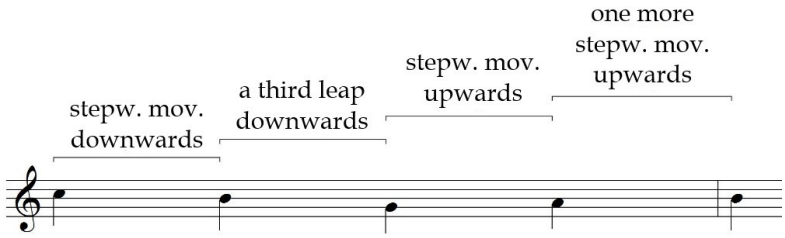
A. Stepwise movement downwards after the first note
B. Leap downwards after the second note
C. Stepwise movement upwards to the third note
D. One more stepwise movement upwards to the last note

Examples:

4. Straight cambiata



5. Inverted cambiata



[3]An important principle is to learn to use the whole figure. But the emphasis is primarily on exercising the leaping dissonance from the second tone - a third upwards and downwards. The simple definition of the cambiata is that before the leaping tone and after it there must be a consonance on the vertical.

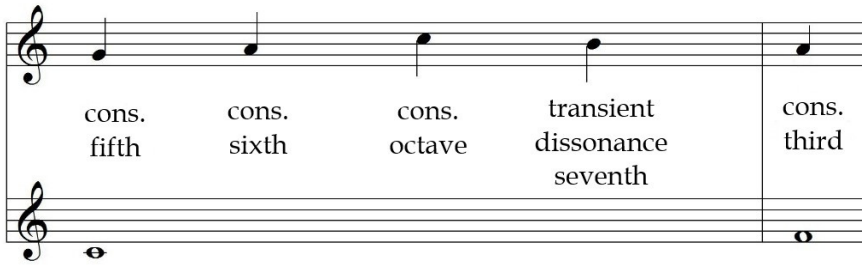
Rules on the vertical - affecting both straight and inverted cambiata[4]

A. The first note must form a consonance with the cantus firmus
B. The second note might[5] make a (leaping) dissonance with c.f.
C. The third note obligatorily must form a consonance with c.f.
D. The fourth note might[6] form a transient dissonance with c.f.
E. The last note must form a consonance with c.f.

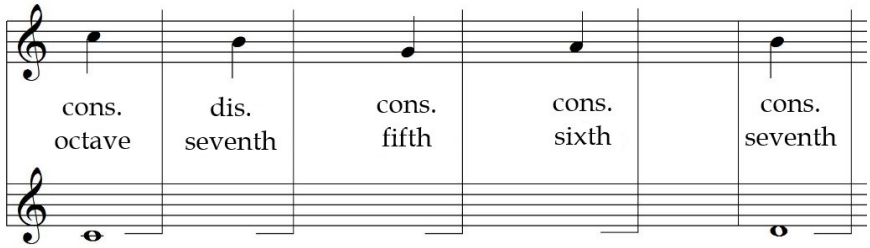
Or, in other words, a consonance is required on the first, third and fifth notes.

Examples of correctly formed consonant and inverted cambiatae:

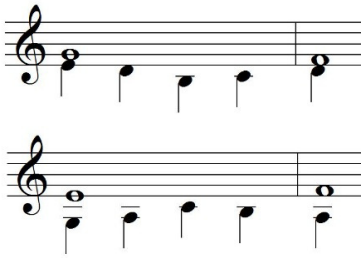
6. Consonant



7. Inverted



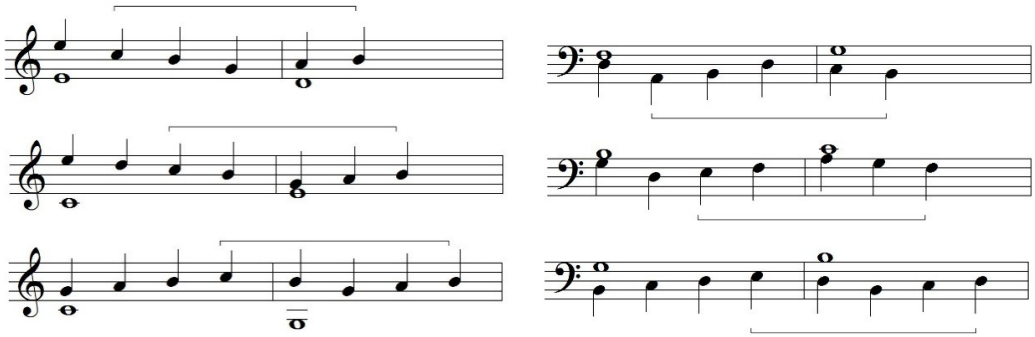
8.



[4] When the cambiata does not start from the first quarter note in the measure, the rules are slightly modified - for this see below.
[5] But if it forms a consonance, it is always allowed. The consonance is never a problem.
[6] Same as footnote 5.

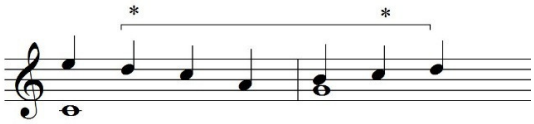
The cambiata can start from any tone, but on the first quarter note of each measure there must be a consonance[7]:

9.



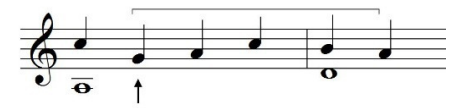
In some cases, the first and last tones of the cambiata can also be interpreted as transient dissonances:

10.

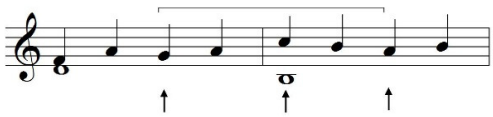


If all the rules for transient dissonance are followed, this cambiata is correct.
Some incorrect examples of cambiatae:

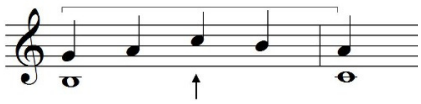
11.



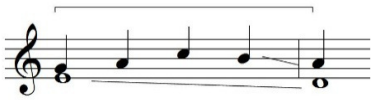
The first note forms a dissonance with c.f.



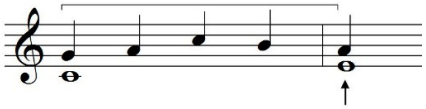
First, third and fifth note - the same



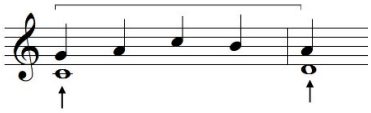
The third note does the same



Parallel fifths



The fifth note does the same



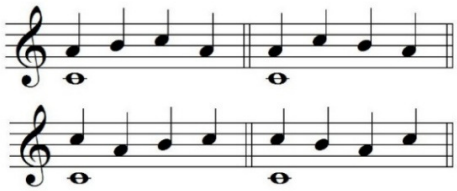
Accented fifths

[7] Position of the different sources on this issue varies somewhat. According to some, if there is no leap of dissonance from the second note, there is no cambiata. But I do not hold to this opinion.

Techniques for gaining time

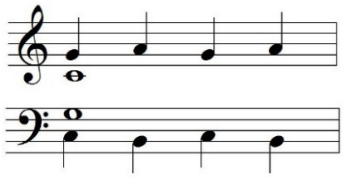
Here I show a melodic figure of "rotation". It allows us to counterpoint an entire measure by ending on the same note we started with:

12. A figure of rotation in its four variants



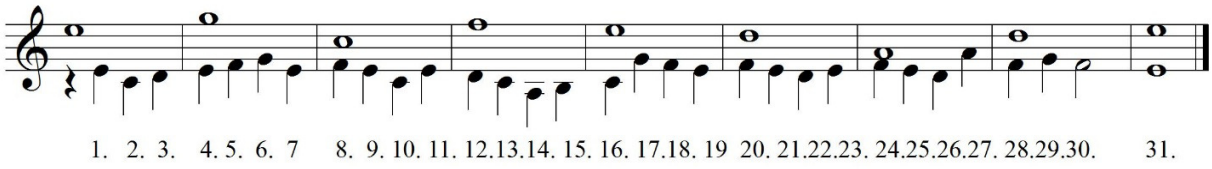
There is only one case in the diatonic tone row where a consonant on the vertical can be found next to another consonant on the vertical - fifth and sixth. This can be used within one measure - a longer use results in an ostinato:

13.



The following is a correct solution to an exercise with a short analysis:

14.



Vertical

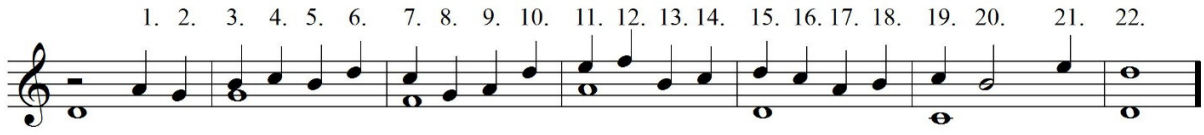
Note 1: lower voice, starting from the first degree
Note 2 – leap in tenth, lateral motion
3 – transient dissonance (ninth)
4 – tenth in similar motion (measure 2 – rotation)
5 – transient dissonance – ninth
6 – an octave reached in lateral motion
7 – tenth in lateral motion

8 – fifth in contrary motion
9 – sixth in lateral motion
10 – octave leap in lateral motion
11 – sixth leap in lateral motion
12 – tenth in contrary motion and beginning of a cambiata
13 – dissonance - fourth (part of the cambiata), leap from a dissonance
14 – sixth, lateral motion
15 – transient dissonance – tritone
16 – last note of the cambiata, tenth in contrary motion
17 – sixth leap in lateral motion
18 – transient dissonance - seventh
19 – octave in lateral motion
20 – sixth in contrary motion
21 – transient dissonance - seventh
22 – octave in lateral motion
23 – transient dissonance - seventh
24 – third in contrary motion
25 – transient dissonance - fourth
26 – fifth in lateral motion
27 – leap in unison on a weak beat, lateral motion
28 – exit from unison into contrary motion[8], sixth leap
29 – fifth in lateral motion
30 – sixth in lateral motion, simultaneous occurrence of 2nd and 7th degree
31 – first degree final

The first measure begins with a quarter pause. The last measure uses two and a half quarter notes. The voices do not cross, although this is allowed. There are no problems on the horizontal.

Incorrect examples:

15.



[8] As I mentioned, the use of contrary motion in the voices is allowed and even recommended.

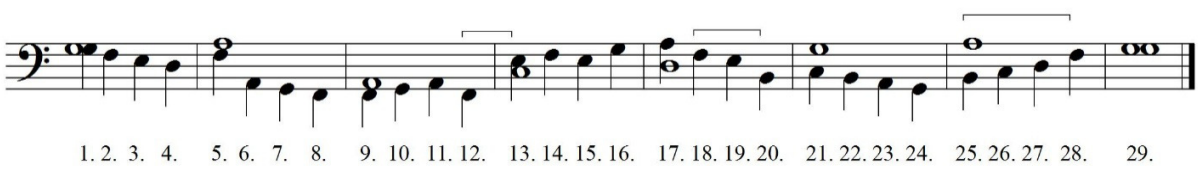
Vertical

Measure 1 begins with a half pause and two quarter notes - this is forbidden
Note 1 - OK
Note 2 is a leaping dissonance
Note 3 – the third is reached with a leap of both voices in a similar motion
Note 4 is a lateral dissonance – it is forbidden
Notes 6-7 – parallel fifths
Note 8 – dissonance leap - a second
Notes 9-10 – no problems here
11 – OK
12 – no problem
13 – a second leap
14 – OK
15 – OK, beginning of the cambiata
16-18 – OK
19 – accented octaves - there is an octave on note 15
20 – impermissible syncope, leaping dissonance (seventh)
21 – OK
22 – third consecutive (accented) octave

Horizontal

Notes 12-13 – descending tritone leap

16.



Vertical

Notes 1-3 – OK
N. 4 – leaping dissonance – fourth
N. 5-8 – OK
9-12 – good vertical
12-13 – both voices leap upwards
14 – lateral dissonance
15-16 - OK

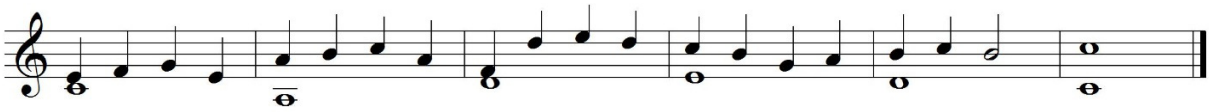
16-17 – parallel fifths
17-18 – OK
19 – leap from a second
20 – OK
21 – hidden fifth
21-24 – OK
25 – seventh on the downbeat

Horizontal

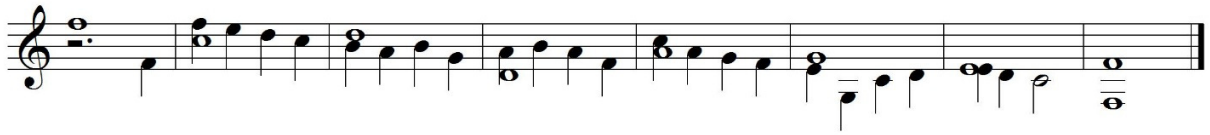
5-6 – sixth leap downwards (unspecified - did you notice it?)
9 – repeated note
12-13 – seventh leap
18-20 – tritone gap - missing D and C
25-28 – tritone gap - missing *E*

Examples for analysis

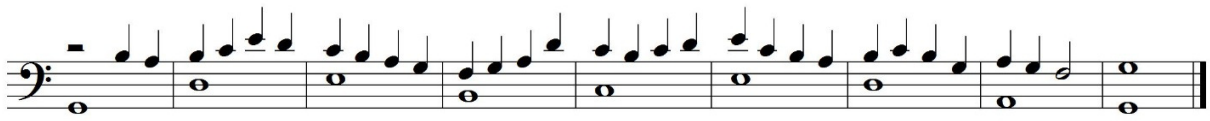
17.



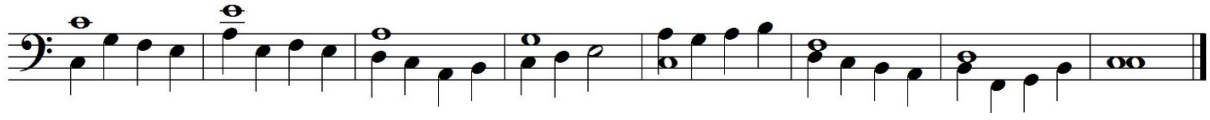
18.



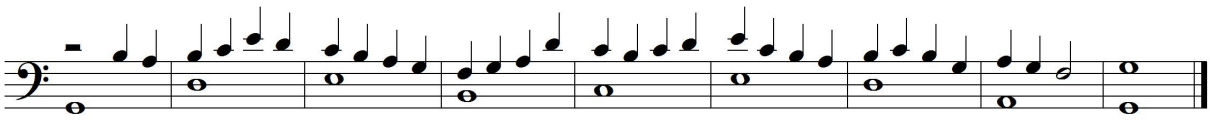
19.



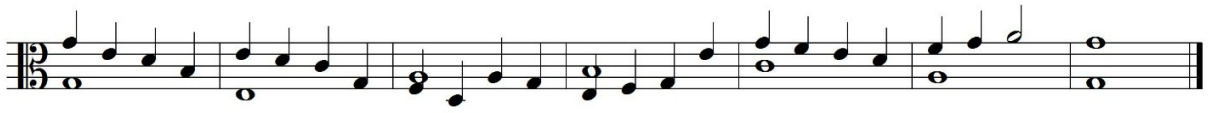
20.



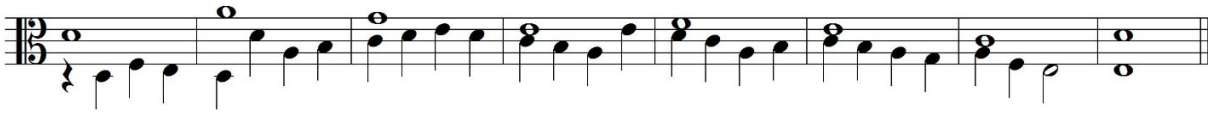
21.



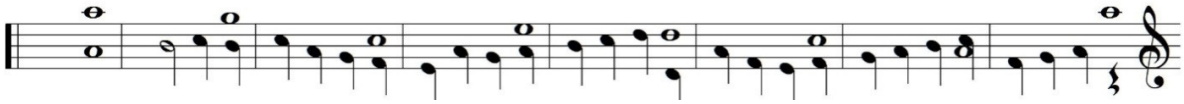
22.



23.

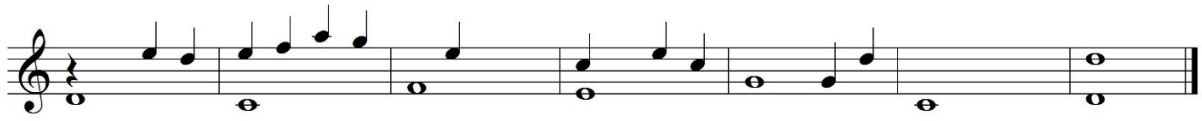


24.

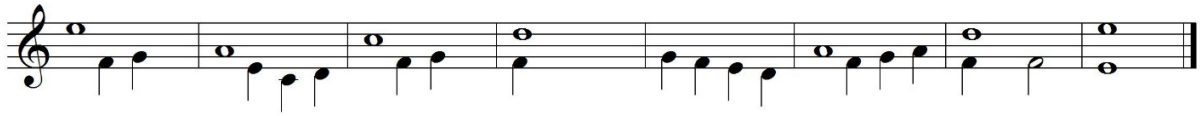


Here I present a few “sudoku-style” exercises. It seems that sometimes I don't remember a note of the cantus firmus, sometimes of the counterpoint, and at times my memory betrays me completely.

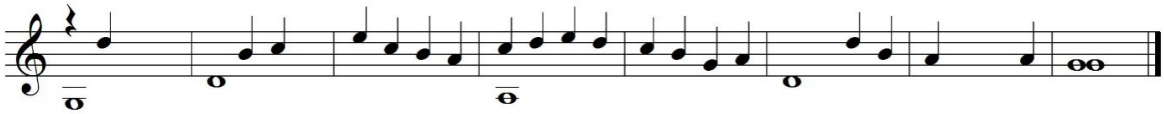
25.



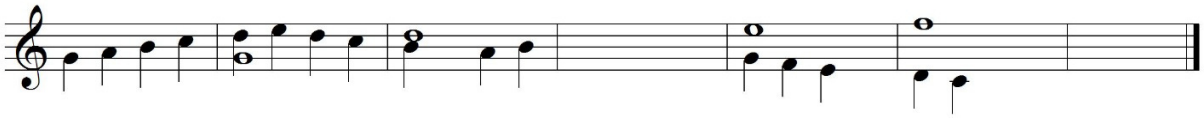
26.



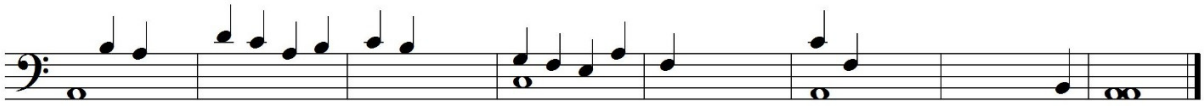
27.



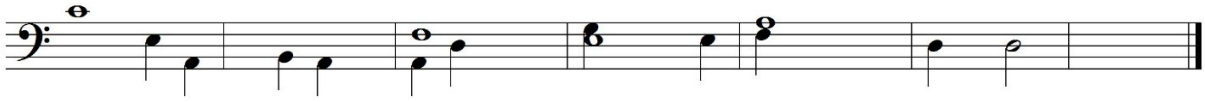
28.



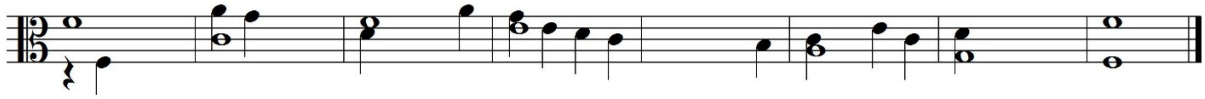
29.



30.

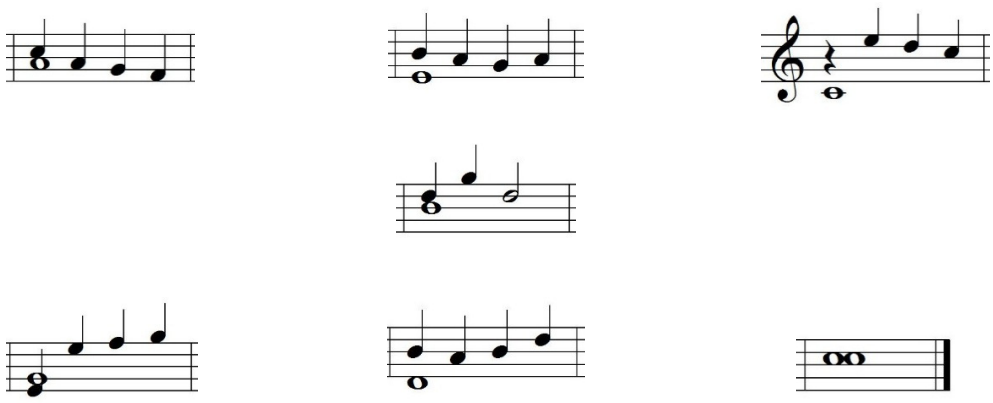


31.



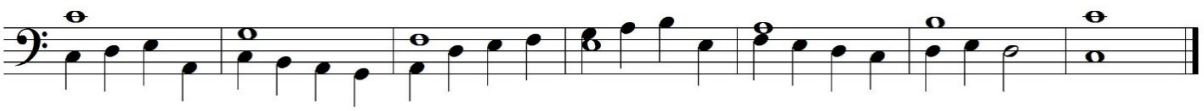
And here the seven measures of the example are presented in shuffled form:

32.



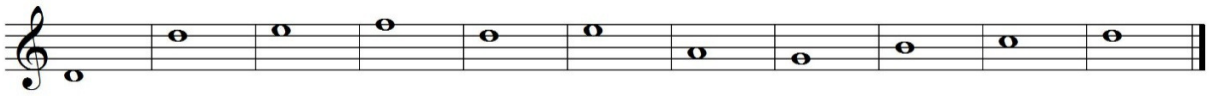
Here I give a solution to example 30:

33.



Correct c.f. for solving:

34.



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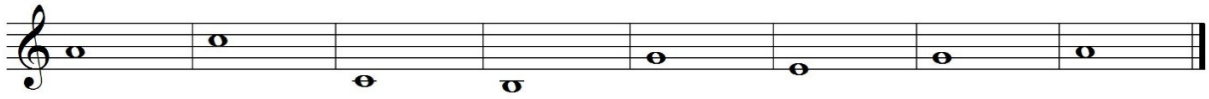
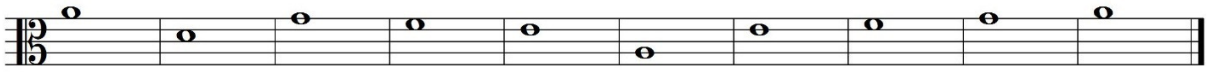
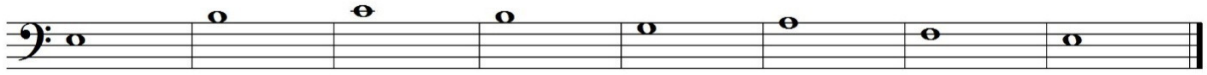


36.



37.

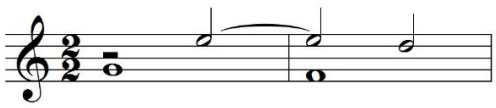


Fourth polyphonic species
Syncopated sequence

This technique consists of half notes in the counterpoint that are tied over the bar line. But first we need to be introduced to the concept of the dissonant syncope.

If we analyse the following example:

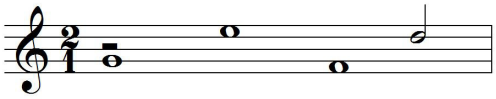
1.



we see that on the vertical at the downbeat of the second measure there is a dissonance, a seventh, which is taken stepwise downwards into consonance - the note D. There is also a consonance at the second beat of the first measure, where is the E. E forms a sixth with G in the cantus firmus.

The same example can also be notated like this:

2.



and then it becomes clear why this phenomenon is called dissonant syncope. There is a syncope in the upper voice. The resulting dissonance is called a suspended dissonance.

The fourth polyphonic species aims to teach this technique.

Fixed rules

I. We start in 2/2 meter and in the counterpoint of the first measure there is a half pause on the downbeat. Then on the second beat, there should be first, third or fifth degree of the mode if the counterpoint is in the upper voice. If it is in the lower voice, only the first degree of the mode can still be used.

II. On the second beat, in the second half of the measure, the tone should always form a consonance with c.f. in every measure.

III. This tone is tied over the bar line, so that the same note occurs on the downbeat of the next measure. With this, the possible vertical options are as follows:

A. There might be a dissonance with c.f. In such a case, the tone is obligatorily taken downwards and stepwise. Thus, at the second beat of the measure, a consonance on the vertical is obtained.

B. There might be consonance with c.f. In that case, we have some liberty - we can write any tone on the second half of the measure, as long as it forms a consonance with c.f.

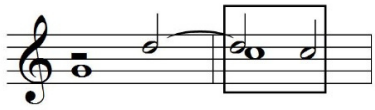
IV. All other horizontal and vertical rules remain in effect. Repeated notes are not allowed, except for those that are tied.

V. In writing these exercises, we find that there is not always a solution (shown below). In this case, at two places in the exercise, we are allowed to not write a tie between the measures and to put a different note. However, it must obligatorily form a consonance with c.f. because it is not tied. It is strongly recommended not to break the syncopated chain in two consecutive bars.

VI. Forbidden are the following two configurations, which do not sound good:

A. The so-called faux (false) second:

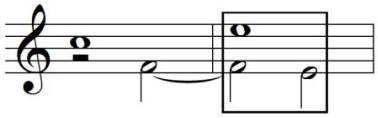
3.



Here, in the second measure the second has been resolved in unison.

B. Faux seventh:

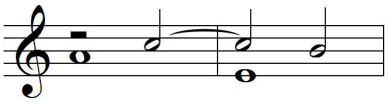
4.



The seventh in the second measure has been resolved in octave.

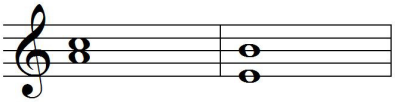
V. If we analyse the following example:

5.



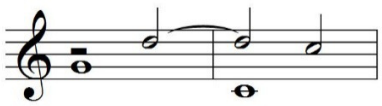
we may discover that it actually appears to be a variant of this:

6.



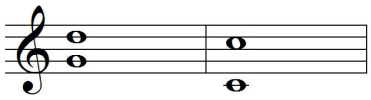
where the upper voice is a little late to go from C to B in the second measure. Example 6. represents an error - there is a hidden fifth in the second measure. On this logic, many sources (but not all) recommend against using Example 5. as well - it is a "disguised", "delayed" hidden fifth. Similarly, a hidden octave is not to be used

7.



because it represents a variant of this:

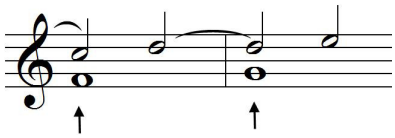
8.



VI. In the penultimate measure, if we want, we can write a whole note.

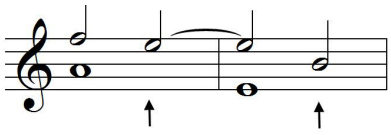
VII. The presence of forbidden fifths and octaves is checked only on the second beats of the measures. In this case there is no problem:

9.



because the fifths are on the downbeat of every measure. However, there is an error here:

10.



because there is a fifth on every second beat.

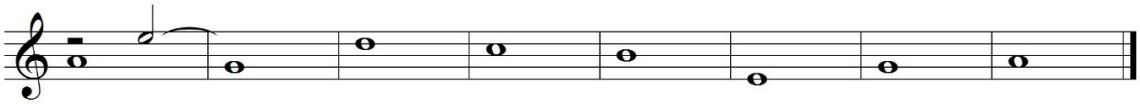
Second more liberal rule

It is recommended that the two voices do not leap in the same direction. But this phenomenon is used in the living music, so we can treat it a bit more liberally.

Strategy of writing

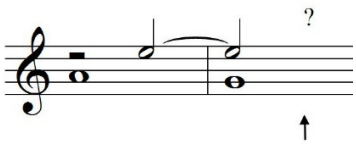
I'm starting to write a counterpoint above:

11.



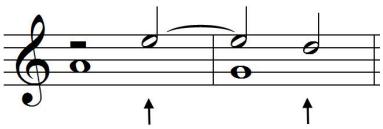
I have chosen the fifth degree of the mode - E. After the note tying, this is what happens:

12.



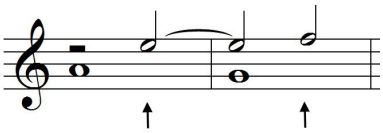
On the downbeat of the second measure, we got a sixth on the vertical, a consonance. I'm not obliged to go downwards stepwise on the second beat. In fact, if I take that move, I am going to run into a problem:

13.



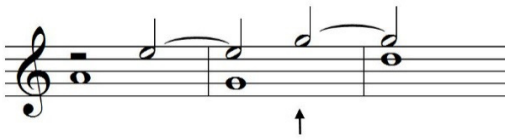
Parallel fifths on every second beat. I can't go upwards stepwise because on the second beat of the measure there will be a seventh, dissonance:

14.



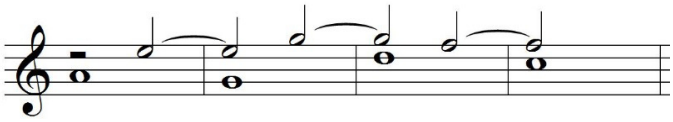
On the same principle, I can not write C as well - it will form a fourth with G. So it comes out that I am rejecting note by note: I can not write F, E (it would be a repeated tone), *D, C*. But I can leap upwards to G - so an octave on the vertical is obtained:

15.



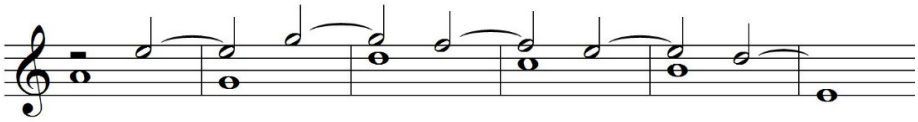
Here I have already tied the G in the upper voice and on the downbeat of the third measure a dissonance occurred, a fourth. I'm taking it downwards stepwise:

16.



and get into the same situation in the fourth measure. I take it downwards again:

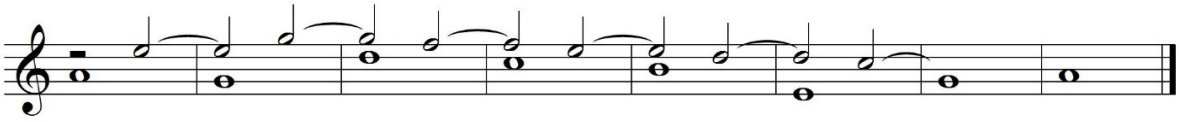
17.



Thus we have three consecutive stepwise movements in measures 3, 4 and 5 - this is good. This type of descending chain sounds pleasing and is even advisable for this situation. Such a chain can be very long without constituting an error.

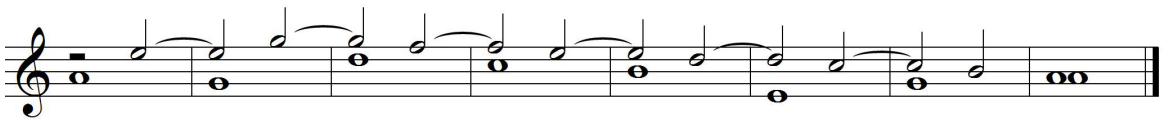
In fact, it seems to me that the chain continues. In measure 6 I have no other way out but to write a *D* (because it's tied over from the previous measure) and take that tone downwards to *C*:

18.



The solution in the penultimate measure is obvious - it is imposed by itself:

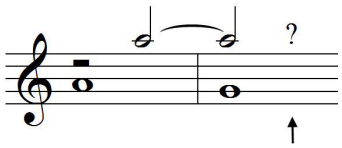
19.



Thus, in the penultimate measure, an elegant opposition of the seventh and second degrees of the mode has resulted, leading to unison in the first degree. The exercise is completed.

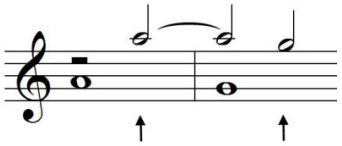
And what will happen if I decide to use the first degree of the mode in my counterpoint:

20.



If there is dissonance on the downbeat, it must obligatorily be taken one degree downwards. But this would produce parallel octaves:

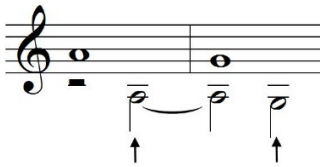
21.



There is no solution. But as I have shown, I can start from the fifth degree of the mode; there is also a solution from the third degree. Therefore, I do not consider it necessary to interrupt the tie in this case. From the first degree there is no solution.

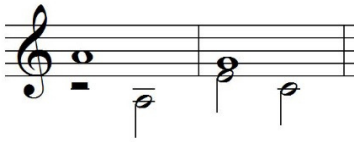
Now I will try to solve the exercise by writing a counterpoint in the lower voice. I get the same problem, because now I cannot start from any other degree than the first:

22.



So I have no choice but to use the clause given to me by Rule V: I don't tie the A and on the next measure I write a different note that forms a consonance with c.f.:

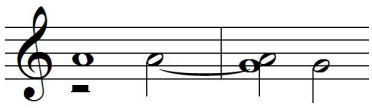
23.



On the second beat of the second measure, I also write a note that consonants with the cantus firmus.

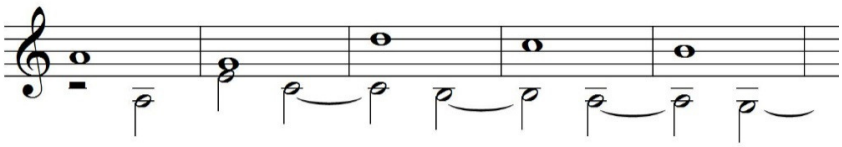
From the same note of the first octave i.e. from the first degree (A) I could not start because the next inevitable move would be a faux second and the appearance of parallel unisons:

24.



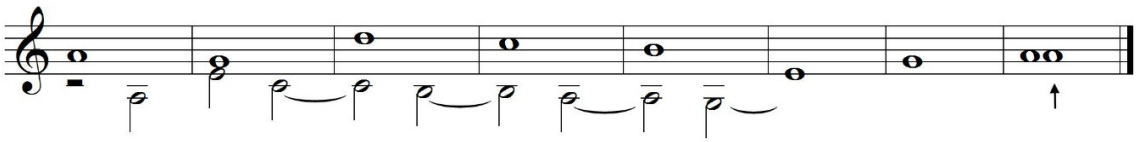
Going back to Example 23, when I tie the C, I find myself at the beginning of a descending syncopated chain, as when I counterpointed above. This is exactly the solution I need:

25.



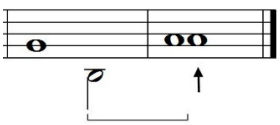
For the last few measures, I could use a little pre-planning. Implementing the exercise from right to left, from the end to the beginning, I place the obligatory tone at the end of the counterpoint, on the last measure:

26.



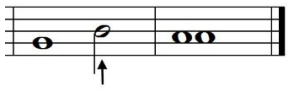
However, I suspect there will be no solution this way. The penultimate tone of the counterpoint should be B, the second degree of the mode, because the penultimate tone of the counterpoint is G, the seventh degree. If in the penultimate measure I write *b*, it turns out that from there I have to leap a seventh on the horizontal in order to reach the last tone A:

27.



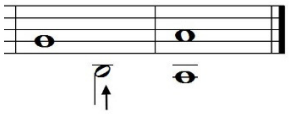
I cannot write B1 in the same place as well. That way the penultimate tone of the counterpoint would be on the top, above the cantus firmus, yet this is a lower voice:

28.



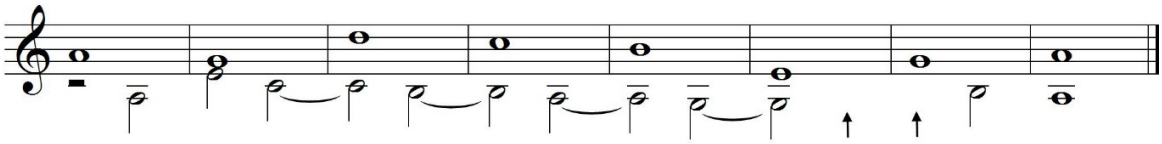
I change the strategy; I write *a* in the last measure. So it becomes possible in the penultimate measure to write *b*:

29.



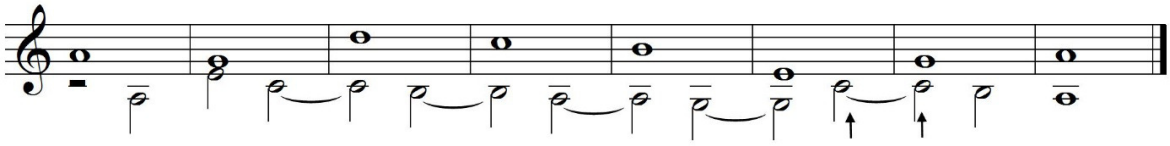
So far so good. Looking at the whole exercise, I find that I have very little work left:

30.



In the sixth measure on the second beat there should be a consonance with the cantus firmus, E. This implied tone in the counterpoint, tied to the first beat of the seventh measure, should then be brought as smoothly as possible to B. I place C:

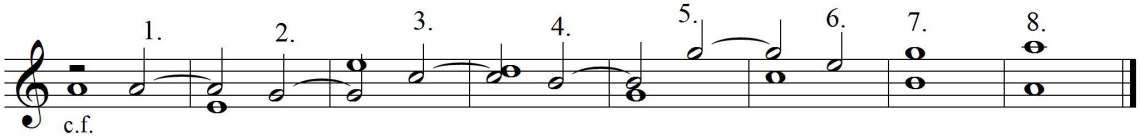
31.



And so I come to a smooth correct resolution.

I provide another correct example with analysis:

32.



After a half pause (obligatory), the counterpoint begins on the first degree with a unison on the weak beat.

In the second measure, on the downbeat, the fourth is led stepwise downwards into a consonance - note 2.

Third measure - I start with consonance on the downbeat. Leap to another consonant on note 3.

Measure 4 - a second on the downbeat, taken downwards into a third - note 4. In measure 3 and 4 the voices are crossed - this is good and normal.

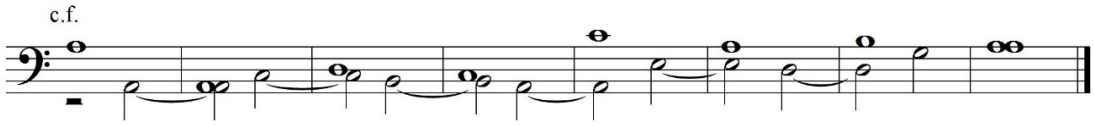
M. 5 – Leap to a consonance, note 5, because there was a consonance on the downbeat.

M. 6 – Same. I know I need G in m. 7, so I prepare it with note 6, E. It can't be tied, because in m. 7 it would have to be taken downwards into D. So I would miss having G on the second beat, and G is a mandatory note.

M. 7 – I'm allowed a whole note. Second degree in cantus firmus and seventh in the counterpoint.

M. 8 – A whole note and first degree in both voices.

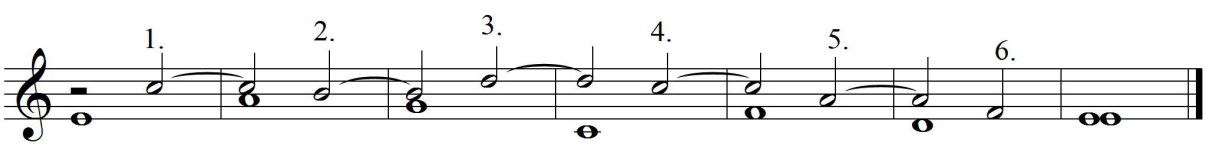
33.



In measures 1, 2, 5 and 7 there is a consonance on the vertical on the downbeat and a resulting leap on the consonance on the second beat. In measures 3, 4, and 6 there is a sustained dissonance that leads downwards. In measures 5-6 I have not followed Rule V. - *D* on the second beat in measure 6 forms a hidden fifth with A.

Here it was not necessary to write a whole note in the penultimate measure.

34.



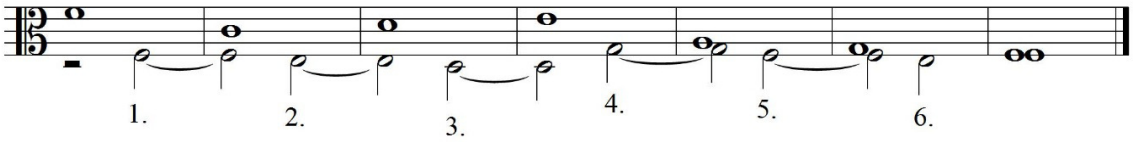
Vertical

Measure 1 – Note 1 (C) is not the first, third or fifth degree of the mode
M. 2 – On the second beat there is a dissonance (A-B). This is not possible. There was no need to lead the C downwards
M. 3 – Looks fine
M. 4 – Hidden octave on the second beat. Again I have not followed Rule V. This octave is heard more clearly than the hidden fifth in m. 6 of example 33.
M. 5-6 – There are no problems on the vertical. The fifths on the downbeats in these two measures are not significant
M. 7 – OK

Horizontal

Two consecutive leaps in one direction in the counterpoint in m. 5-6.

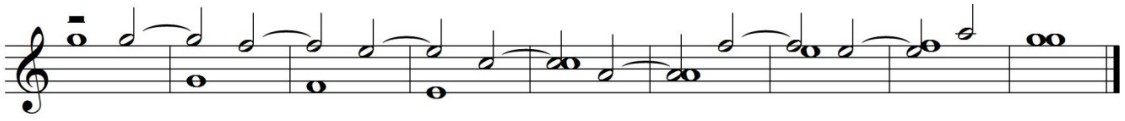
35.



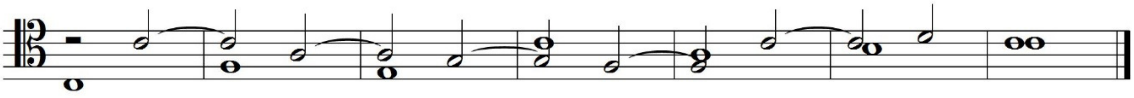
Here there are no problems on the horizontal. There are only two vertical errors. In m. 3 we have a classic faux seventh as shown in Example 4. In m. 4, the dissonance is not taken stepwise downwards; instead, we have leapt from *D* to *G*.

Examples for analysis

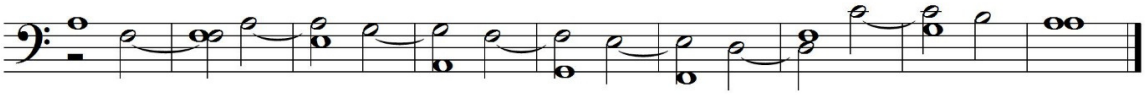
36.



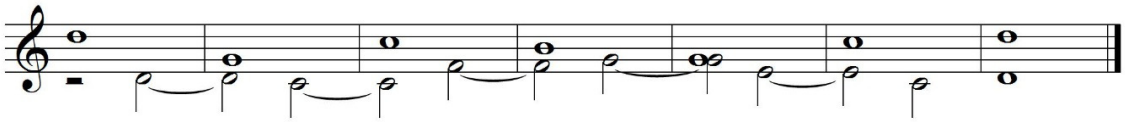
37.



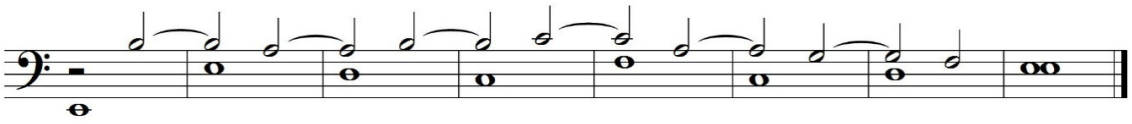
38.



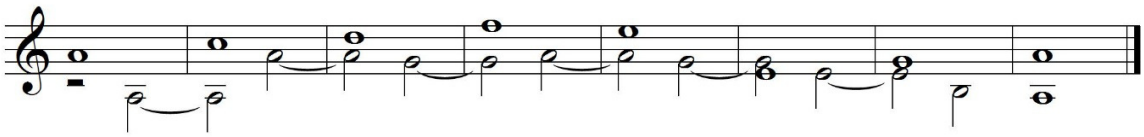
39.



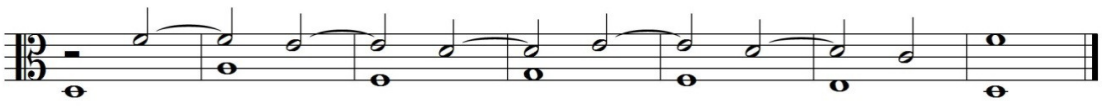
40.



41.

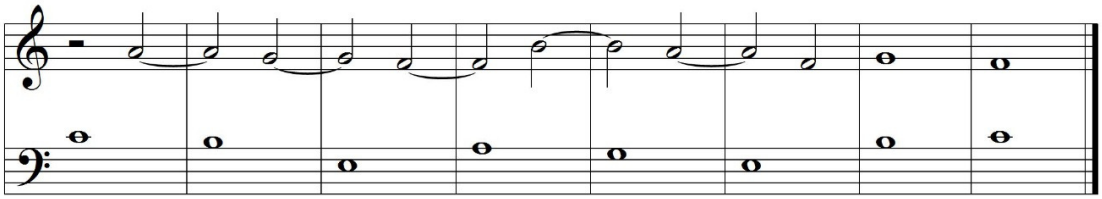


42.



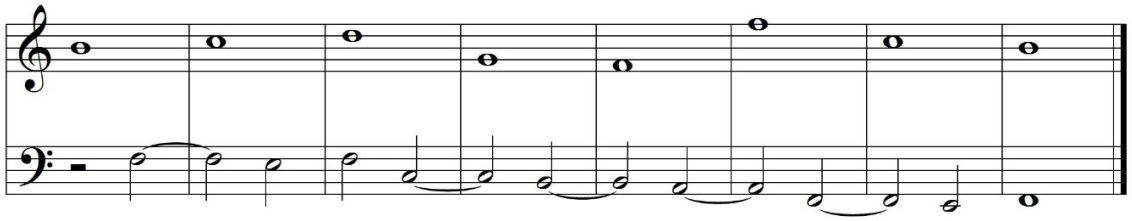
In the following example, the counterpoint seems quite wrong. But I am sure that if it is transposed to some interval, it will fit perfectly to the cantus firmus:

43.



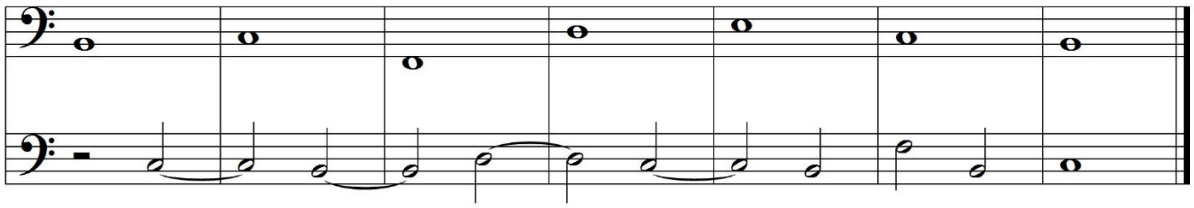
And there seems to be a problem with the cantus firmus here. It cannot start from B. Maybe it should be transposed?

44.

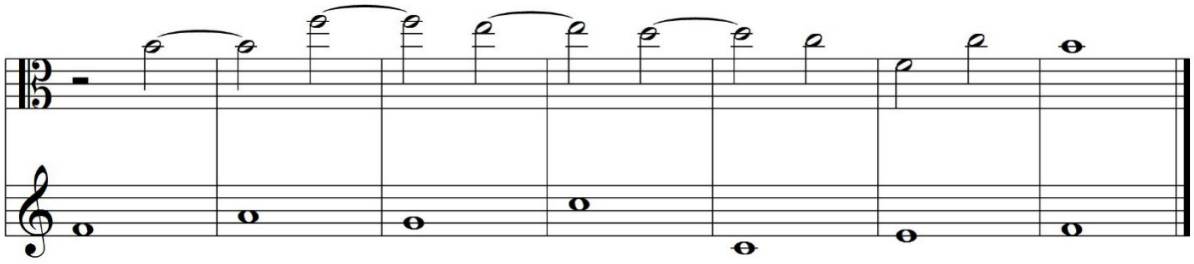


Another two similar examples:

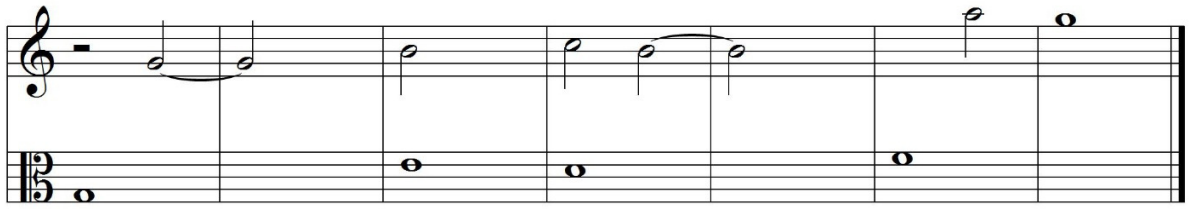
45.



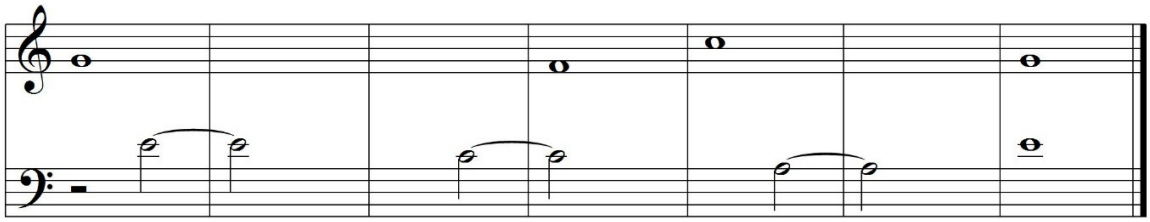
46.



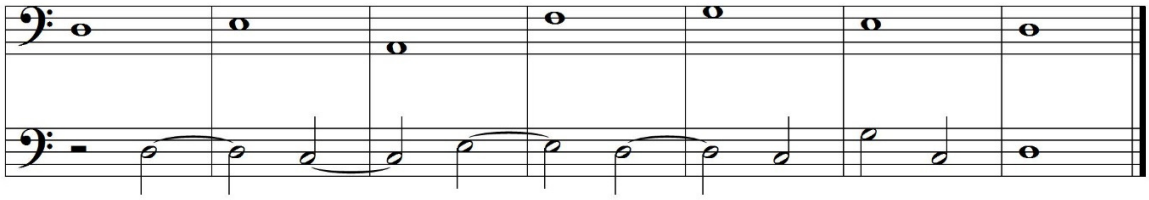
47. No need for transposition here, but there are missing notes:



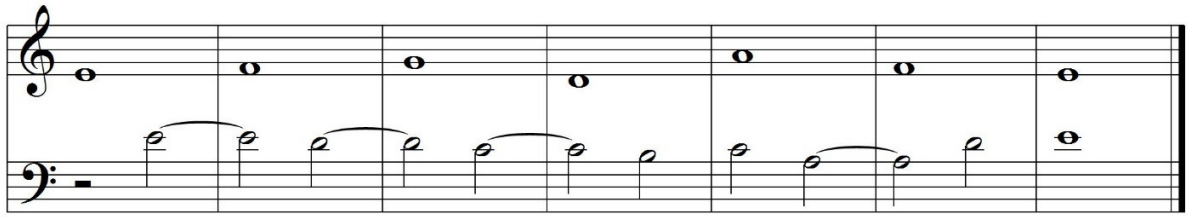
48. And here the missing notes have to be filled in and also one or two of the voices have to be transposed:



49. Solution to example 45. (not the only one possible):

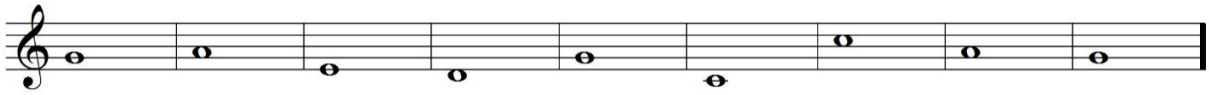


50. Solution to Example 48. (same):

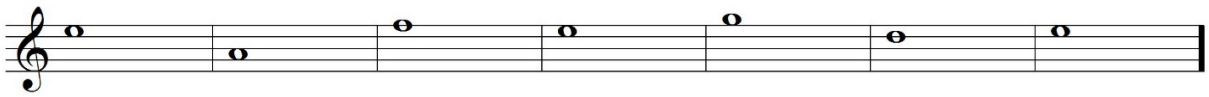


Correct c.f. for solving:

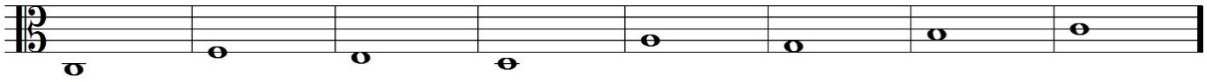
51.



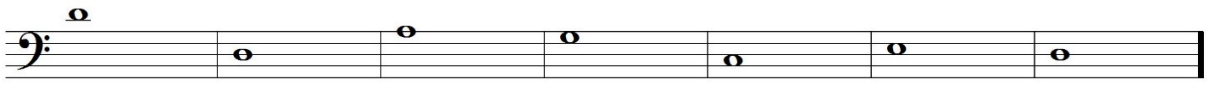
52.



53.



54.



Fifth Polyphonic Species
Counterpoint in a Florid Melody

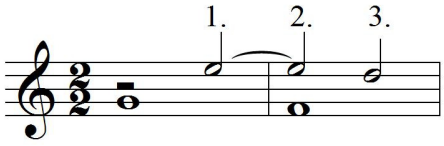
These species use new rhythmic and melodic combinations with a partly Renaissance aesthetic, obeying fixed rules. One approach to explaining this species is that it uses elements from the other species as well as new elements including eighth notes and dotted notes.

In preparation, we need to demonstrate a new principle, which refers to the fourth polyphonic species:

Fragmentation of a dissonant syncope

As we showed in the previous chapter, dissonant syncope consists of the following three elements:

1.



Element 1 - Consonance on the vertical of the second beat in measure 1. The note of the counterpoint is tied over the bar line - note 1.

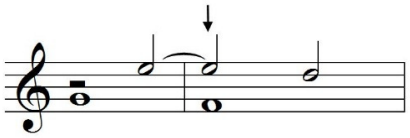
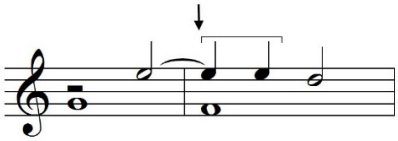
Element 2 - Dissonance on the downbeat in the second measure. This dissonance is called a suspended (held) dissonance. Note 2.

Element 3 - Leading the dissonance stepwise downwards into a consonance. This is obtained on Note 3.

The fragmentation of the dissonant syncope refers to note 2. Notes 1 and 3 don't change. This includes four variants or four formulas:

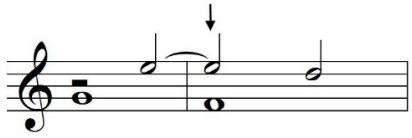
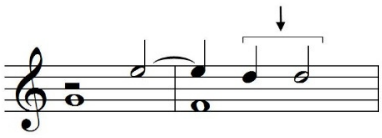
First formula

2.

Model	First formula
	

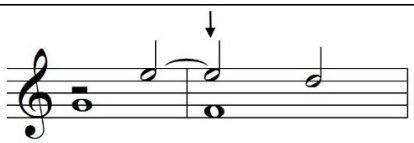
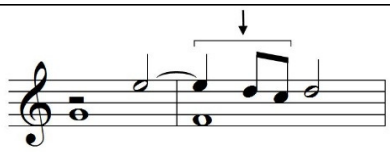
Note 2, the suspended dissonance, is fragmented into two identical fourth notes. This is the first of the two ways to use a repeated tone.

2.

Model	Second formula
	

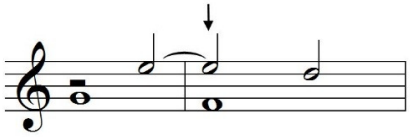
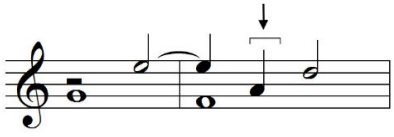
Here the fragmentation is also on note 2. But now the second quarter note is the same as note 3. So note 3 has appeared a little earlier. That's called an anticipation. This formula will be used a lot in the future with extremely beautiful results. This is the second way to use the repeated tone.

3.

Model	Third formula
	

The second quarter note of the second formula (D) is further fragmented, into two eighth notes that follow a descending stepwise line - then we reach note 3 again. This descending stepwise line is obligatory.

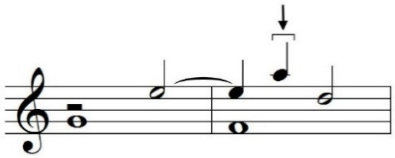
4.

Model	Fourth formula
	

The second quarter note is again modified. It leaps from the suspended dissonance (note 2) into consonance with the cantus firmus. This is the second and final opportunity to leap from a dissonance (the cambiata represents the first opportunity). The leap into a consonance is the characteristic feature of this formula, and then we reach note 3 again.

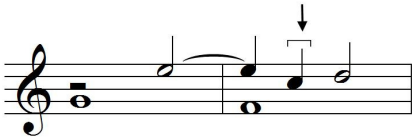
If all the horizontal and vertical rules are followed, the leap can also be upwards - to any interval that will form a consonance with c.f.:

5.



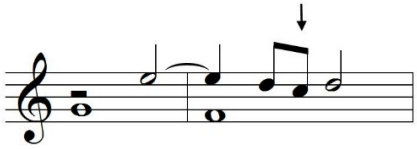
Or:

6.



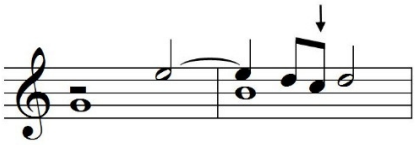
A few more words about the third formula. As shown here again:

7.



on the second eighth note there is a consonance with the cantus firmus, F. But on the same eighth note there can also be a lateral dissonance:

8.



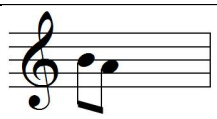
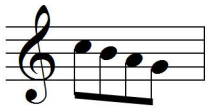
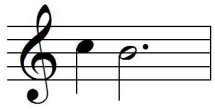
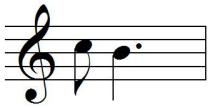
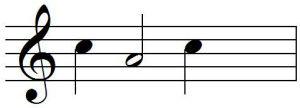
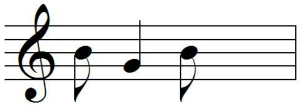
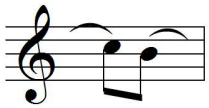
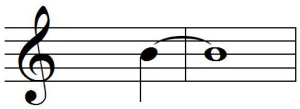
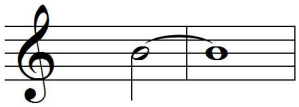
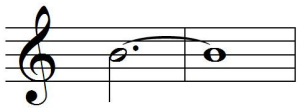
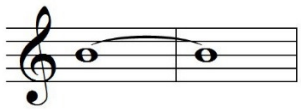
Different textbooks have a different opinion on whether or not a lateral dissonance is allowed to be there. It occurs in the living music, so I consider it allowed.

Fixed rules

All rules remain in force except the repeated tone which is allowed in formulas 1 and 2.

I. Some new note values - dotted quarter and half notes, eighth notes - are now allowed. But the following rhythmic combinations are forbidden:

9.

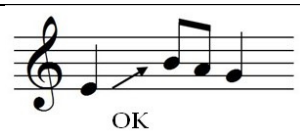
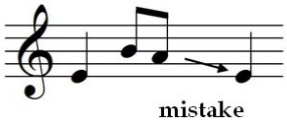
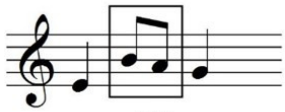
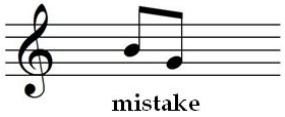
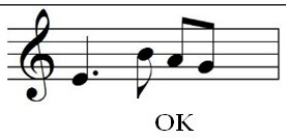
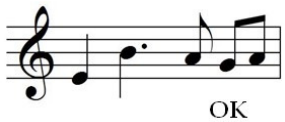
A. Two eighth notes on the downbeat	
B. Four eighth notes anywhere in the measure	
C. A quarter note and dotted half note - the so-called Lombard rhythm[1]	
D. An eighth note and dotted quarter note - also Lombardian rhythm[1]. Anywhere in the measure	
E. Syncope	
F. Syncope – anywhere in the measure	
G. Tying of an eight note or from an eighth note. Anywhere in the measure or over the bar line	
H. Tying of a smaller note value to a larger one[1]	
I. Same[1]	
J. Same[1]	
K. Tying of a whole note with a whole note[2]	

All the fixed rules here are synthetic, for practice only. They are not necessarily followed in the living music. But for the purposes of the textbook they must be obeyed.

II. Rules regarding the eighth notes

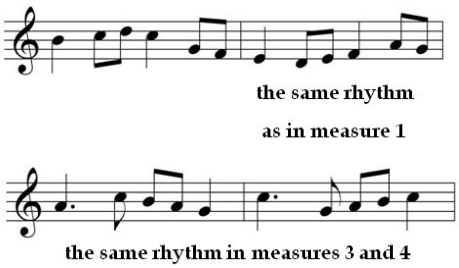
[1]The reverse combination is allowed.
[2]A quarter-and-quarter and half-and-half note ties are allowed.

10.

A. There can't be a leap after a group of two eighth notes	 OK  mistake
B. When the two eighth notes are in a group, they must move in a stepwise motion	 OK  mistake
C. Three eighth notes may be present if the first is part of a preceding group - of a dotted quarter note and an eighth note followed by two more eighth notes	 OK or  OK

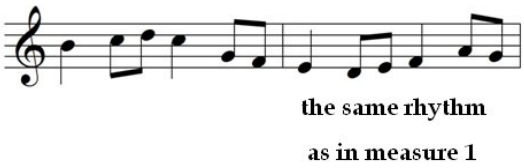
III. Avoiding identical rhythmic patterns in two consecutive measures is strongly recommended. Symmetrical and square constructions are to be avoided[3]:

[3]By a square construction I mean one in which the rhythm of the second measure is the same as that of the first, the rhythm of the fourth measure - the same as that of the third, and they follow the first and second (1-1, 2-2):



Another possible variant is that the rhythm of the third measure is identical to the first, and that of the fourth measure is identical to the second(1-2, 1-2).

11.



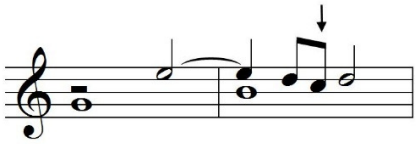
IV. There may be a pause (only a quarter or half) only in the first measure, according to the rules for the third or fourth polyphonic species.

More lenient rules and recommendations

1. It's a good idea to start the counterpoint melody more smoothly, more calmly. Then it can speed up a little. Towards the end it can slow down again in terms of note values.

2. As we said, at times we can also use formula 3. in its modified variant - when the lateral dissonance of the second eighth note with the cantus firmus occurs[4]:

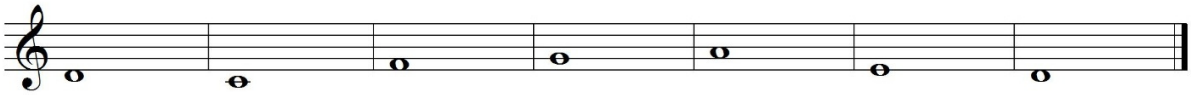
12.



3. It is advisable to use a whole note as a counterpoint (first counterpuntal species) very carefully and sparingly. It is best to write it at the very beginning of the exercise and perhaps - in the penultimate measure. If we have already achieved a certain motion, it is not good to suddenly "freeze" it, especially if we are in the middle of the exercise.

Example

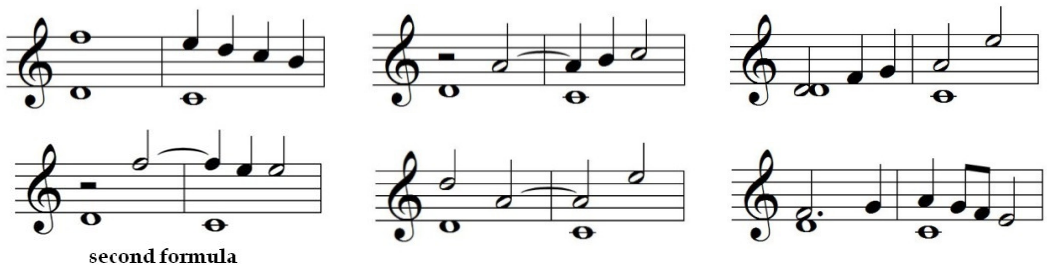
13.Here is the assigned firmus cantus:



[4]The use of this type of dissonance is still prohibited in all other cases.

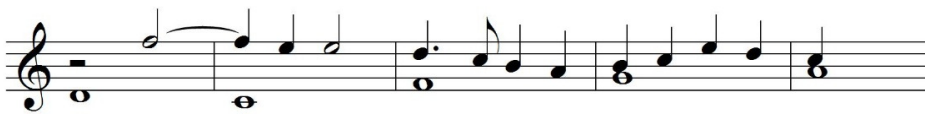
There are many solutions - over a hundred. I show a few variants for the first two measures:

14.



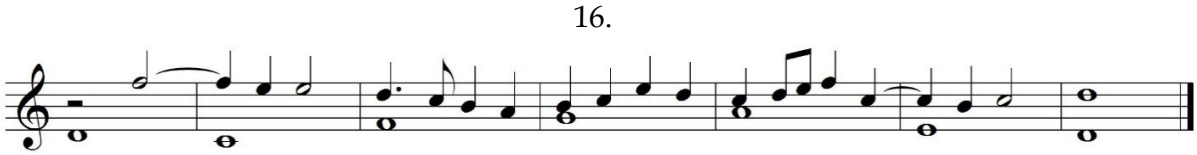
I choose the formula and proceed further.

15.



In the third measure I used the third polyphonic species. Because there is a fourth-note pulsation in this and the next measure, I have added a dot to the D in the third measure for more rhythmic variety; in the fourth measure and at the beginning of the fifth I have placed the cambiata.

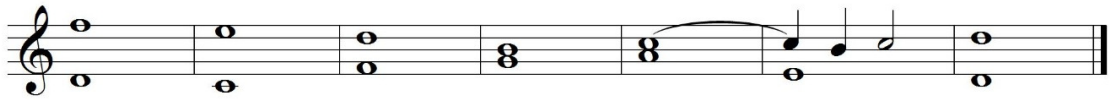
16.



There are two eighth notes in the fifth measure - the vertical is used on exactly the same principle in both the second and third polyphonic species (D is a transient dissonance). Then I go back to C - the last quarter note in measure 5. I will also need the C in the penultimate measure. I use the fact that I can go one degree downwards and still have consonance with my cantus firmus. At the end of the measure I have C again, which is the seventh degree against the second in c.f.

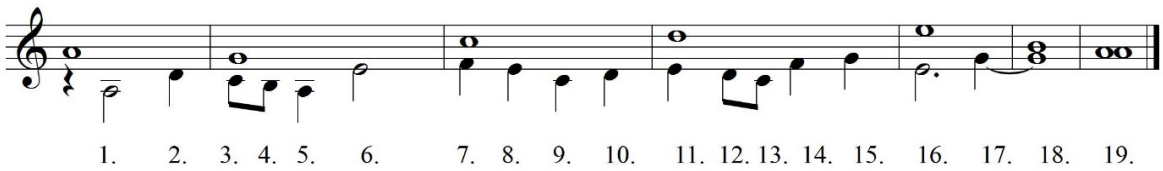
As an experiment, this exercise can be solved similarly, but with less rhythmic motion, less smoothly. Such a schematic solution perhaps better demonstrates the consonant vertical of Example 16.:

17.



Following are two incorrect solutions with an impressive number of errors:

18.



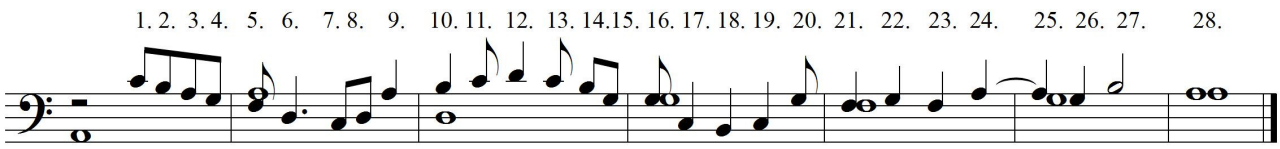
Vertical

Parallel fifths, notes 2-3.
Leap from a dissonance, note 5.
Hidden fifth, note 7.
Note 10. doesn't make a transient dissonance and so parallel sevenths on notes 10. and 11 are formed.
Dissonance on the downbeat (unsuspended), note 11.
Leap from a dissonance, note 13.

Horizontal

A quarter pause is followed by a half note and a broken syncope on notes 1. and 2.
Two quarter notes on the downbeat, notes 3.-4.
Incorrectly constructed cambiata: its last note - 11., forms a dissonance with the cantus firmus on the downbeat.
Note 17. is tied to a larger one - note 18.

19.



Vertical

Note 15. - leap into a fifth with c.f.

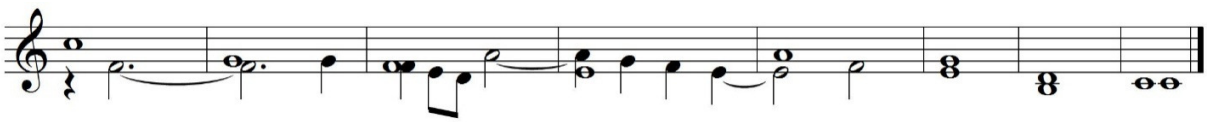
Unison on the downbeat, note 16.
Parallel unisons, notes 20.-21.
Note 22. - lateral dissonance
Note 26. - faux second

Horizontal

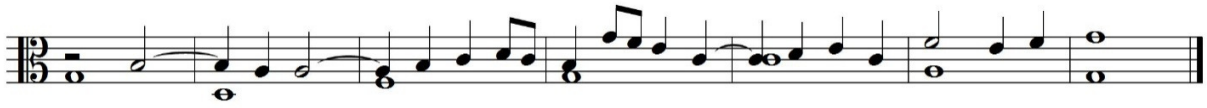
Eighth note after a half pause (it can't be started at all with eighth notes), measure 1, notes 1. -4.
Four eighth notes one after another, notes 1.-4.
Eighth note on the downbeat, note 5.
Lombardian rhythm, notes 5.-6.
Syncope, note 5.-7.
A leap after a group of two eighth notes, note 9.
Syncope, notes 11.-13.
Two eighth notes in a group that have not been led stepwise, notes 14.-15.
Repeated note – 16.
Chain syncopation, notes 16.-20.

Exercises for analysis

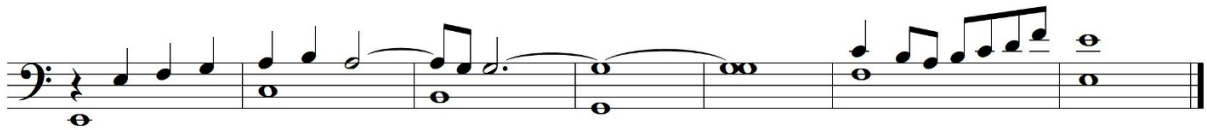
20.



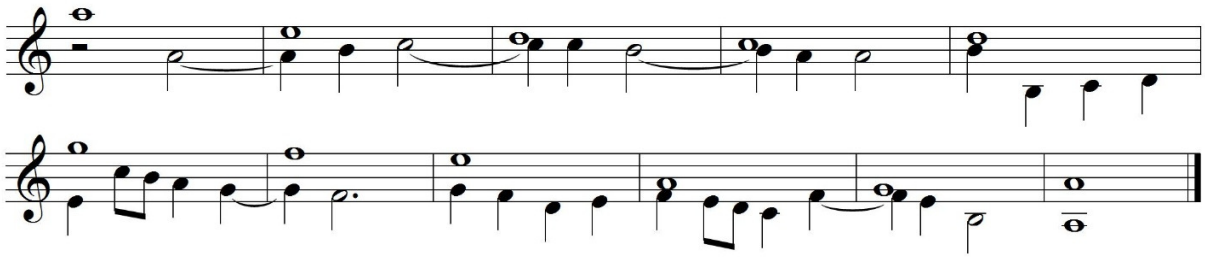
21.



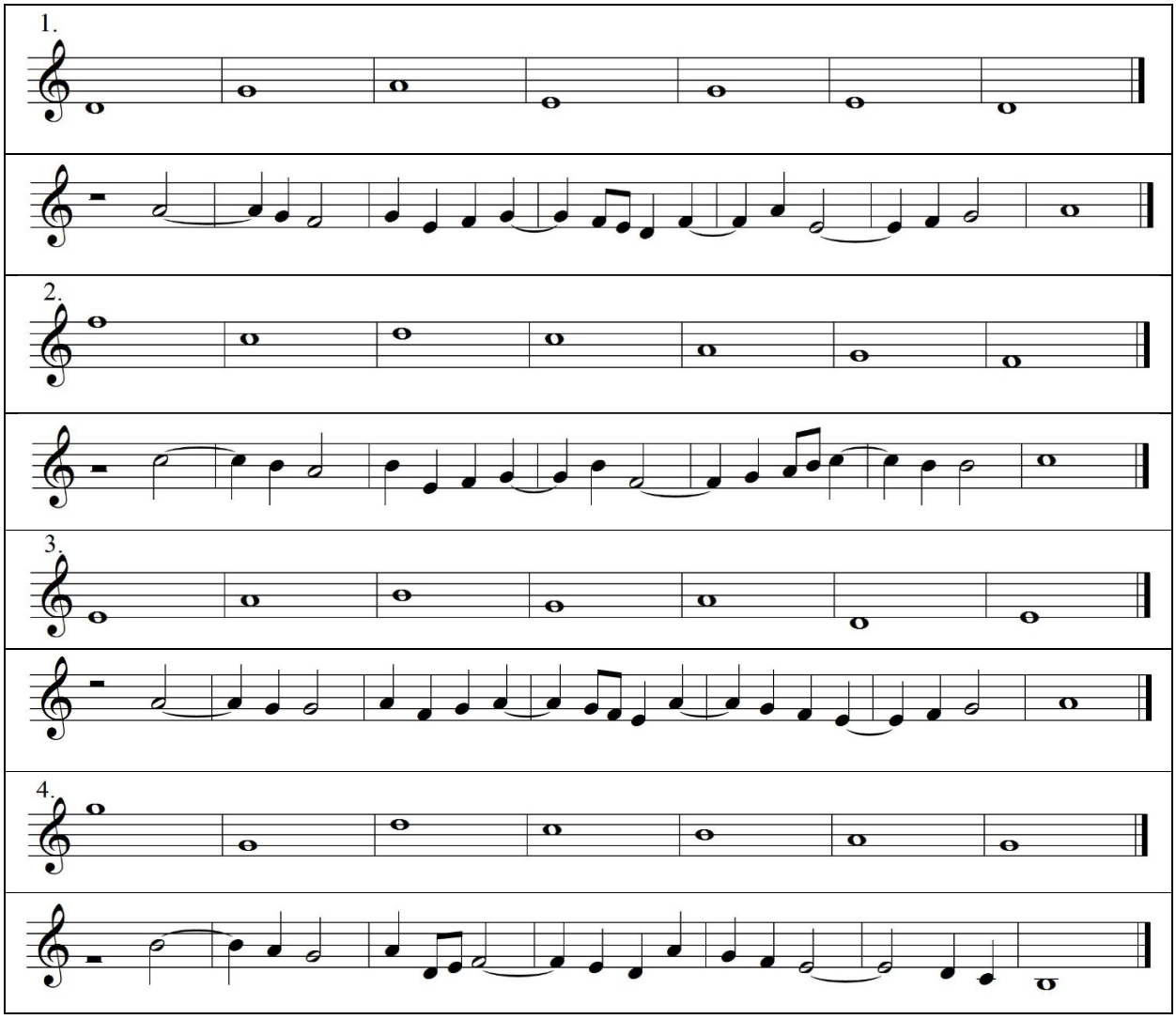
22.



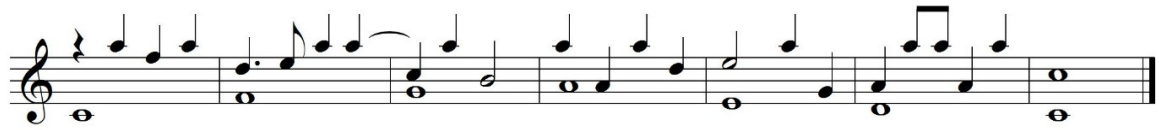
23.



24. Here there are four canti firmi with four counterpoints. But the counterpoints seem to be scattered, not suiting to the cantus firmus under which they lie. It is not clear whether they are upper or lower voice. In addition, unfortunately, they have been transposed. Can you find which counterpoint belongs to which cantus firmus and in which transposition?

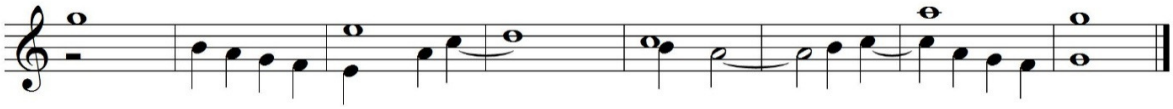


25. Here in the counterpoint each note A2 represents
a wrong note. Which is the correct one?

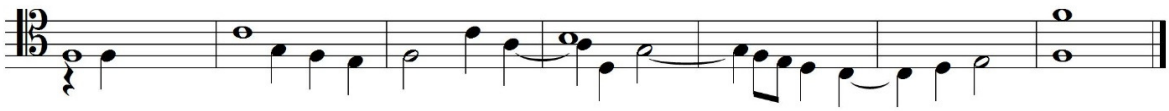


Exercises with missing notes[5]:

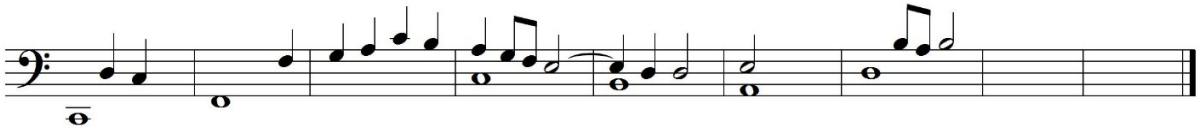
26.



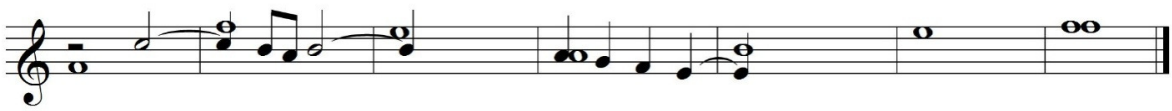
27.



28.

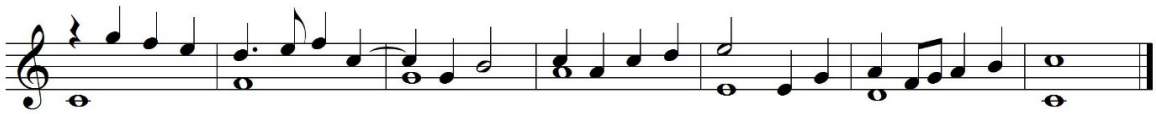


29.Treble clef on the first line of the staff:



Solution of example 25:

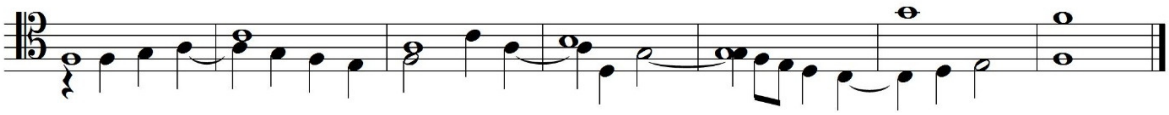
30.



[5]There is more than one solution.

Solution of example 27:

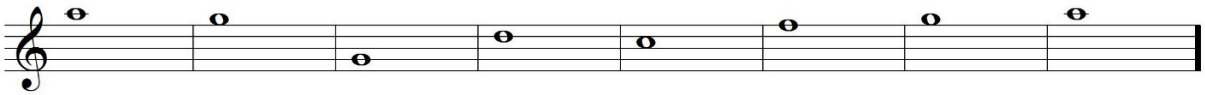
31.



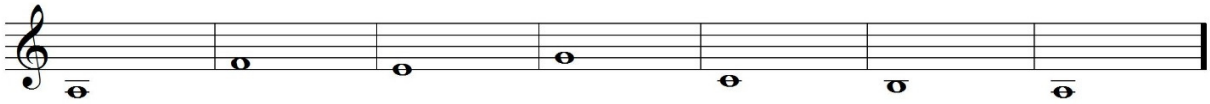
Regarding Example 24. I am pretty sure that the first cantus firmus would somehow be suitable to the second counterpoint.

Correct c.f. for solving:

32.



33.



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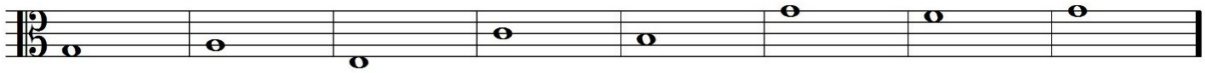
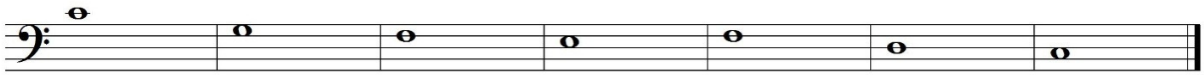


35.



Two florid melodies

Using one florid melody as a predefined one, we add to it another as a counterpoint.

There are three new fixed rules, as well as easing and transformation of some others.

Fixed rules

I. We can't write a tie against a tie - one that runs simultaneously in two voices.

II. We can't write two eighth notes (and eighth notes in general) against two other[1].

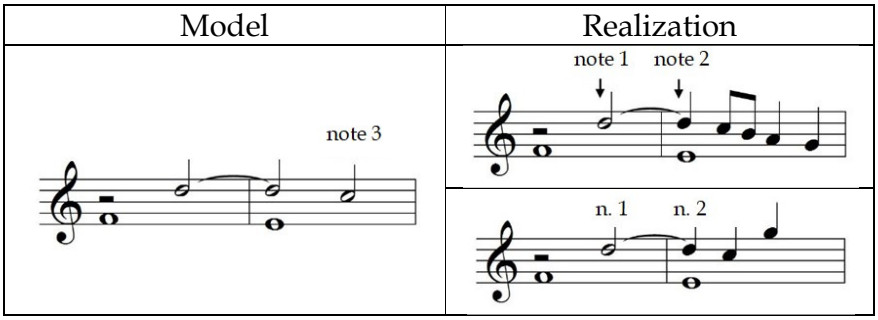
III. It is also not good to write the same rhythm in both voices at the same time[2].

Transformation of the dissonant syncope rules

The four formulas for fragmenting a dissonant syncope can still be used of course, but now in the counterpoint they can be further modified. This affects several aspects starting from their model.

1. If the suspended dissonance is resolved stepwise downwards, we might not reach back to note 3 as a half note:

1.

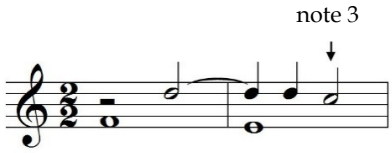
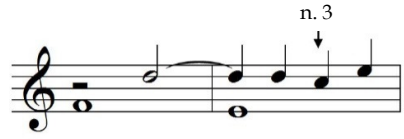
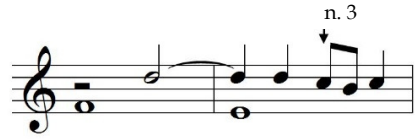
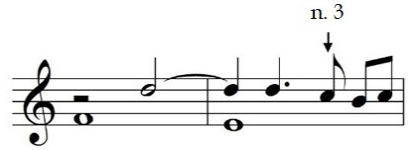


2. In formulas, there might be a shortening of the length of note 3:

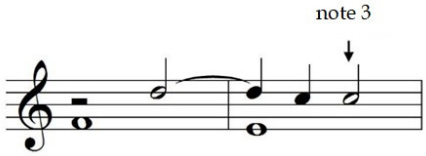
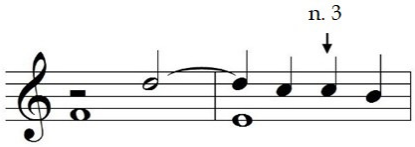
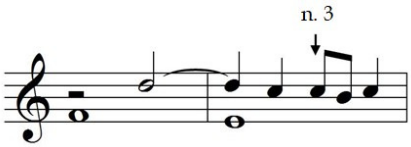
[1]It is also not recommended to have a pause versus a pause in the first measure. In general, if there is a pause in one of the voices, it is better not to have one in the other. This is not a stylistic recommendation; rather a synthetic, "technical" one.

[2]It can only last for a short time. It is acceptable to have fourth notes in both voices for about one measure; less acceptable - half notes, even less - whole notes or identical rhythmic groups.

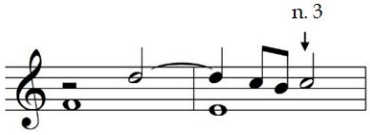
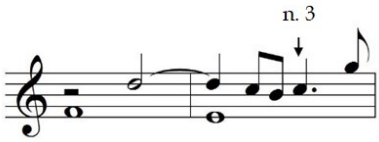
2.

Formula	Realization
 First formula	  

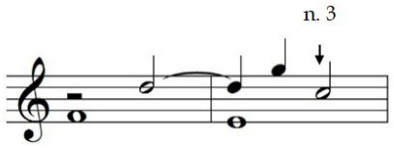
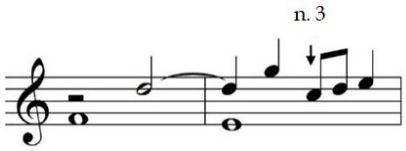
3.

Formula	Realization
 Second formula	 

4.

Formula	Realization
 Third formula	

5.

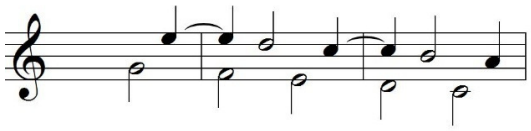
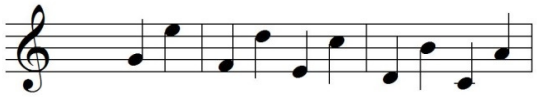
Model and name	Realization
 Fourth model	

Two more recommendations follow before we move on to writing:

1.Overall motion and complementary rhythm.

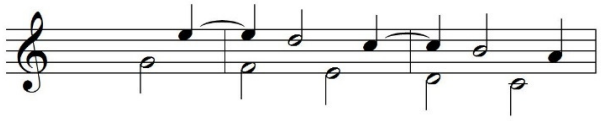
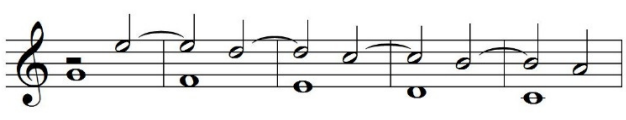
It is recommended that the solution is not static, that there is more or less constant motion, which is mainly maintained in quarter notes. However, this doesn't apply to both voices at the same time; if one voice moves mainly in fourth notes, the other may be less flexible. For that purpose, the principle of complementary rhythm is taught, in which there may be half notes in both voices with a horizontal shift of one fourth note:

6.

Initial construction	Effect of a resultant rhythm
	

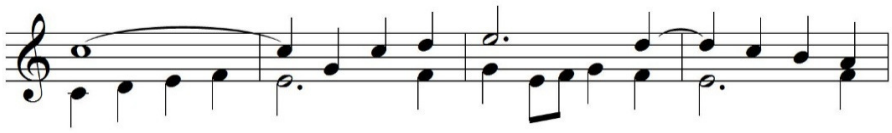
This technique doesn't conflict with the prohibition against using syncope in the counterpoint. Such a chain syncopation can be used by both the teacher (who defines the first melody) and the student who writes the second melody. The reason for this is that such a musical text is in fact a texture in fourth polyphonic species with note valuestwice as small:

7.

Initial construction	Same construction with twice as long note values
	

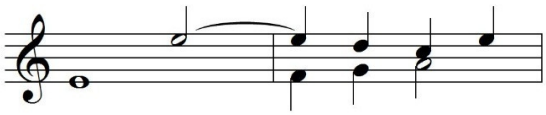
As a direct result, the two voices can “split their work”. If one voice moves (mainly) in fourth notes, the other can “rest” for a while:

8.



2. In some cases, the suspended dissonance can be resolved even if the two voices move simultaneously and contrary:

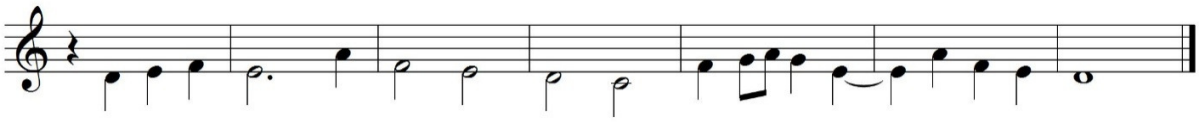
9.



This is possible because the condition that the suspended syncope should be resolved downwards stepwise in one of the voices - is fulfilled.

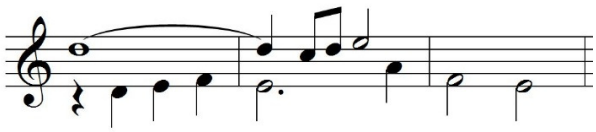
Strategy of writing

10.



Following the recommendation in footnote 1, I start from the downbeat. Also since there is a quarter note pulsation in the first measure, I am going to approach it a little “lazily” by writing a whole note there. In the second measure I will use a modified fragmentation of a dissonant syncope.

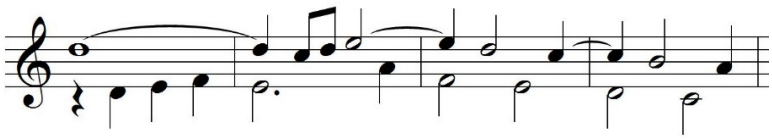
11.



In measure 2, I resolve the suspended syncope downwards; the second note D in the upper voice thus appears to be a transient dissonance between C and E. I plan to tie

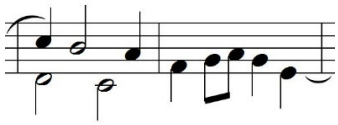
the last note as well: the half notes in the lower voice in measures 3 and 4 suggest to me that there might be a chain syncopation:

12.



I have stumbled on a bit of a tricky spot here - what to do between measures 4 and 5?

13.



In the fifth measure - which in this example appears second from left to right - I have difficulty deciding what tone to write.

I can't repeat the tone A from the previous measure. I can tie it with a quarter note, but later I would have a problem again.

I can't write B - it would form a tritone with F in the lower voice.

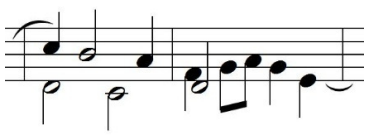
I can't write C and any tone over it at all. So the two voices would leap in the same direction.

I can't write F - we get a unison on the downbeat (there is also an F in the lower voice).

I can't write E - it would form a second with F.

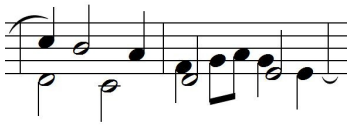
I can write D (but for how long?) I decide it to be a half note:

14.



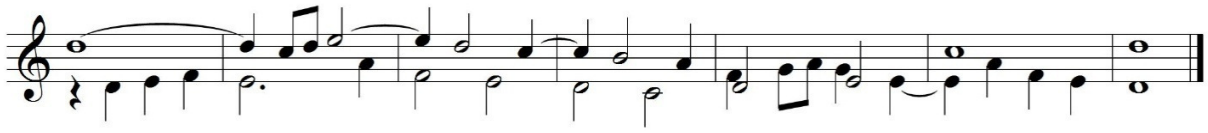
So the voices have leapt in opposite directions and they have crossed. The first note G appears to be a transient dissonance to D. The next note could be E - also a half note. The unison that is obtained on the last quarter note is not a problem because it is on a weak time:

15.



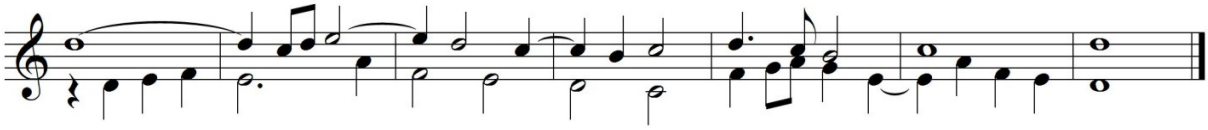
And since the penultimate measure moves in fourth notes, I consider that I can allow myself a whole note again and complete the exercise:

16.



The troublesome spot discussed above occurred because there is an A at the end of measure 4 in the upper voice. After that note, there appears to be one possible correct solution[3], which I have shown. But if there was no A there, then I could use a slightly different strategy:

17.



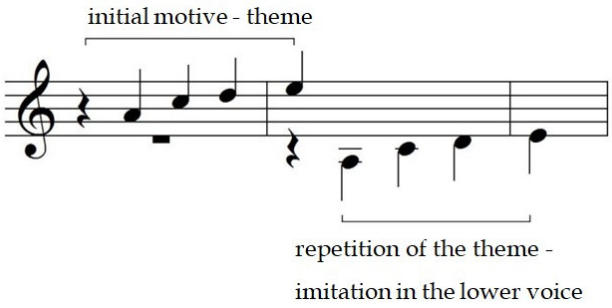
Thus, stopping the dissonance chain a little earlier, I have solved the problem without a single leap in the upper voice and without crossing the voices[4].This shows the game of variants and the consequences resulting from each move – "what could happen if...?" In this respect, the logic of the strict polyphonic style has common features with the logic of some games - chess, for example.

Alternative strategy

A free two-voice polyphony can also be written in another way. We can compose both voices virtually simultaneously. One useful exercise is to write an imitation[5]to a theme that is set initially in one voice:

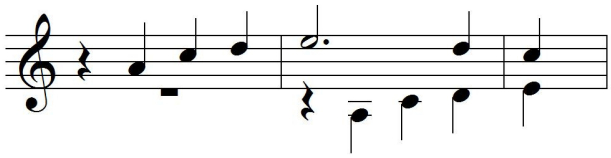
[3]There is another possible correct solution - an octave leap downwards in the upper voice.
[4]But this can in no way be considered forbidden or even undesirable. The solution is simply different.
[5]This is a brief introduction to imitation polyphony, which is touched on only vaguely here.
It is explained in more detail in the chapters on imitation and canon.

18.



So the lower voice pauses legitimately on the downbeat. Next, we can write the upper voice in the second measure so that it suits the lower:

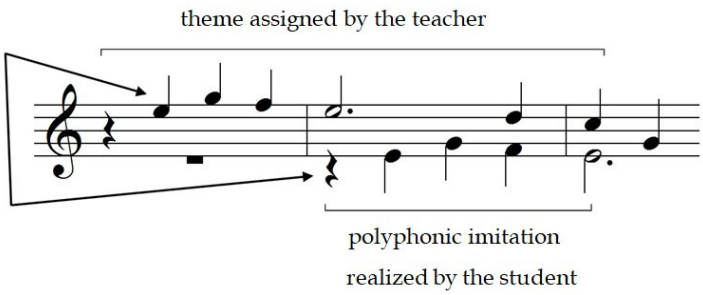
19.



Afterwards we can continue to write freely both voices.

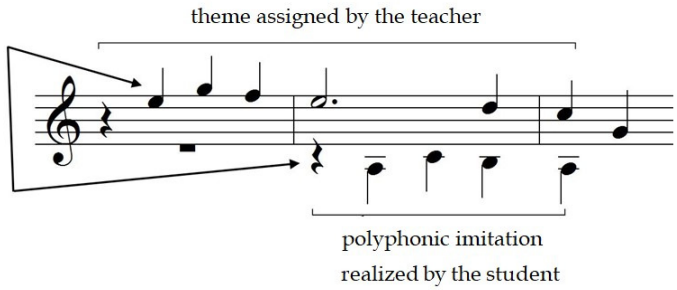
Another approach is to already have two written measures as a theme - usually assigned by the teacher. The student can attempt to create a response (polyphonic imitation) of the theme from the first measure of the upper voice:

20.



In this example, the "student" has written the theme from A and the lower voice in the second measure suits the upper voice perfectly. But "another student"could have written the theme from a different tone, not from A, and would again obtain a correct result:

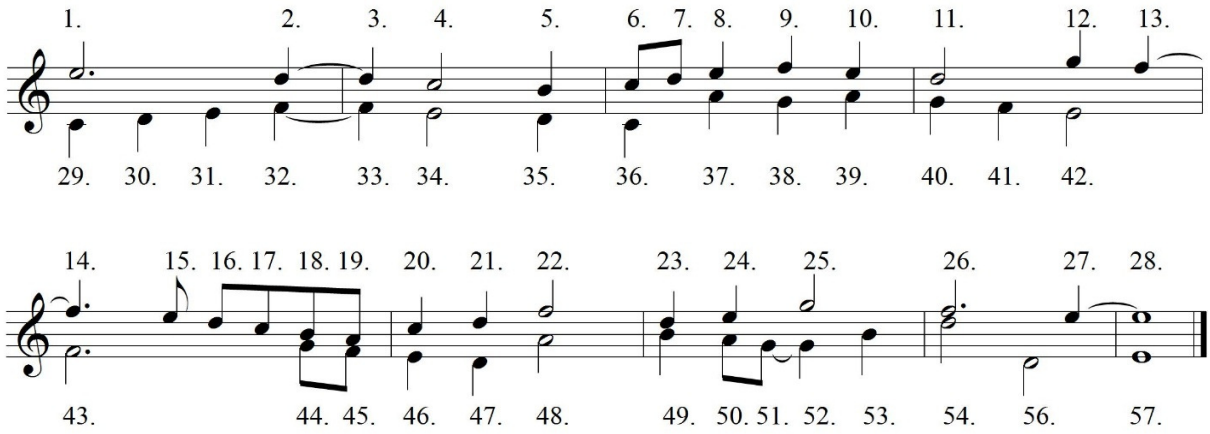
21.



Again, the solution can freely proceed further.

Incorrect examples with analyses:

22.The assigned melody is above



To our amazement, mistakes were made by both the "teacher" who wrote the assigned melody and the "student" who added the counterpoint in the lower voice.

Vertical

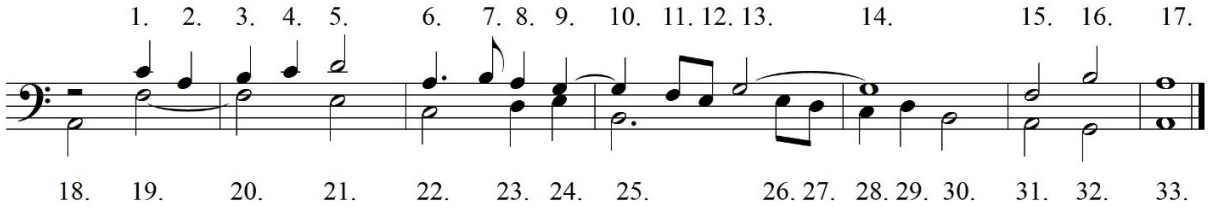
A tie simultaneously in both voices - notes 2.-32. and 3.-33.
Identical rhythm in both voices in notes 3.-4.-5.and 33.-34.-35.
Hidden fifth - notes 8.-37.
Lateral dissonance (seventh) - notes 9.-38.
Parallel fifths between 10.-39.and 11.-40.
A dissonance (ninth) in notes 13.-42. It is not properly resolved in the next measure

Eighth notes simultaneously in both voices - notes 18.-19.and 44.-45.
Identical rhythm in both voices in notes 20.-21.-22.and 46.-47.-48.
A leap of both voices in one direction - from notes 21.-47.towards 22.-48.
Forbidden dissonance and anticipation (ninth) on the vertical from notes 27.-56[6].
As a result, the second degree is missing in the penultimate tone of the assigned melody (there is a seventh degree in the counterpoint)

Horizontal

The counterpoint doesn't begin at the first degree of the mode - note 29.
A smaller note tied to a larger one - notes 13.-14.
Four consecutive eighth notes – 16.-17.-18.-19.
A sequence - notes 20-21-22 and then 23.-24.-25.
A smaller note tied to a larger one - notes 27.-28.
Ascending leap of a large sixth - notes 36.-37.
A tie from an eighth note - from note 51. And as a result of this:
Syncope - notes 51.-52.(total duration a dotted quarter note)are enclosed by eighth note at the back and quarter note at the front
Two leaps in one direction - notes 52.-53.-54.

23.The assigned voice is below



Vertical

The dissonance that note 3. forms with note 20. is led upwards instead of downwards.
Unresolved dissonance - notes 5. and 21.
A leap of both voices in one direction - notes 5.-21. and 6.-22.
Lateral dissonance, note 7.
Note 11. - it is going downwards into a dissonance. Since note 6. is not a dissonance, this is not necessary.Thus we get the opposite of the desired situation – a dissonance is led into a dissonance

[6]And I am overlooking the fact that note 27. is tied with a larger note - note 28.

Note 12. - one more dissonance with note 25.

Again note 12. – we have leapt from it. Here the error is twofold. First we have leapt from a dissonance and, second, we have leapt after a group of two eighth notes

Again a leap from a dissonance to note 29. This note can't be changed because it is assigned, which means the error is in the counterpoint, not in the cantus firmus

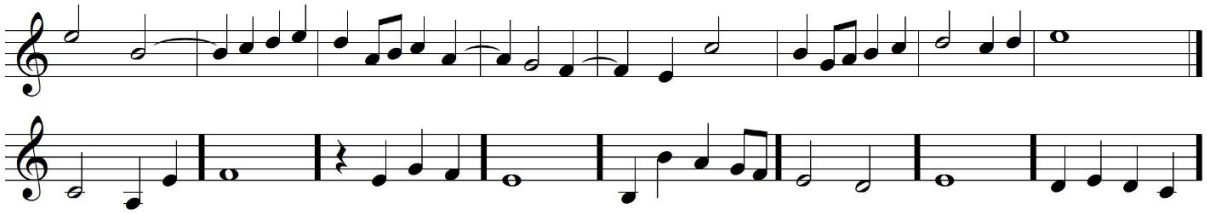
Horizontal

We can't start with two quarter notes after a half pause in the counterpoint

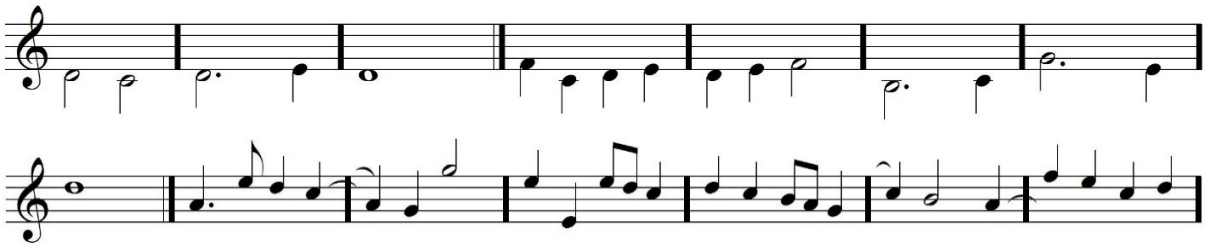
Tritone leap - notes 15-16.

Exercises

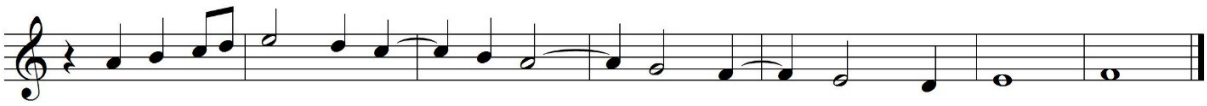
24. In this example the counterpoint is presented in a shuffled order. What is the correct arrangement?

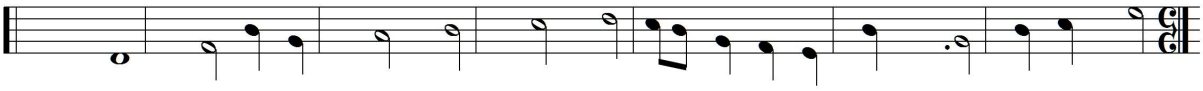


25. Here, for some reason, both the assigned and the contrapuntal melodies are shuffled

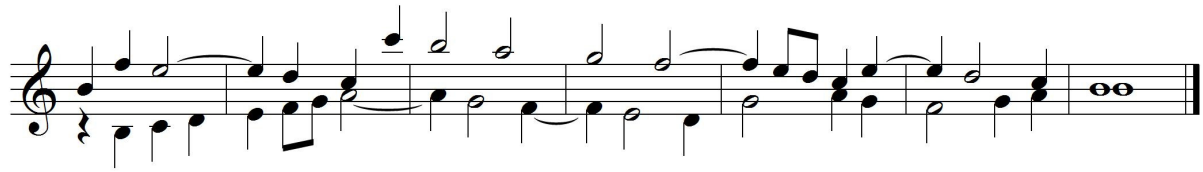


26. The counterpoint is written backwards. Will it suit the assigned melody?

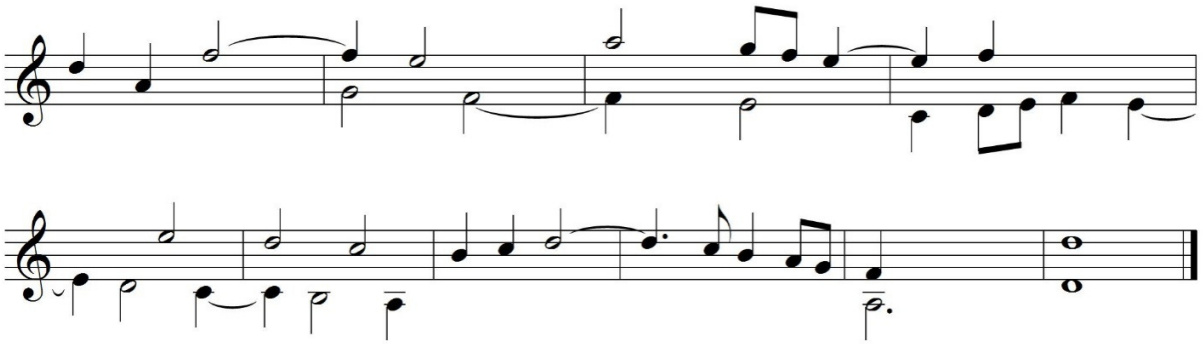




27. For this exercise, I am sure that it is correct. The notes should stay exactly the same, without changing their places on the staff. But I suspect that a different clef should be written...

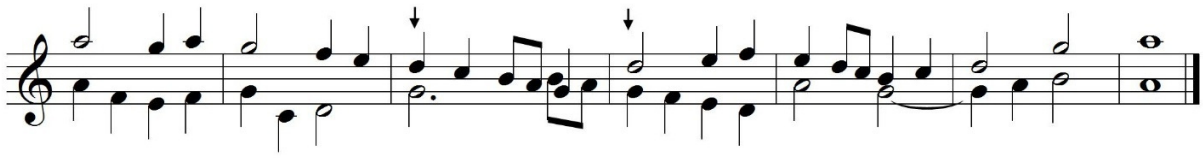


28. Exercise with missing tones



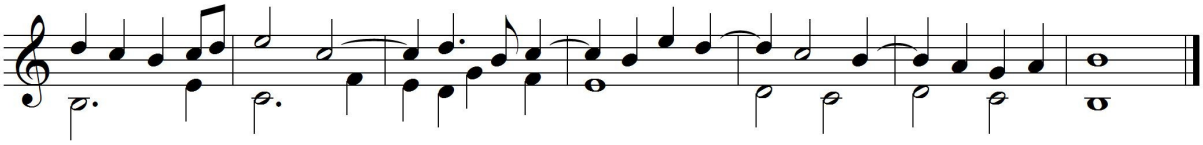
For analysis

23.

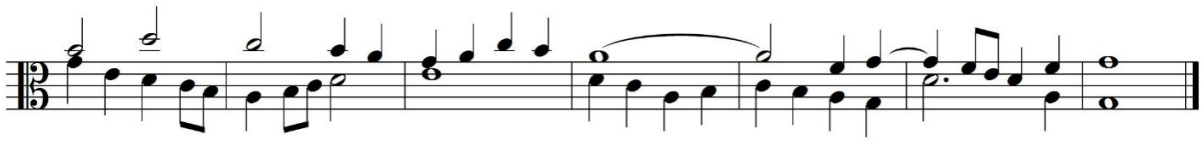


The fifths on the downbeat, shown in m. 3-4, are not considered to be incorrect accented fifths because they are identical, i.e., fixed. However, I do not bet that the exercise is correct.

24.



25.



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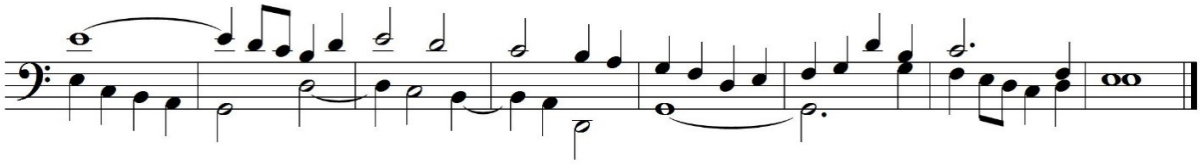
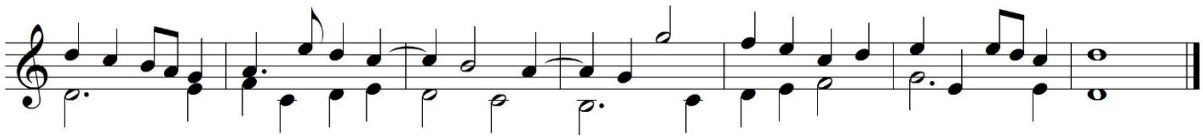
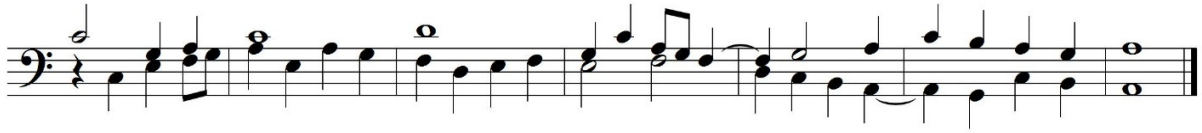


27. Solution of example 25.

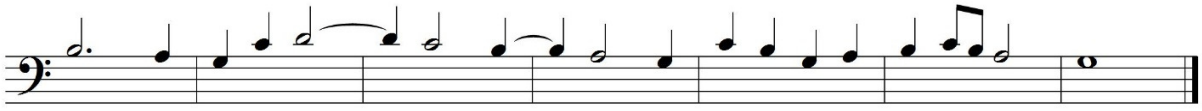


28. Solution of example 31.

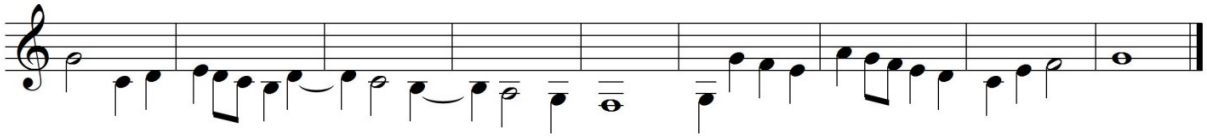


Assigned melodies for solving

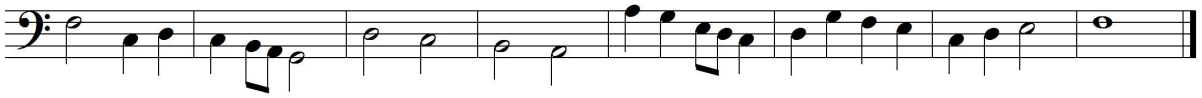
29.



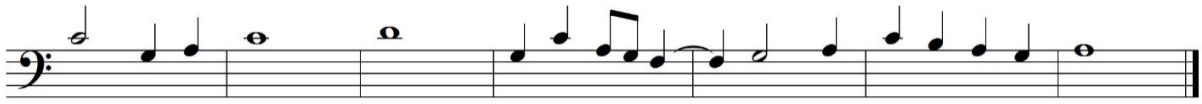
30.



31.



32.



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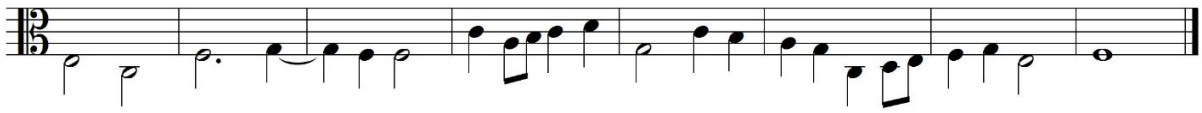

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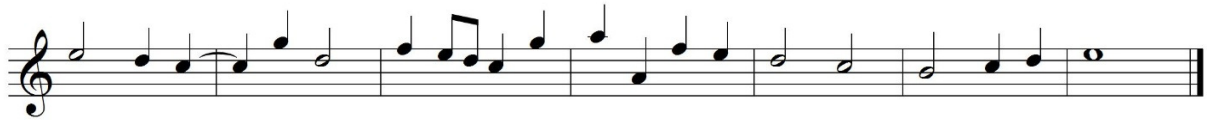

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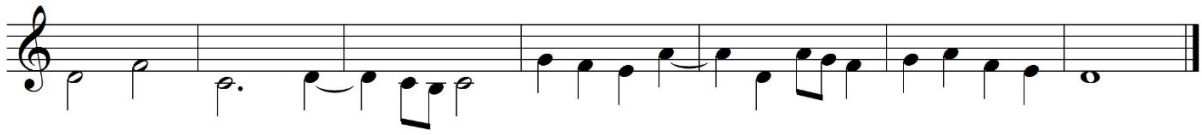

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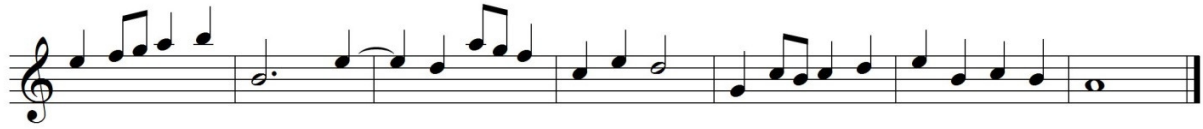


First Polyphonic Species

In three-voice polyphony, the practice is to write two counterpoints to an assigned cantus firmus. One counterpoint is always in whole notes. Thus, together with the cantus firmus, two voices in whole notes are obtained. The third voice (second counterpoint) is successively written in the first species, then in second, in third and so on. It follows that in this chapter all three voices will be in whole notes - first polyphonic species.

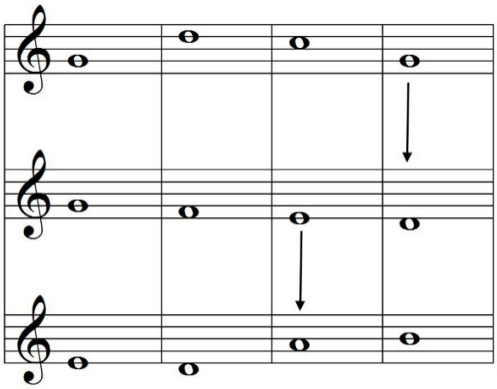
Of course, it is possible to also use other combinations. On the assigned cantus firmus one can write a counterpoint in the second species, and for the third voice - a counterpoint written in the third species, for example. This is called a contrast three-voice polyphony. There is a separate chapter on it where it is taught and explained.

Fixed rules

I. It is forbidden to have a fourth on the vertical with the lowest voice[1] – we can call it very conditionally a tenor. The middle or even the upper voice can be crossed with the lower voice and so temporarily they become the lowest voice. Then this rule applies to them as well.

II. A fourth is allowed between all voices, as long as it doesn't affect the lowest (in general or currently):

1.



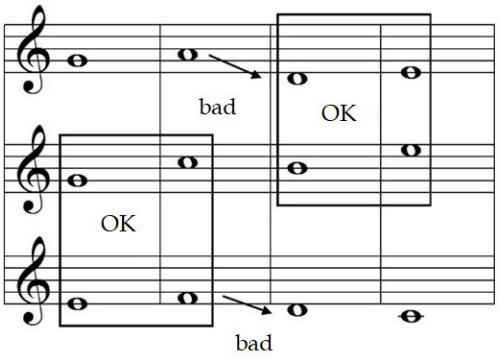
This is an error.

III. Hidden fifths, octaves and unisons are allowed as long as they are not in outer voices, i.e. between the lowest and highest voice[2]:

[1]But the transient fourth is allowed.

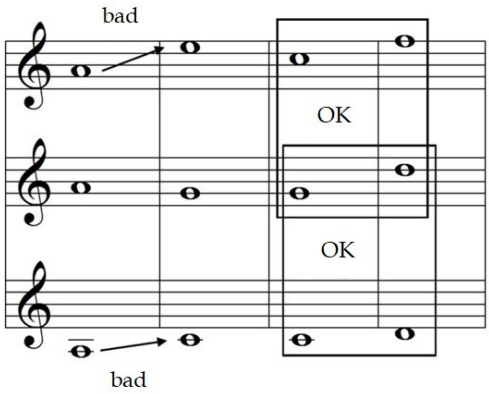
[2]There is a connection between Rule III. and Rule VIII. - see below.

2.



IV. It is also allowed for two voices to leap in the same direction, as long as they are not outermost[3](hidden octave between middle and lower voice in the third measure):

3.



V. As in a two-voice polyphony (first species) there can be two identical tones next to each other in the two counterpoints.

VI. Unisons are allowed in every measure. This is a new rule. But it must be used carefully - only if there is no other option for a solution. It is shown in example 3.

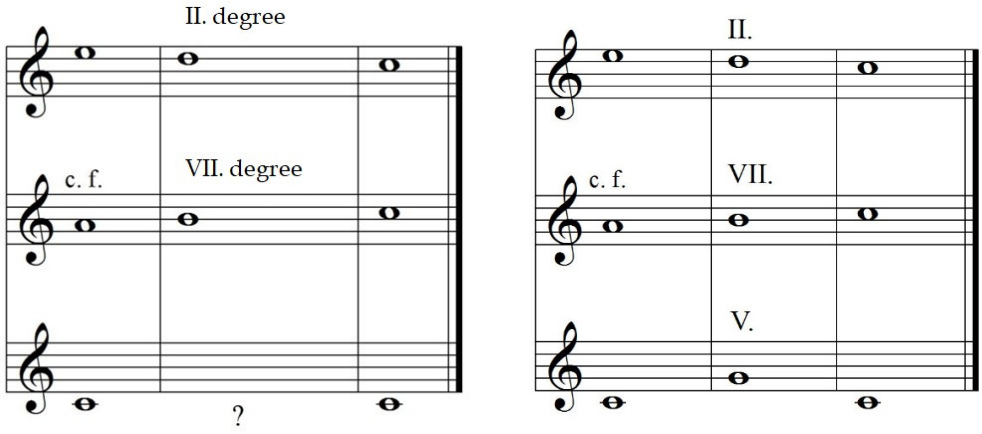
VII. If the counterpoint is in the lowest voice, it should begin from the first degree. The middle and upper voices don't have to do this - they can begin from the third or fifth degree - but they can also begin from the first if it suits the logic of the solution.

VIII. One important innovation is that now in the penultimate measure the fifth degree of the mode can be used if it is in the counterpoint rather than in c.f.

If the cantus firmus uses the seventh degree, then one of the counterpoints must use the second against it. The third voice could thus use neither seventh nor second - parallel octaves or unisons would result:

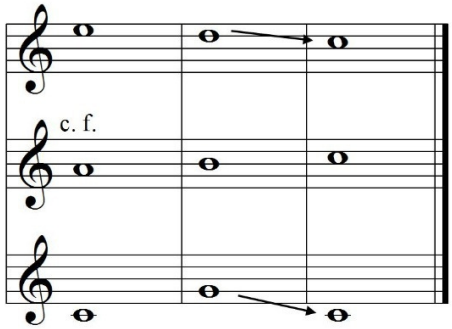
[3]Two leaps in one direction in one voice - on the horizontal - remain forbidden.

4.



There is an ambiguously taught exception to Rule III. It is sometimes forbidden to have a hidden octave or fifth in the last measure, in the cadence, as a direct consequence of Rule III.:

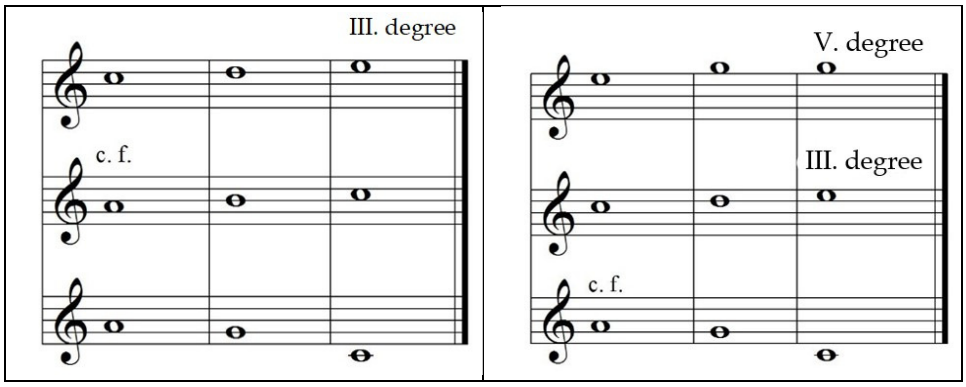
5.



Some teachers - for example, Jeppesen - allow it in their examples without explaining it. It occurs in the living music. Sometimes avoiding it is easy, sometimes not so much. But if it is used by Palestrina, then we can use it too.

IX. Sometimes the last tone of the counterpoint may not be the first degree. It may be third, less often fifth. But third and fifth degrees - together with first in the cantus firmus - can give a beautiful ending (shown in example 6). The presence of a fifth degree without a third is not desirable.

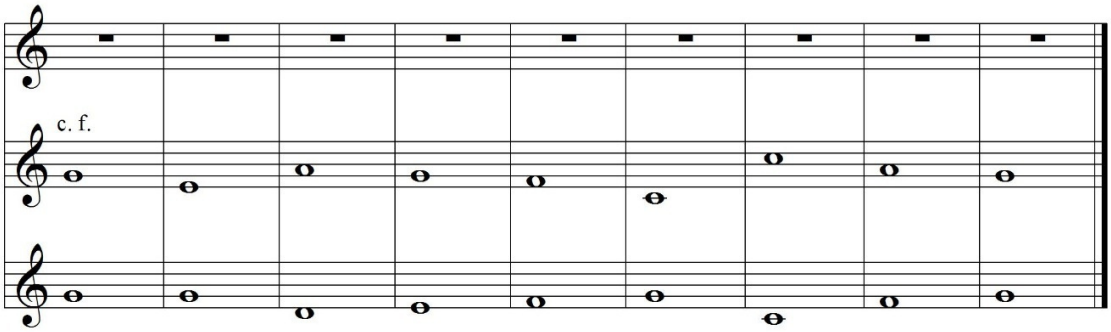
6.



Writing

If the c.f. is not in the lowest voice, I recommend that this voice be the next to be written. In the example below this has already been done:

7.



The first tone of the counterpoint has begun from the first degree, in unison with c.f.

The second tone is repeated, crossing with c.f.

The third tone has leapt downwards; c.f. has leapt upwards, in contrary motion.

With the exception of measures 1-2, I have used the contrary motion throughout. Only at the beginning is there lateral motion because the counterpoint is on the same tone. The contrary motion is a recommendation, not a requirement.

In m. 6-7 there are also contrary leaps - from fifth to octave on the vertical.

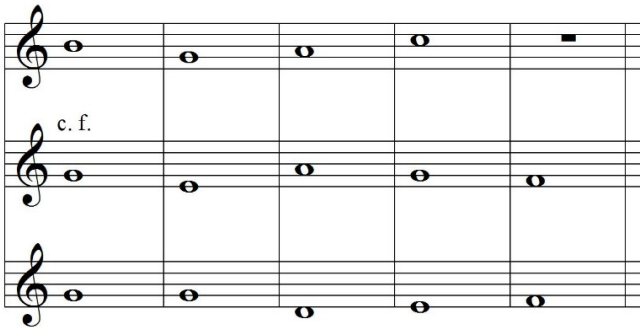
The penultimate tone of the counterpoint has occupied the seventh degree against the second in c.f.

I am starting to write the uppermost voice.

Each tone must be calculated to form consonance with the tones of the other two voices, observing all the rules for first polyphonic species, together with the rules for three-voice polyphony shown above.

The first four tones of the second counterpoint are shown below.

8.



In the uppermost voice, I have started with the third degree of the mode. This forms a third with both the lowest voice and c.f.

In the second measure I went to a position of third with c.f. and unison with the lowest voice. So the c.f. and the uppermost voice have leapt in the same direction, which is good - they're not outermost voices.

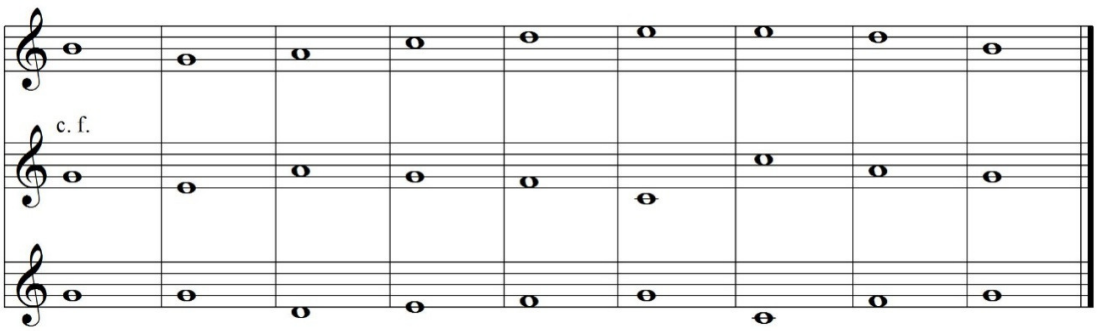
The third tone of the uppermost voice is in unison with c.f., and the fifth with the lowest voice is achieved in contrary motion.

The fourth tone forms a fifth with c.f. (allowed) and a sixth with the lowest - OK.

No voice has leapt out of or into dissonance (the fourth doesn't count).

I also have to follow all the rules for the horizontal. No dissonant leaps or tritone gaps, nor two leaps in one direction.

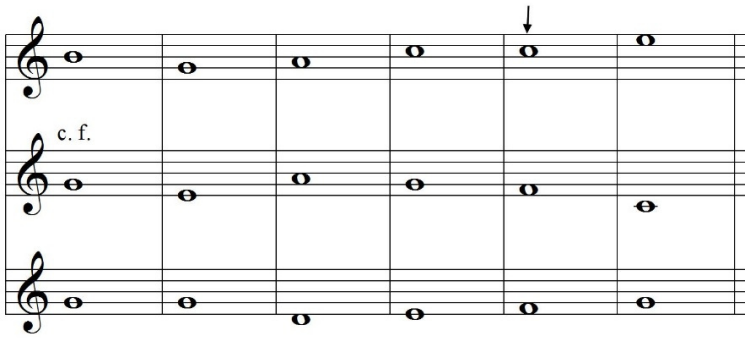
9. Solution to the end



In m. 5 there is again a fourth in the internal voices.
In m. 6 c.f. is temporarily the lowest voice - there is no fourth above it.
The last tone of the uppermost voice is the third degree of the mode.

Why couldn't I write C instead of D in m. 5:

10.



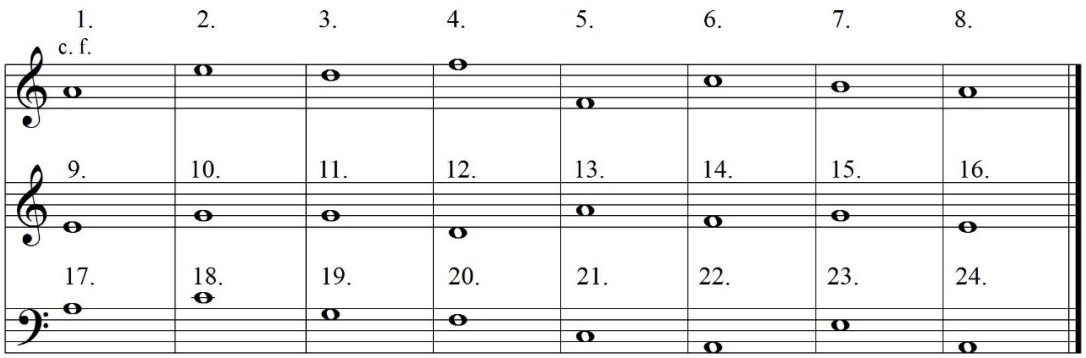
Thus it would appear that there are two leaps in one direction in the uppermost voice - measures 3-4-5-6. That there is the same tone in m. 4 and 5 doesn't change that fact. If the voice is not moving, then the two tones count as one.

Can I write G in the last measure of the uppermost voice? I can - and there would be no mistake. But if I write B, there would be no leap - here I have used the fact that I am allowed to write B in order to have a smoother motion.

Writing such an exercise shouldn't pose any particular problem. But there's an important procedural rule - after I have written everything, I will necessarily check the vertical between all pairs of voices - 1-2, 2-3, 1-3. Then I will check the horizontal of the two counterpoints, assuming that c.f. was correct. Only then I do consider my work to be completed.

Incorrect solutions

11.



The cantus firmus is correct

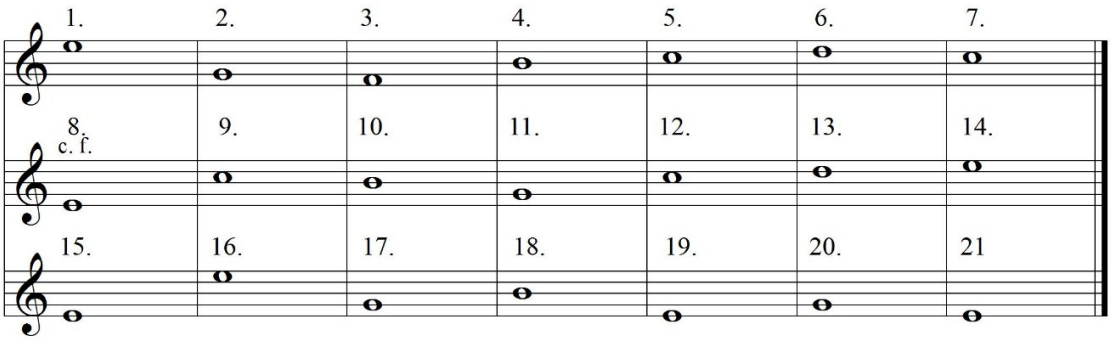
Vertical

A leap of both voices in one direction in outermost voices - notes 1.-2. and 17.-18.
Parallel fifths – notes 9.-10. and 17.-18.
Hidden fifth in the outermost voices – notes2.-3. and 18.-19.
Leap of the two voices in one direction in the outermost voices – notes 4.-5. and 20.-21.
A fourth with the lowest voice – between notes 5. and 21.
Hidden fifth in the cadenza (not severe) – notes 7.-8. and 23.-24.
Another "secondary" problem - in the last measure there is a fifth degree of the mode in the middle voice and no third degree anywhere. Not recommended.

Horizontal

Two leaps in one direction – notes 20.-21.-22.

12.



Vertical

A fourth with the lowest voice - between note 2, which is currently the lowest, and note 9.

Dissonances with both the c.f. and the lowest voice. Tritone between notes 3.-10. and a second between notes 3.-17.

The leap in one direction in the two voices - between notes 3-4 and 17-18 is not considered an error, because the lowest voice temporarily crosses with the middle voice.

Hidden fifth between outermost voices – notes 5.-6. and 19.-20.

There is nowhere a second degree (F) against the seventh of c.f. – notes 13[4].

There is no fifth degree also in the penultimate measure[5].

Parallel unisons – notes 5.-6 and 12.-13.

The last note of the upper voice – note 7. – is neither first nor third or fifth degree of the mode.

"Secondary" error - in the last measure the uppermost voice is below the middle voice – note 7 lies below note 14. This is insignificant.

Horizontal

A sixth leap downwards – notes 1.-2.

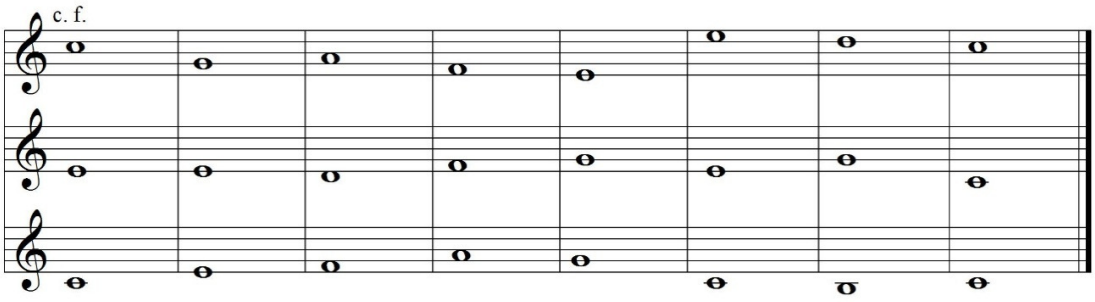
Tritone leap – notes 4.-5.

Sixth leap downwards – notes 16.-17.

Not a single stepwise motion in the counterpoint in the lowest voice. This I consider incorrect because there are many theoretical variants in stepwise motion; I don't seem to have the necessary skill to apply them.

Examples for analysis

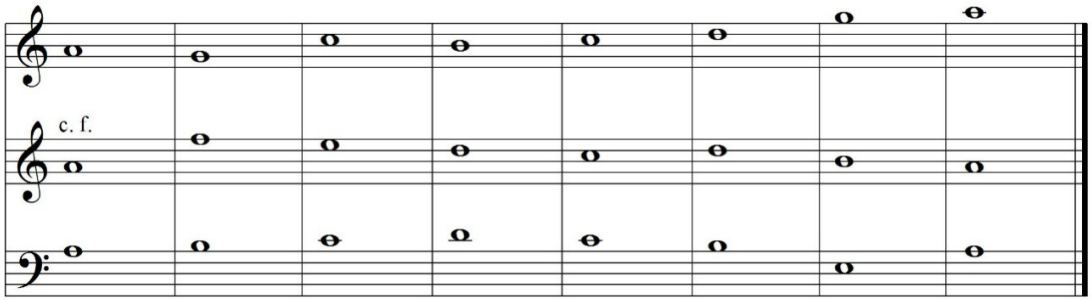
13.



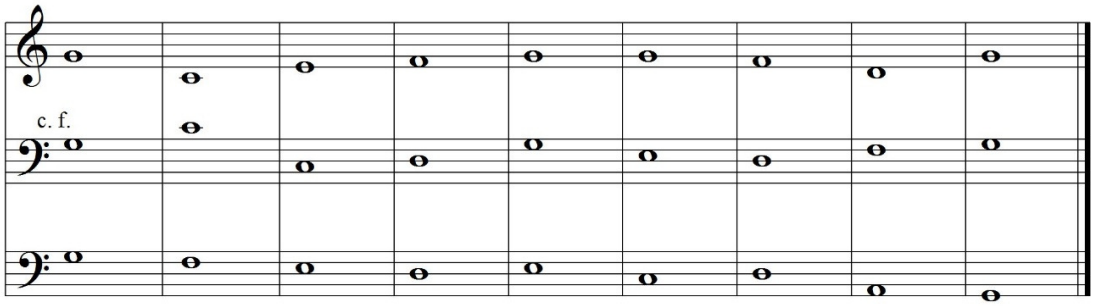
[4]Measures 6-7 represent a plagal cadence in thirds. It is characteristic of the Middle Ages (mainly 13th-14th century), but is now "obsolete" in the Renaissance. In other words, it is inappropriate for the strict style.

[5] In the Phrygian mode (which begins with E), the second and fifth degrees could not be used at the same time in the penultimate measure, because there is a tritone interval between them - F and B (can use ficta).

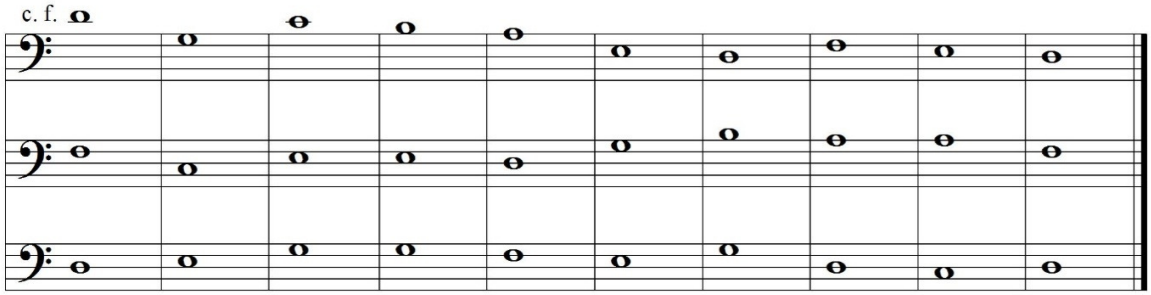
14.



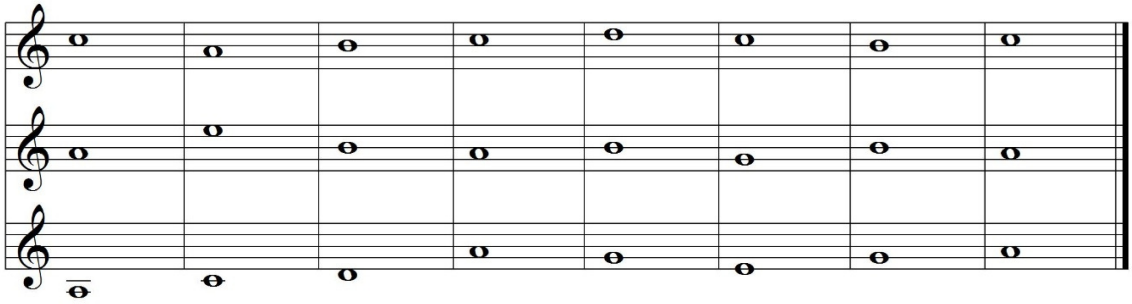
15.



16.

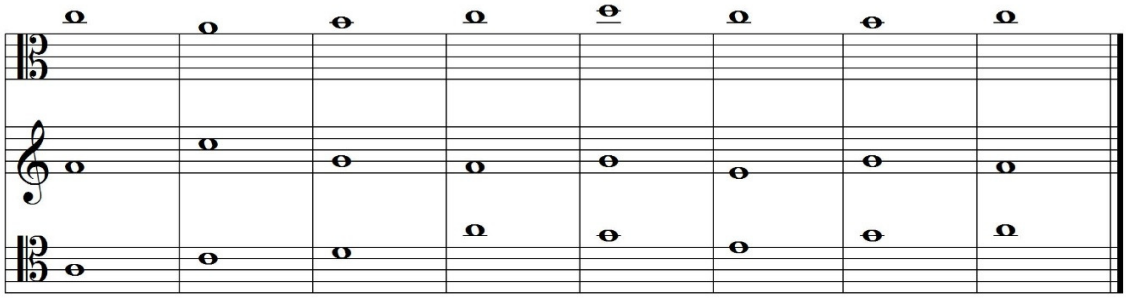


17[6].

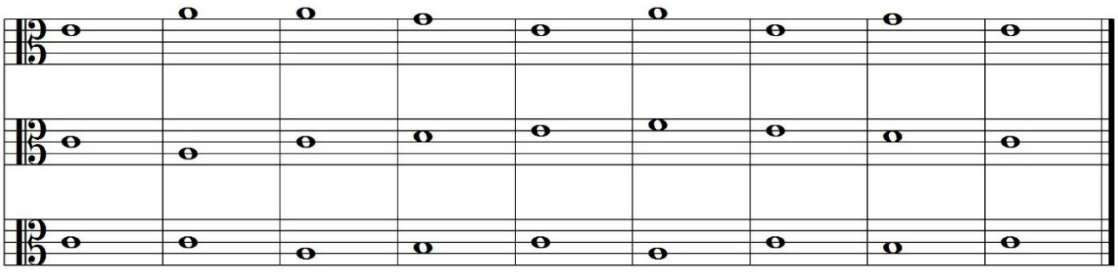


[6] From here to the end: at least one voice shall have the characteristics of a cantus firmus.

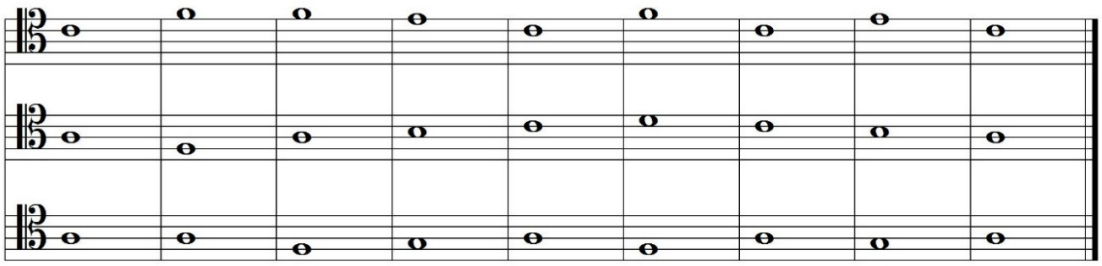
18.



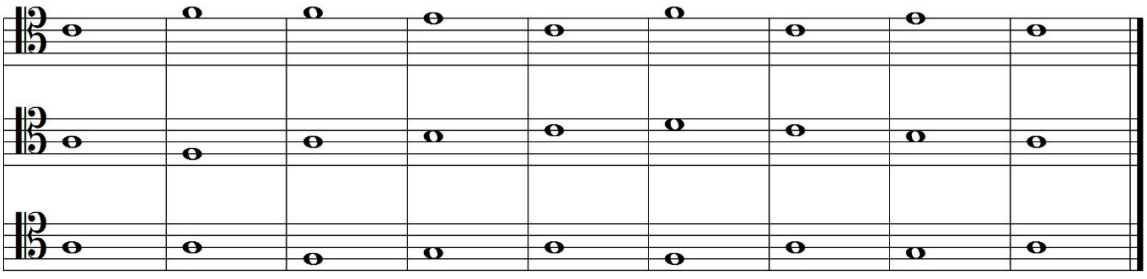
19.



20.Examples 19. and 20. seem somewhat similar. Are they correct?

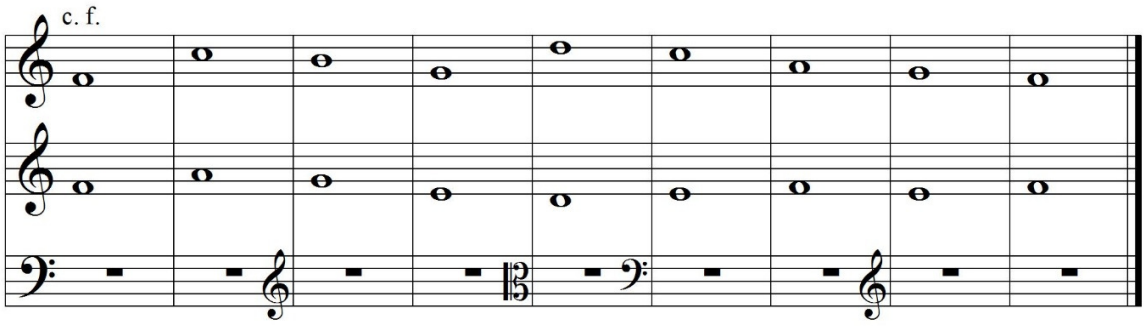


21.

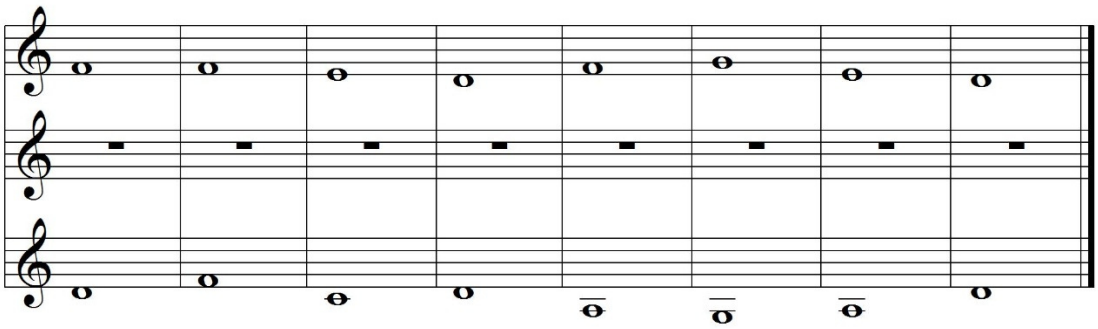


Exercises

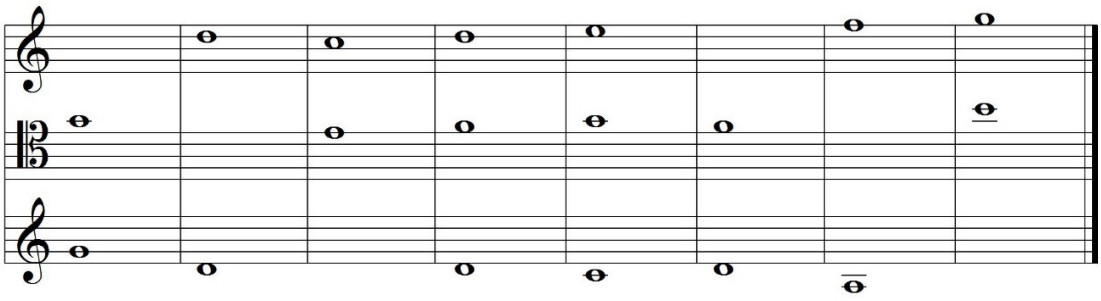
22.



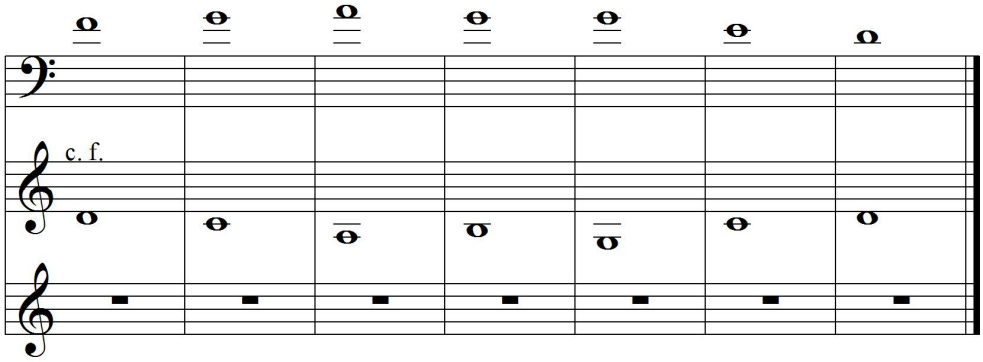
23.



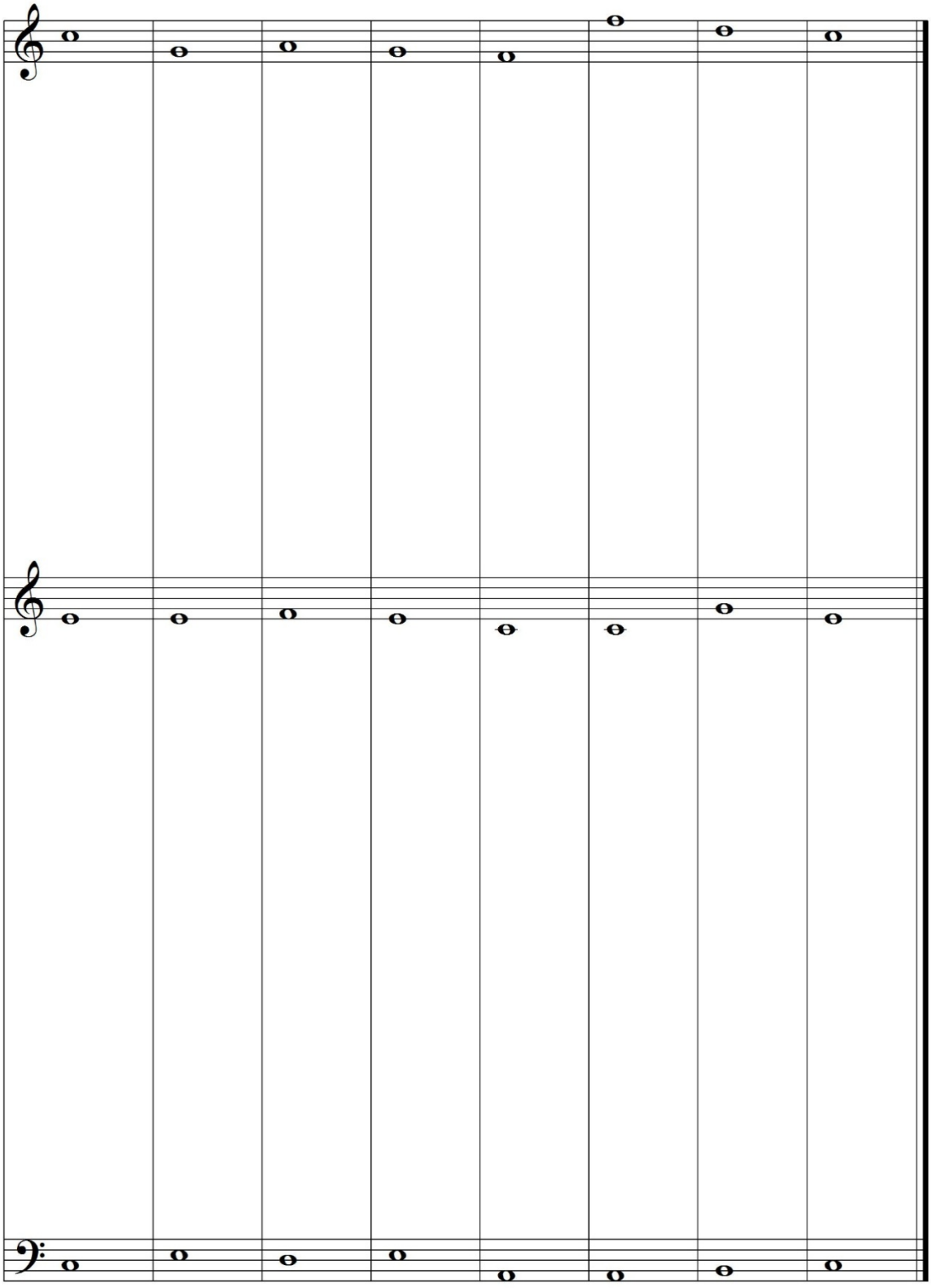
24.



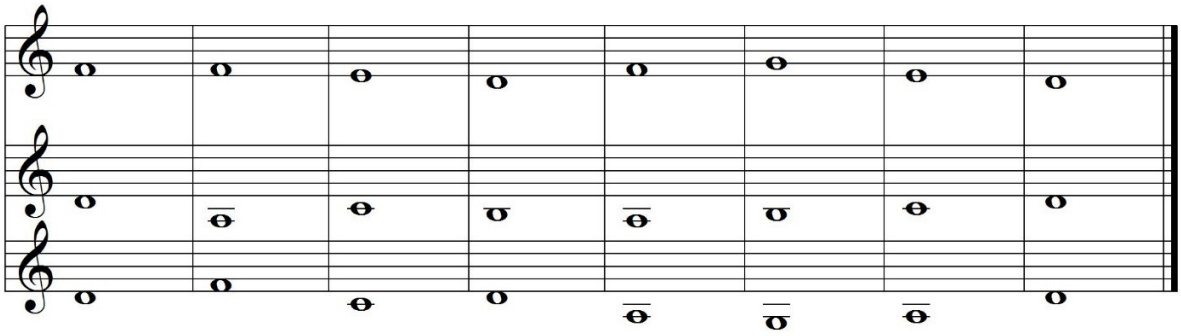
25.



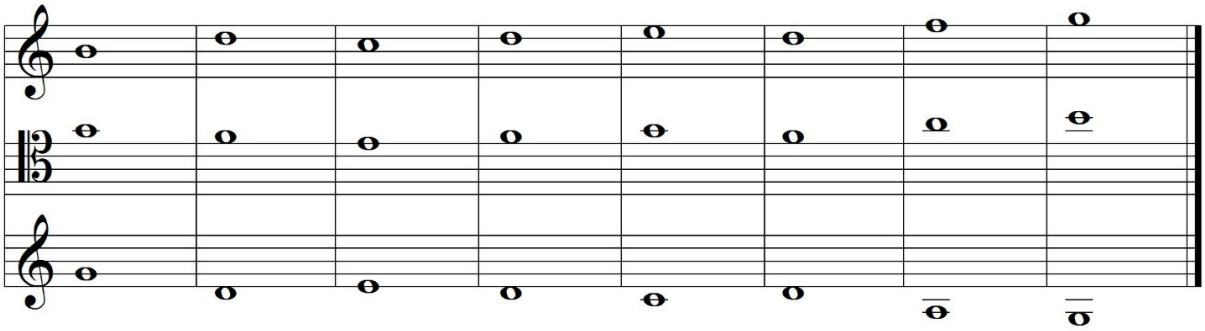
26. Maybe it's easier to read that way? Or on the contrary - maybe it's harder? Is the exercise correct?



27. Solution of example 23.



28. Solution of example 24.



Second Species

There are no new rules. No tone can be repeated in the half-note part.

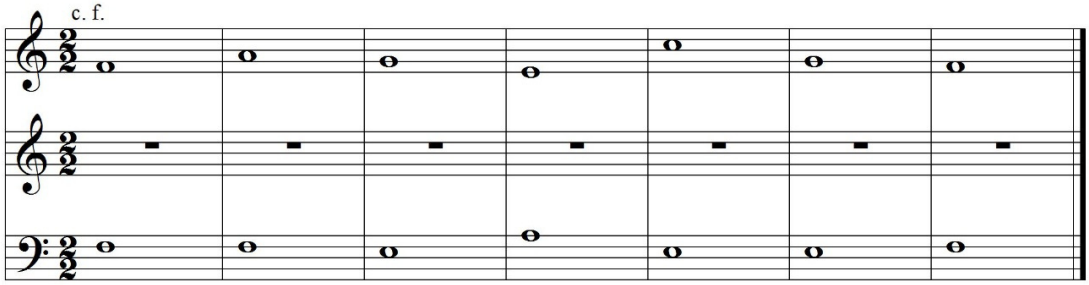
Exemplary solution

I purposely don't put the c.f. in the lowest voice. The first counterpoint will be at the bottom, and the second polyphonic species - in the middle.

It is important to note: it is not possible to change the c.f., but it is possible to do so with the counterpoints - even "post factum".

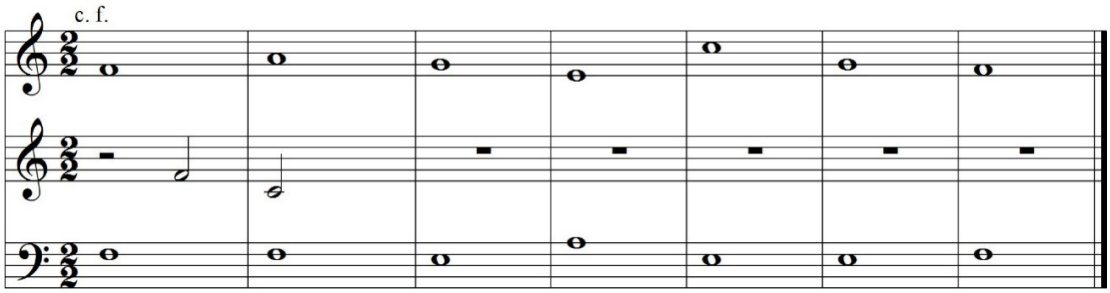
In the example below I have already written the lowest voice as a counterpoint to the c.f. I have no clue whether the counterpoint in the middle voice is possible - that remains to be seen.

1.



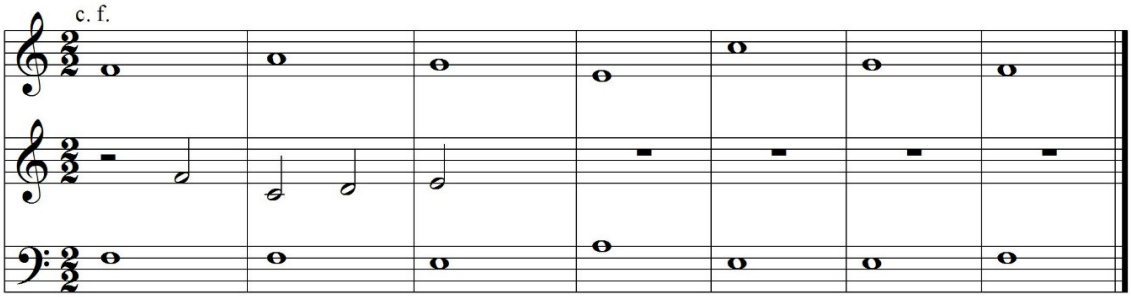
If I begin with a half pause and then an F, I can no longer write E (dissonance with the lower voice) or G (same with the upper voice). D or C can be used:

2.



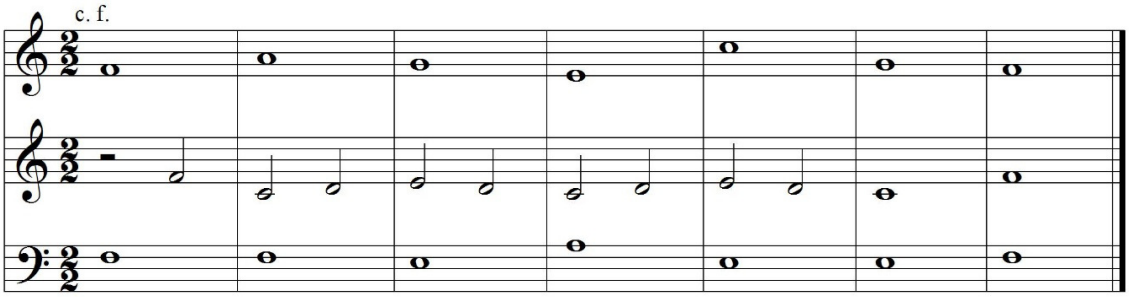
As I mentioned, it might be a D. I write it after the C. Then I logically add E - in the third measure it is in harmony with both the upper voice and the lower:

3.



It seems peculiar to me (but it is true) that the problem could be solved so smoothly:

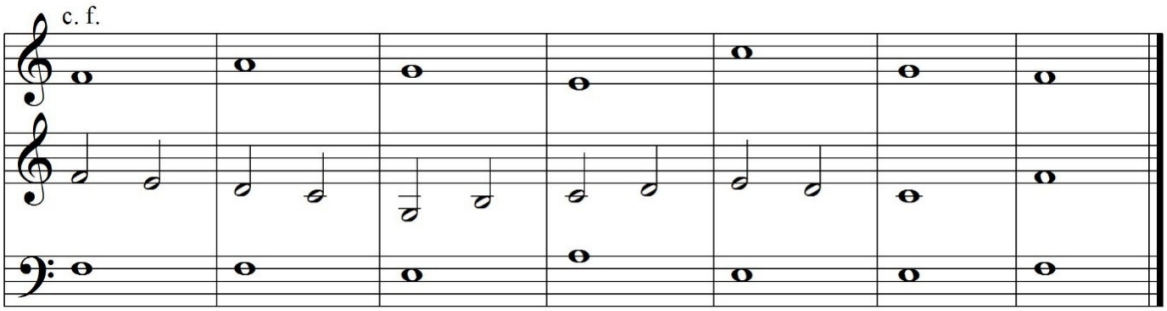
4.



Unfortunately, I'm breaking the rule about the ostinati - the middle line is definitely an ostinato. It is also a palindrome - it reads the same from left to right and right to left. Of course, I didn't plan anything like that. Anyway, this version is polyphonically correct, but it is not acceptable.

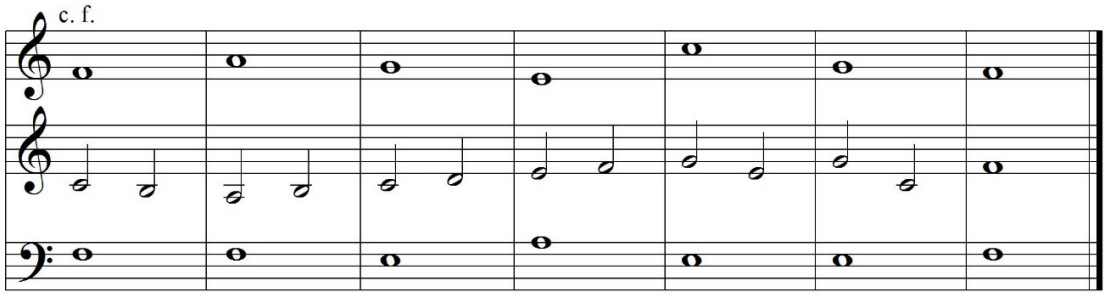
If there is no half pause, yet I still start again from F, the first half of the exercise could be varied as follows:

5.



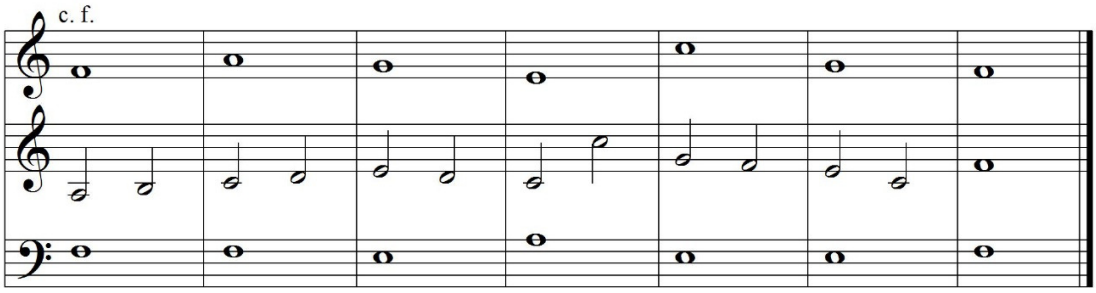
Here are also solution variants from C and from A:

6.



In the penultimate and last measures there are accented unisons between the upper and middle voices. The accented perfect unisons, fifths and octaves are a mistake, but in the polyphony they can sometimes be treated a little more liberally.

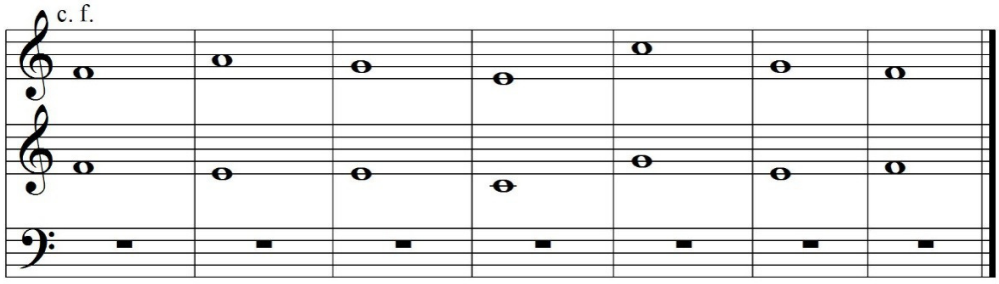
7.



Solution "from top to bottom"

I can try to solve the same problem in an altered order of both voices and actions: I will write a counterpoint in whole notes in the middle voice, and then write last the lowest one in half notes.

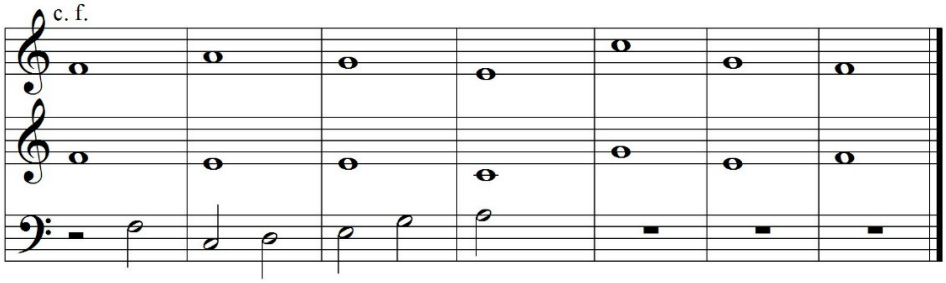
8.



I've deliberately allowed a fourth between the voices in two places - in order to see what would happen.

I begin the "pair" after a half pause. After the F I can't go stepwise to the first note of the second measure:

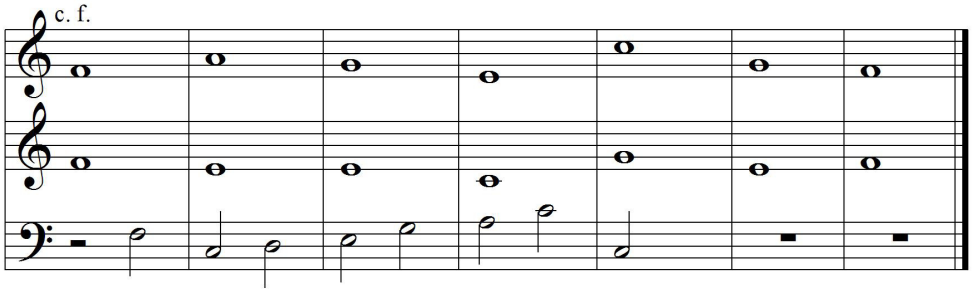
9.



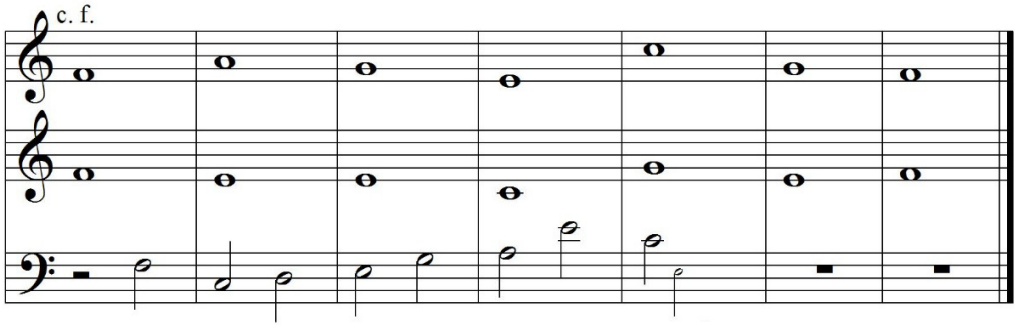
The fourth measure appears to be problematic.
After *A* I cannot write:
B upwards (second with the middle voice) - after it, a *C* should follow.
There would be a hidden octave with the upper voice. I cannot go back to A – then the B would be a lateral dissonance.
A – I cannot repeat the A.
G – forms a fourth with the middle voice and then I cannot go to F, because it will form a dissonance with the middle voice in the fifth measure.
F – I would leap into a dissonance with the upper voice, E.
E – I can write an E, but after it there is no solution. D is not possible in the fifth measure, it forms a dissonance with both voices. A second leap downwards is not allowed. I cannot leap from E upwards – the uppermost voice leaps upwards.
D – it would form a dissonance with both upper and middle voices.
C – the sixth leap downward is not allowed.
B – forbidden seventh leap and a dissonance with the middle voice.
A– the same situation as with the E. It might be written, but after that there is no solution.

However, there is still a solution - in fact, they are even two. Here is the first one:

10.



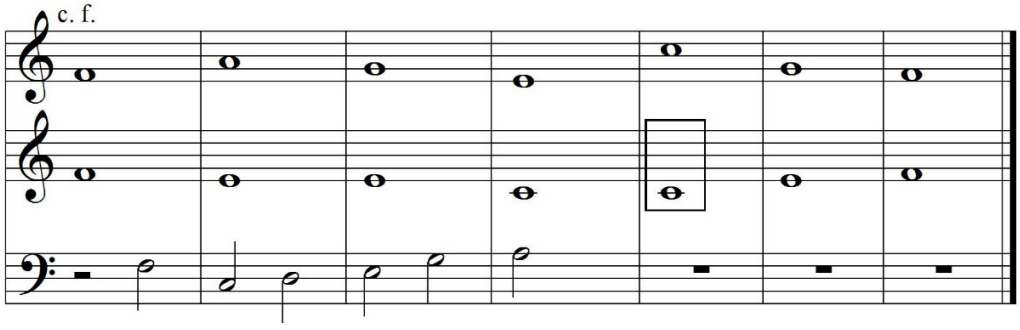
11. The second:



or:

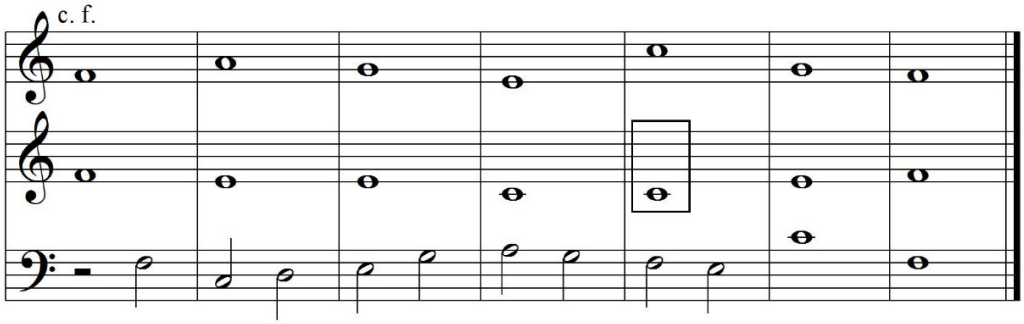
Instead of using these two variants, I present another strategy. In the fifth measure, I replace the G in the middle voice with a C (the other option would be F):

12.



This way I change the condition of the exercise with an unchanged c.f. and make my work easier:

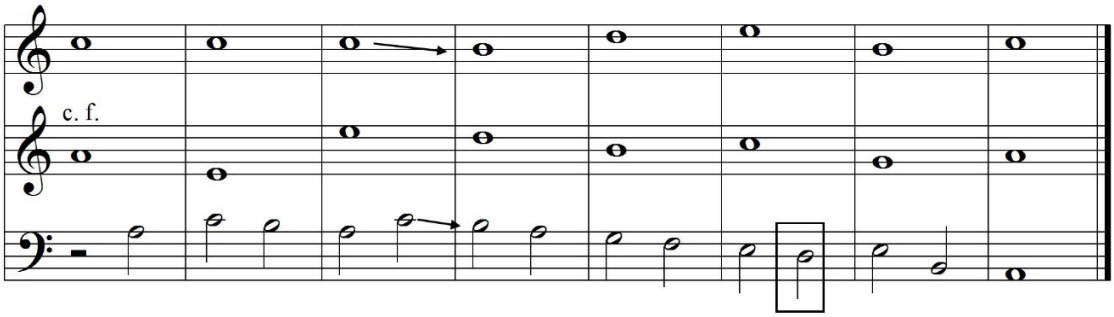
13.



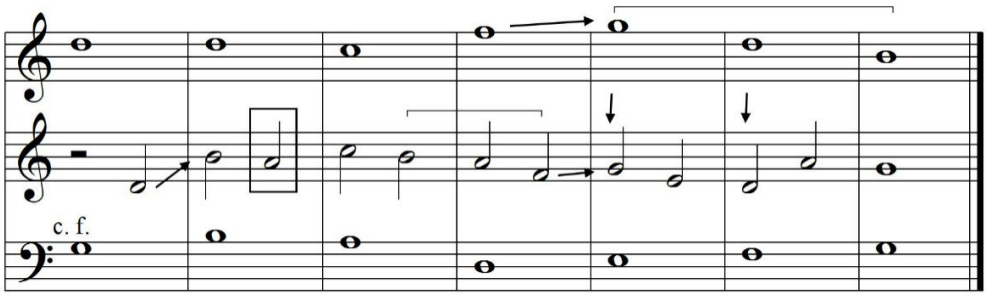
There are antiparallel fifths between the outer voices in m. 4-5-6. This is not a major issue. The fact that sometimes there is a fifth between the upper and middle voice practically changes nothing.

Incorrect solutions

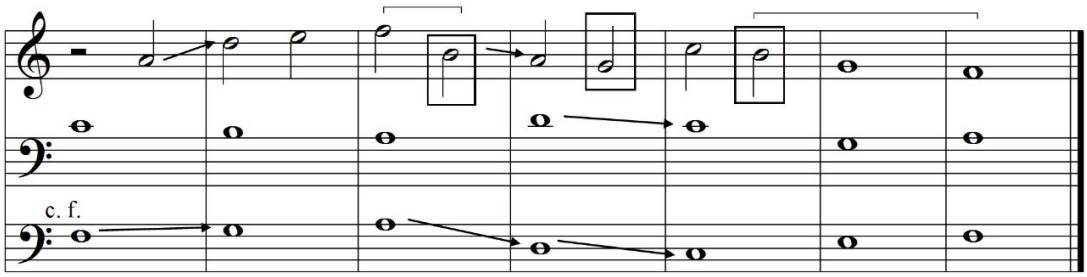
13.



14.



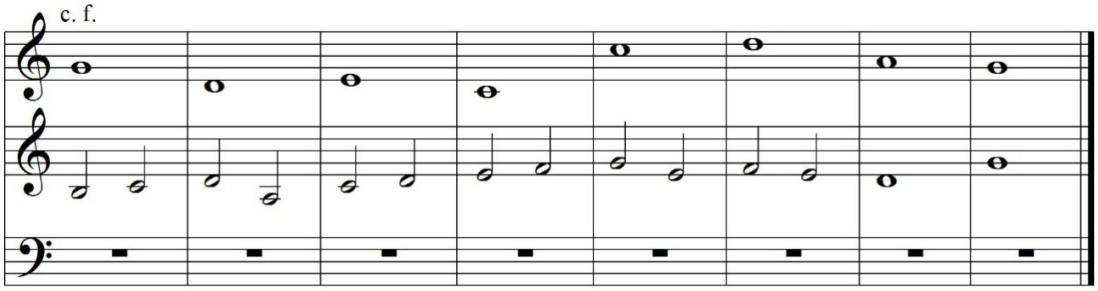
15.



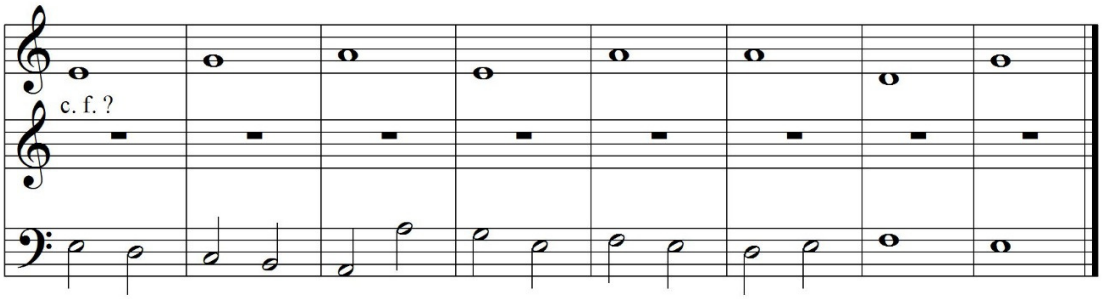
The arrows indicate hidden, parallel or accented intervals, as well as forbidden intervals horizontally. The rectangles indicate tones that dissonate incorrectly (not transient dissonances). The segments indicate incorrect intervals, two leaps in one direction, or tritone segments in the horizontal.

Exercises

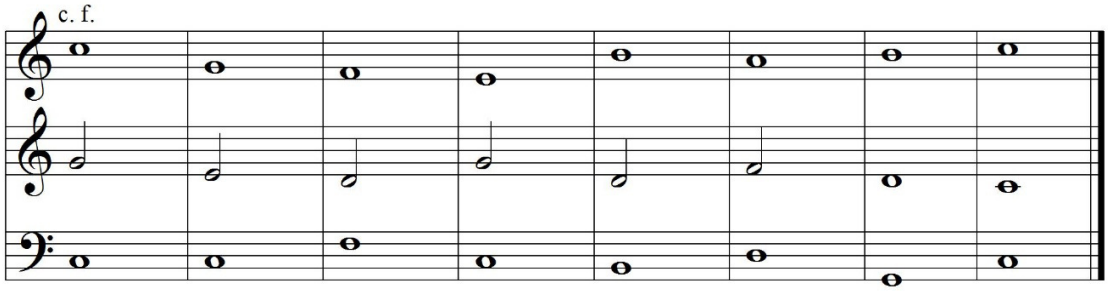
16.



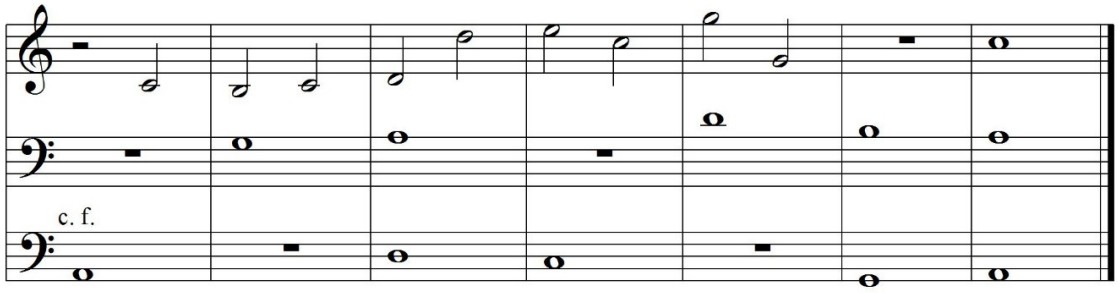
17.



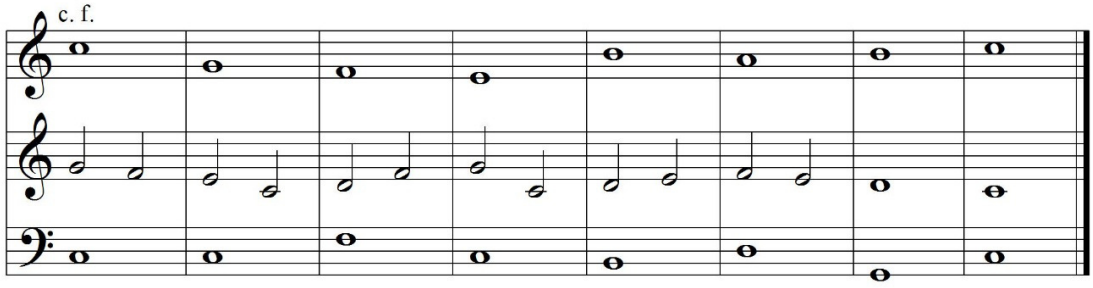
18.



19.



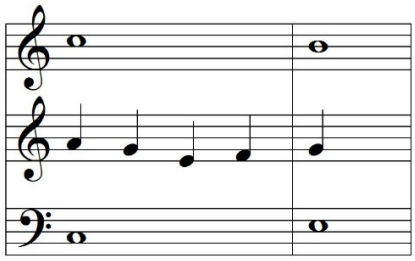
20. Solution of example 18



Third Species

It's very impressive if we are able to incorporate a cambiata into a three-voice polyphony:

1.



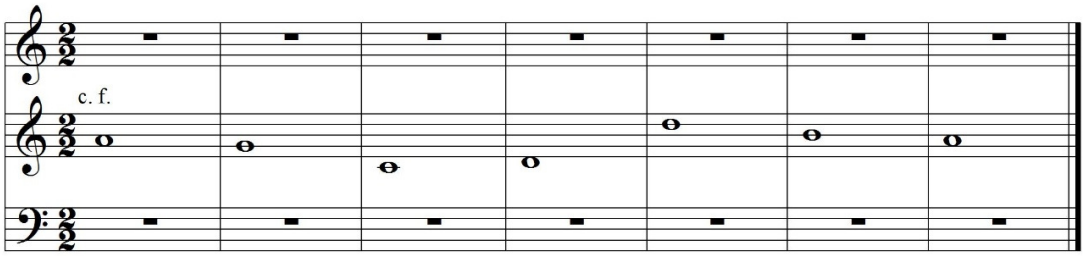
Example

The principle solution strategies are generally the following:

1. After the cantus firmus I write the counterpoint in whole notes.
    1. A. In the lowest voice, if the c.f. is no longer there
    1. B. In another voice
    Then I write the part in quarter notes.
2. After c.f. I write quarter notes.
    2. A. In the lowest voice
    2. B. In another voice
    Then I complete the counterpoint in whole notes.
3. Vertical approach: I write the exercise measure by measure on the vertical.

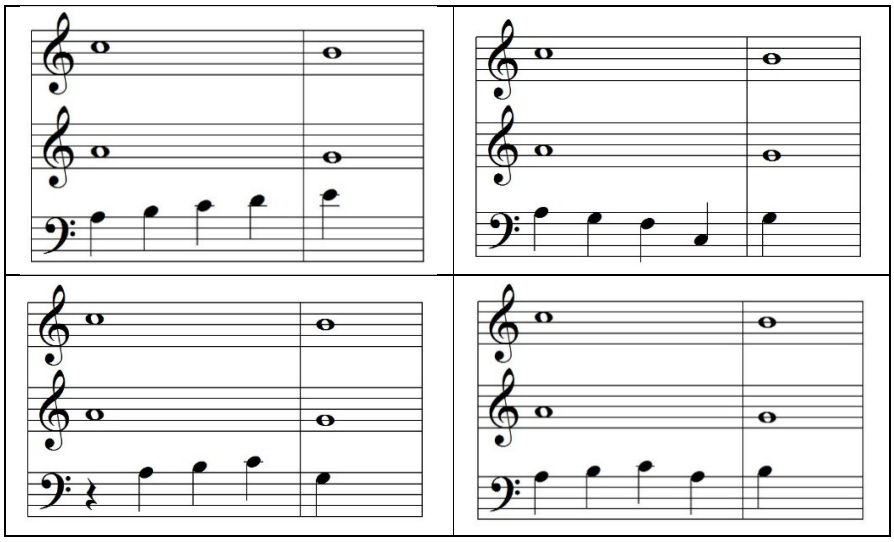
I have already shown to some extent how to do the first two methods. Now I will try to demonstrate the third approach.

2.



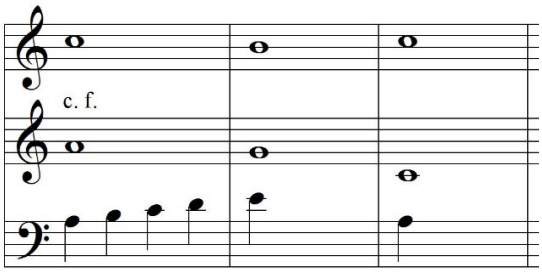
If I have marked the lowest voice for the third species (generally harder), I can place C in the uppermost voice in the first measure. Bearing in mind that a B may follow, I try to construct a motion in fourth notes from below. There are several options - not all possibilities are shown:

3.



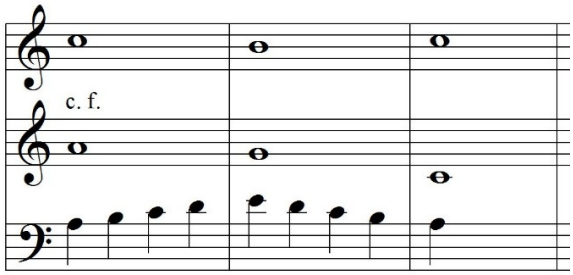
Choosing the first variant, I move on. Again I will add a tone in the uppermost voice and calculate the quarter note part by it.

4.



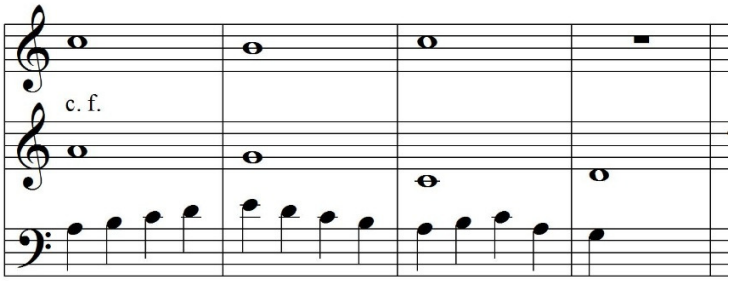
My strategy is as follows - provided that I can't change the c.f., I choose a stepwise motion at the top which can be counterpointed by a stepwise line at the bottom, i.e. I try to move as smoothly as possible.

5.



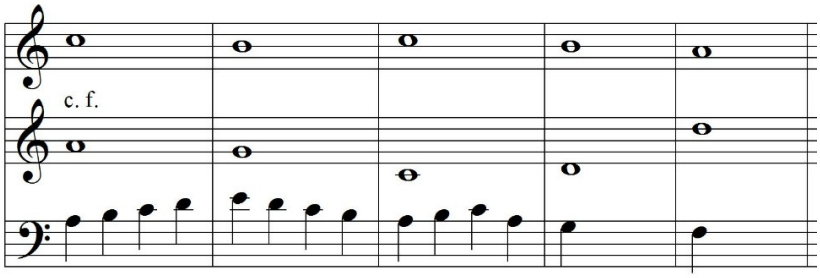
And now that I have "walked" stepwise in the lowest voice, I undertake a form of "turning" in m. 3:

6.



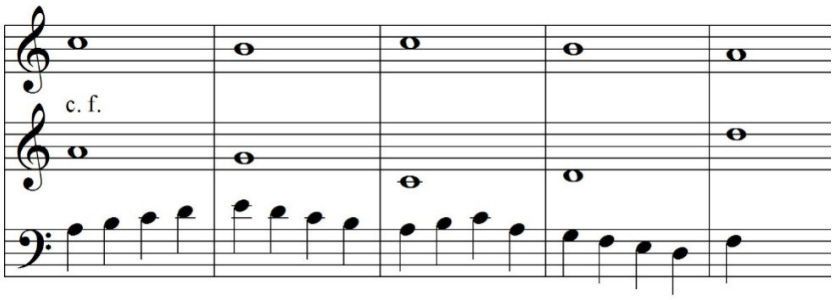
So the smoothest move at the top reappears to be the B. I write it and go on, adding after it an A. So I can mark an F at the bottom as the first note of the measure.

7.



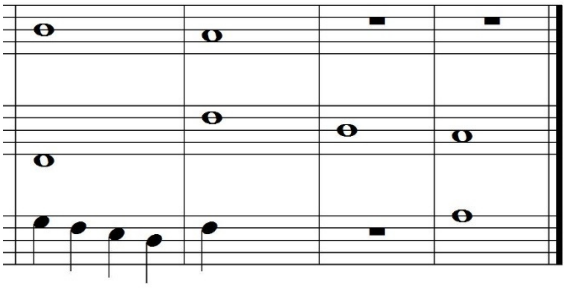
The remaining quarter notes can be arranged again in a straight line:

8.



And planning my finale, I place preliminary an A at the bottom of the last measure and make an analysis[1]:

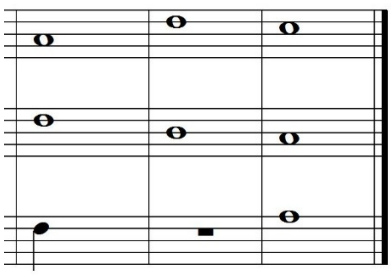
9.



If I insist on ending the uppermost voice above the middle voice (not obligatory), I might write a D in the penultimate measure. If I write B, then my last measure can't be A - parallel perfect unisons with c.f. would result. I can't write an F - it will form a tritone with c.f. G (upwards) doesn't work either - so there will be a seventh leap on the horizontal from A.

Then, having already decided to write a D, I then place C in the last measure. I can end on the third degree of the mode in counterpoint.

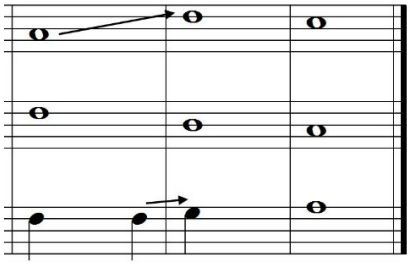
10.



[1]Examples 9-12 are written on the same clefs – treble for the upper and middle voice, bass for the lowest.

Here I feel I have set myself a little trap: if there is a G in the lowest voice in the penultimate measure, the fourth note immediately before the G can't be F:

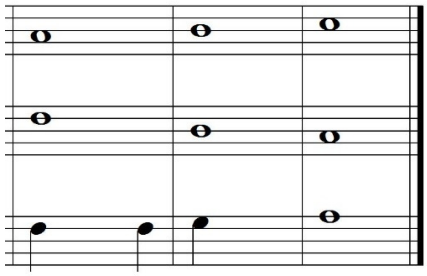
11.



Thus, in the penultimate measure I got a hidden fifth between the outermost voices.

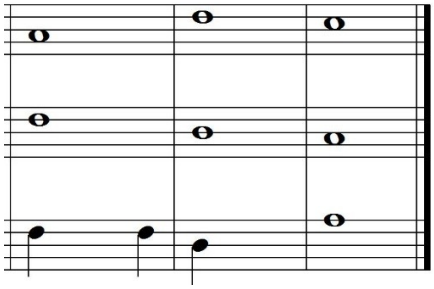
Possible resolution №1:
I don't write D at the top in the penultimate measure. Instead I place a B:

12.



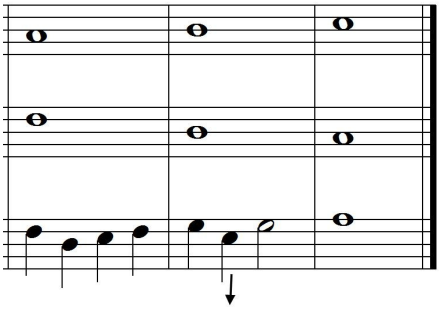
Resolution №2:
D stays in place at the top in the penultimate measure. Instead I write a D in the lowest voice:

13.



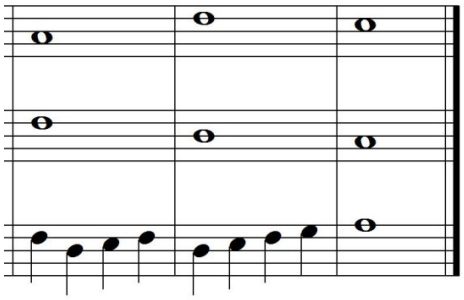
Here I show the resolution of the last three measures according to variant №1:

14.



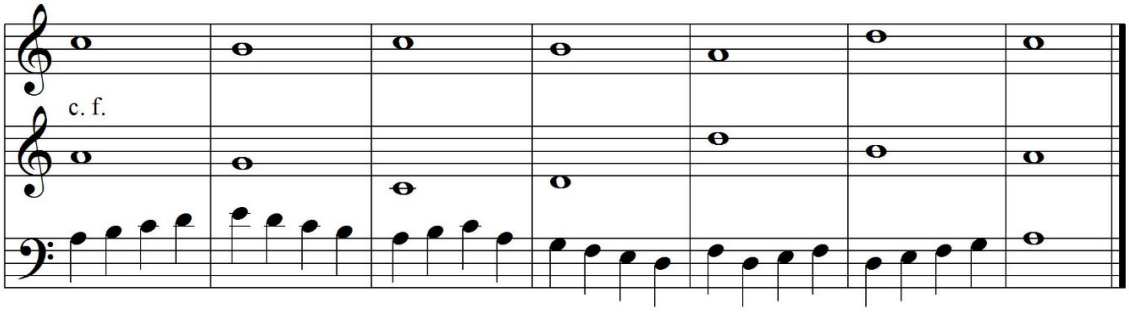
Permission according to variant № 2:

15.



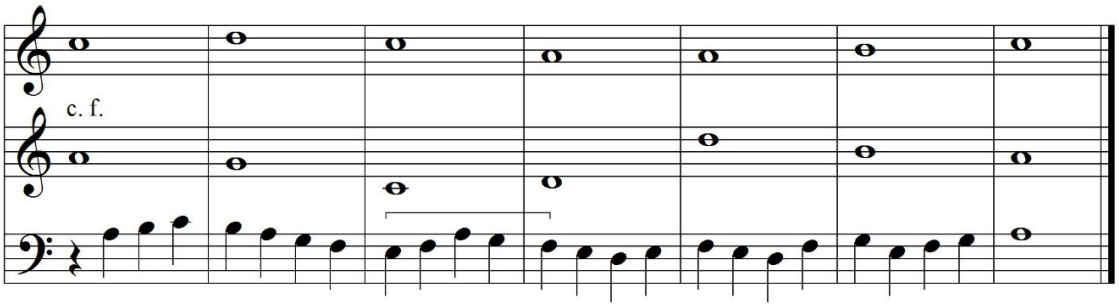
The whole exercise:

16.

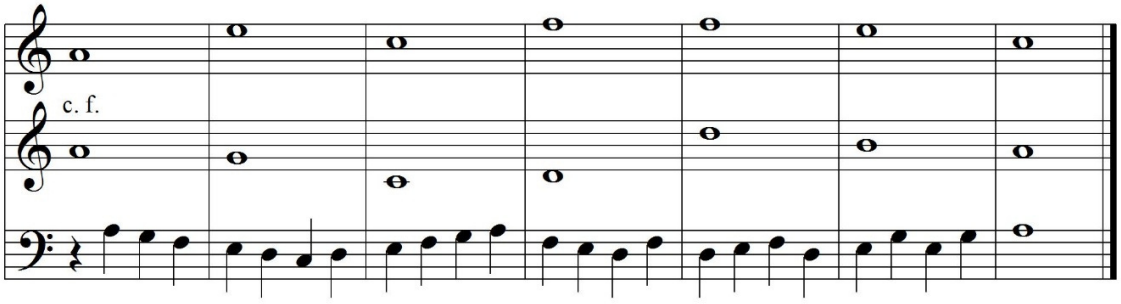


Another three possible solutions:

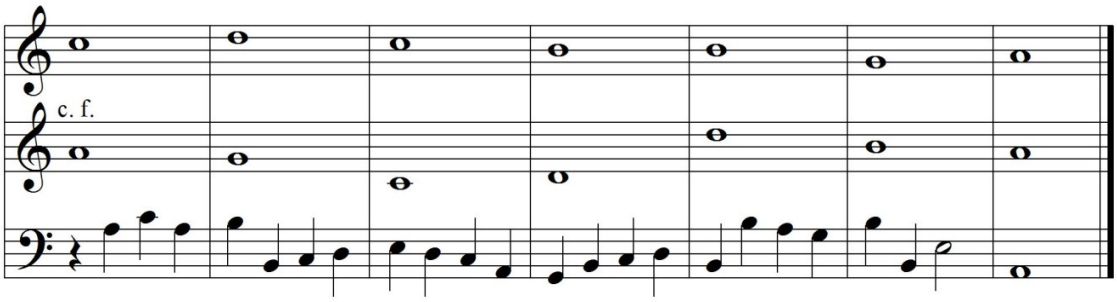
17. With a marked cambiata



18.

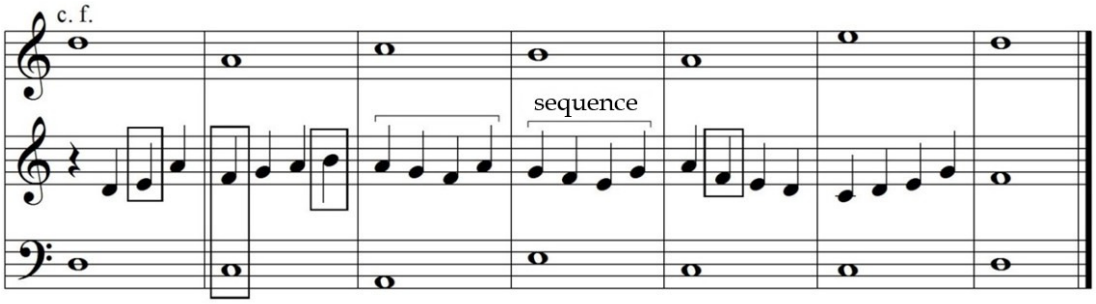


19.

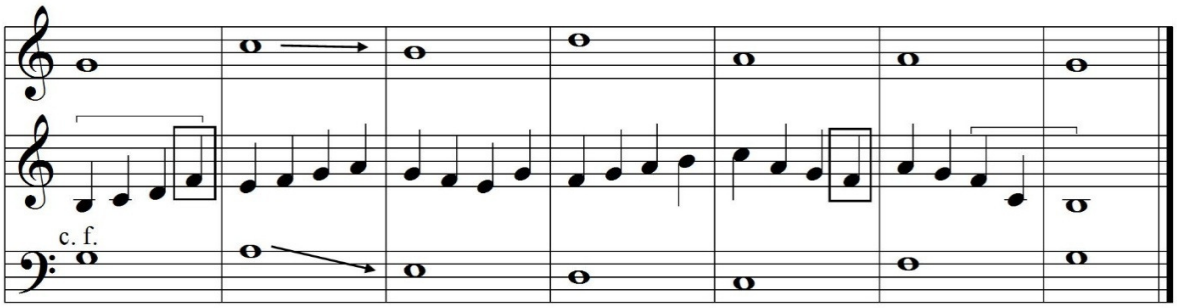


Incorrect examples

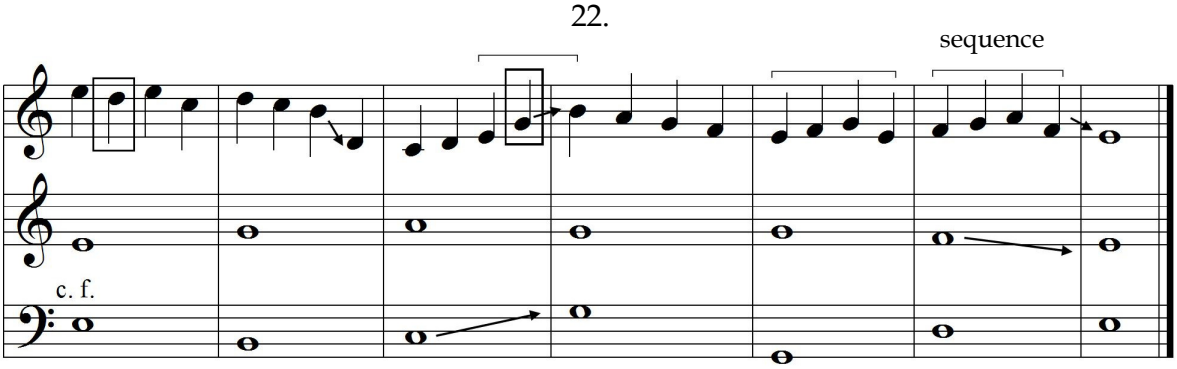
20.



21.

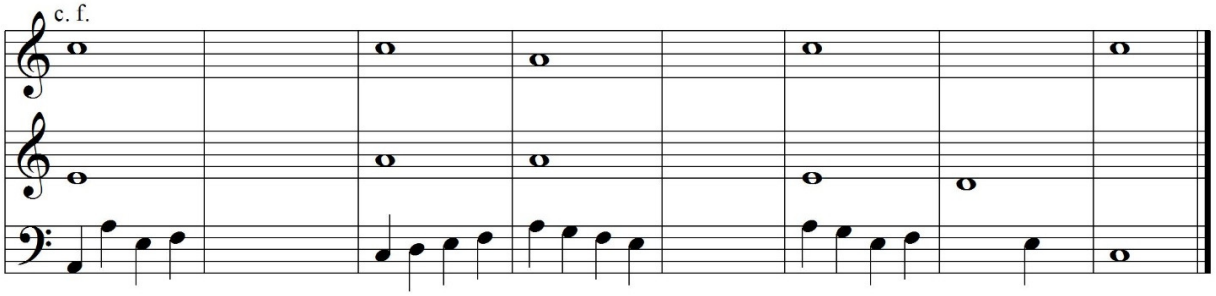


22.

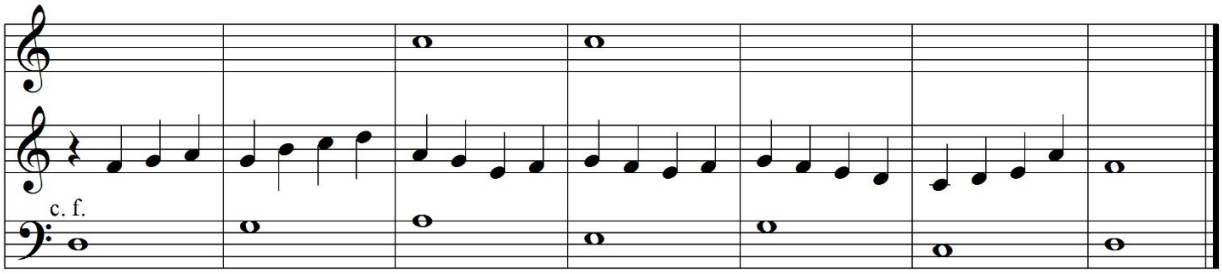


Exercises

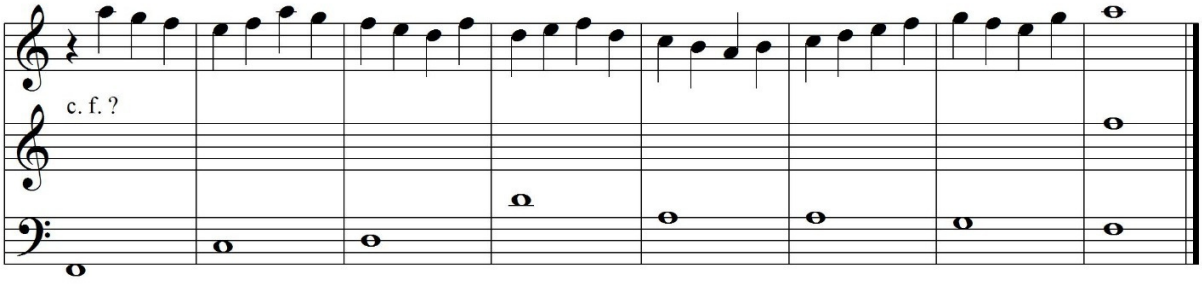
23.



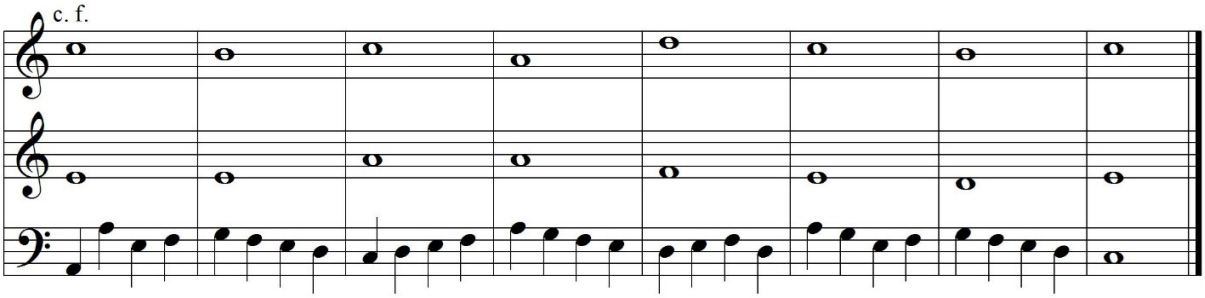
24.



25.



26. Solution of example 23.

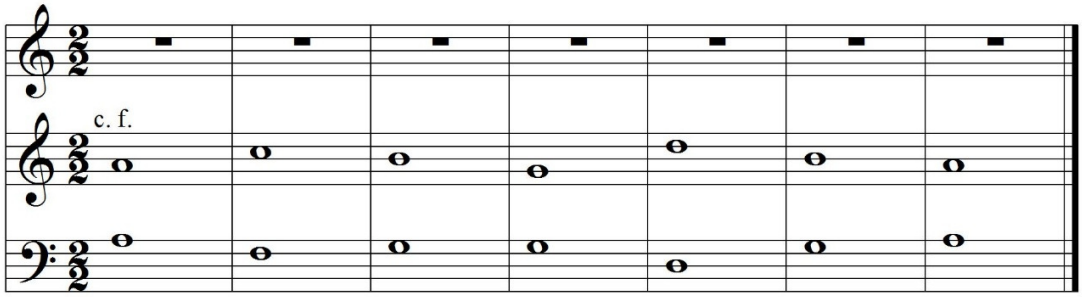


Fourth Species

It may seem difficult; in fact it isn't at all. However, here the "retrospective" changes might be a bit more.

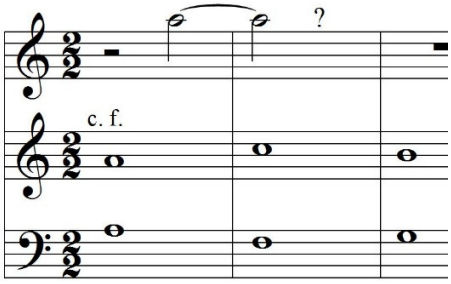
Starting the easier way, I first add a counterpoint in whole notes in the lowest voice. I don't try to plan anything in advance.

1.



If I start from A2 (to make sure that this is the uppermost voice), I find myself in a curious situation after the tie:

2.



There is no solution! I can't use B, D, G, or E on the second beat - there has to be a note on each second beat that consonants with the others below (but the fourth in three-voice polyphony is not prohibited if it's between the inner voices)[1].

I can't repeat A.

I can't write an F - parallel octaves are formed with the lowest voice[2].

I can't leap upwards to C3 - apart from going too high (a relatively minor problem), there are parallel octaves with the c.f.[3].

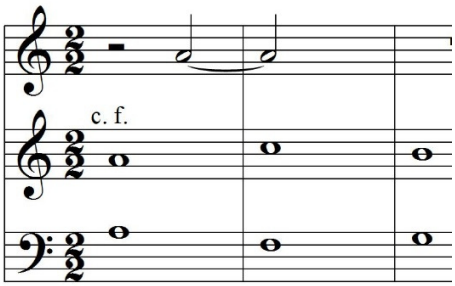
---

[1]May I also remind that I can neither resolve a dissonance upwards nor leap from a dissonance.
[2]Only the second beat of the fourth species is regarded in relation to the whole note in another voice.

I can't leap downwards to C - the descending sixth leap is prohibited.
I can't also leap to A1 – that way the outer voices are leaping downwards.

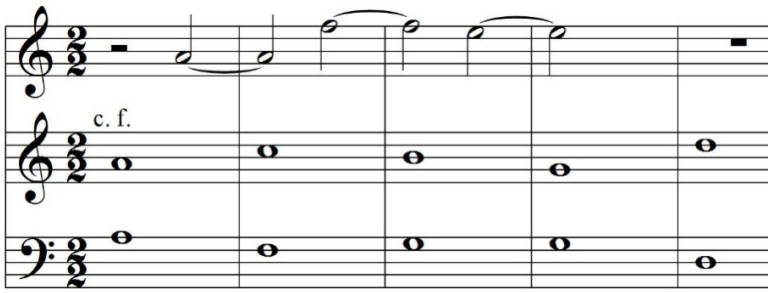
If I am in the mood for compromise, I would probably use the last option[4]. But instead I will now try to begin from A1:

3.



There are even two options here, not just one - F2 or A2. First, I will try to develop the variant with F.

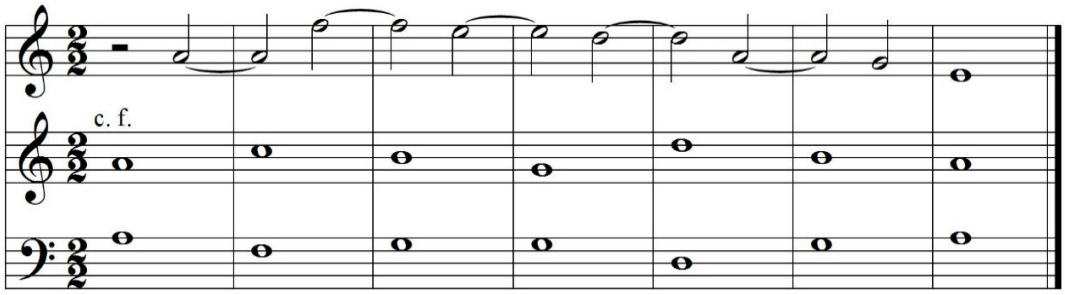
4.



It may seem curious, but the fourth between the upper and middle voice is now legit. This is a correct solution. The fourth is only considered "taboo" in a two-voice polyphony; in a three-voice polyphony it's only forbidden if it's formed with the "bass".
I continue further.

[3]By making the same check as in footnote 2.
[4]Although both voices formally leap in the same direction (by the logic of the fourth polyphonic species), the leap in the uppermost voice is a bit delayed. Therefore, I would use this variant. It occurs in the living music.

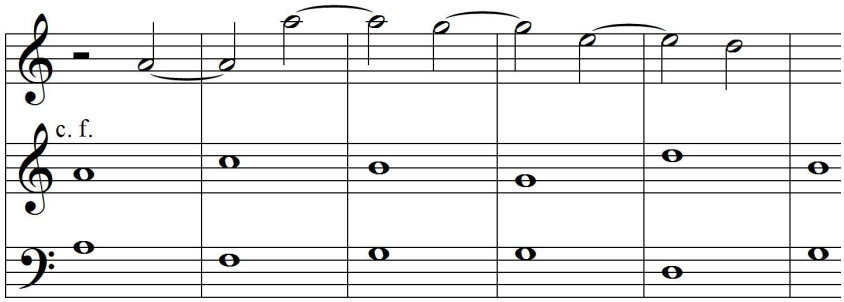
5.



In this solution, the uppermost voice ended up below the middle voice. The last note could be C2 as well without any error. In m. 5 there is a D in the uppermost voice, tied over from the previous measure. It forms a unison with the middle voice on the downbeat. I show this simply as a curious fact - there are no mistakes or any adverse consequences.

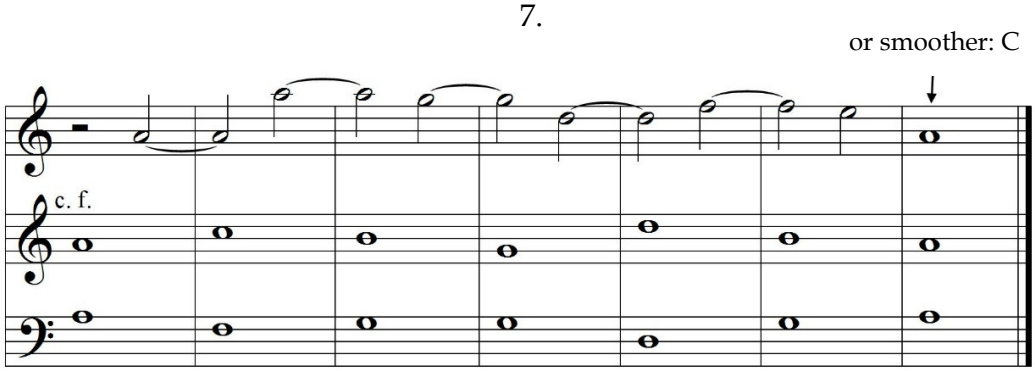
I now undertake an attempt to solve the exercise, using A2 in the uppermost voice from measure 2.

6.



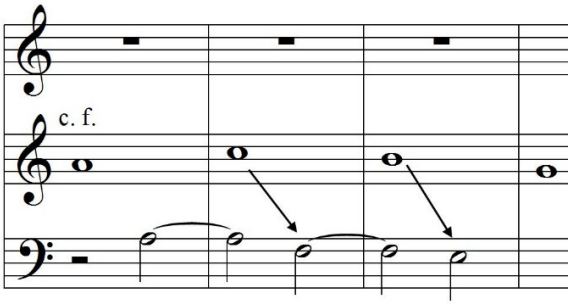
However, I have made a mistake here. There is a faux second in m. 5 - E in the uppermost voice has resolved with a stepwise motion downwards in D of the middle voice. I am correcting it. I am also trying to finish the exercise differently.

7.



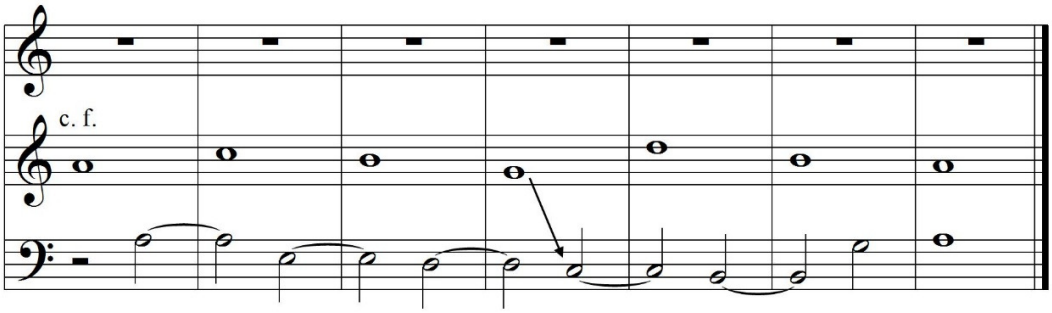
In the next solution, I will place the fourth species in the lowest voice, using the same c.f.

8.



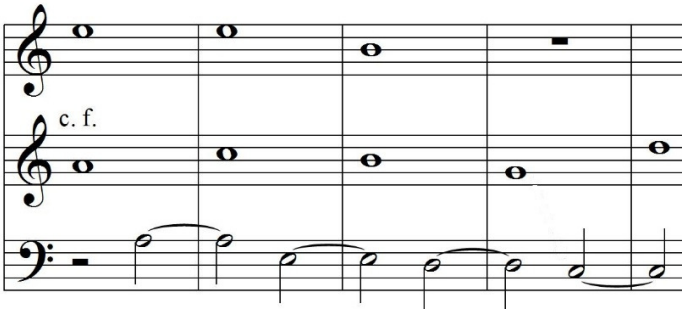
It doesn't look good that way - I have allowed parallel fifths.

9.



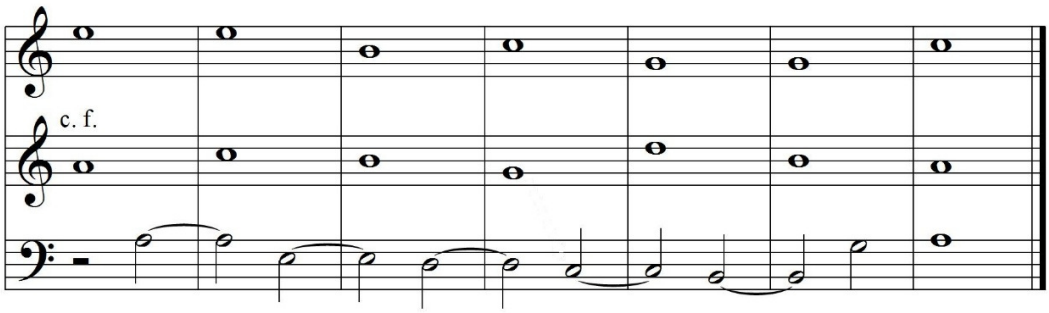
It's okay now. The marked hidden fifth is allowed in three-voice polyphony because it is not formed between the outermost voices.

10.



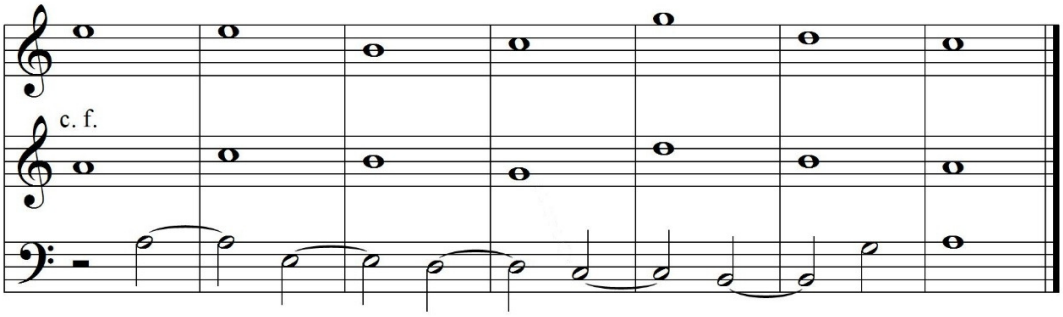
In the uppermost voice there is a B in the third measure. I can't write G - it will form a fourth with the lowest voice. D is also not possible - so would be formed parallel octaves with the lowest voice as well (m. 2-3).

11.



Another variant of completion - only the voice at the top is changed.

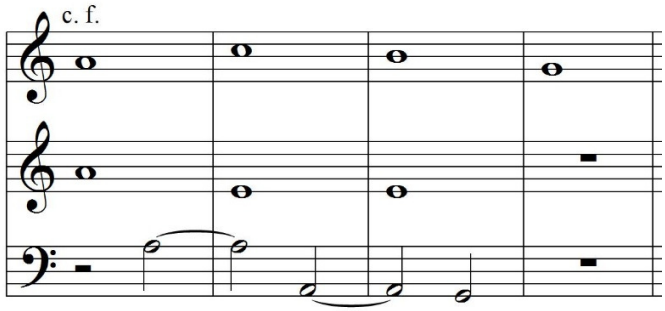
12.



For this species it is particularly valid to add smaller note values after larger ones, not the other way around. If I have written a cantus firmus, this rule is automatically followed. If I were to follow the irrational idea (e.g., when writing a contrast multi-voice polyphony) of starting with the fourth species and then adding the first to it, it would become very difficult, verging on impossible.

Solution with a "vertical approach" - c.f. is at the top.

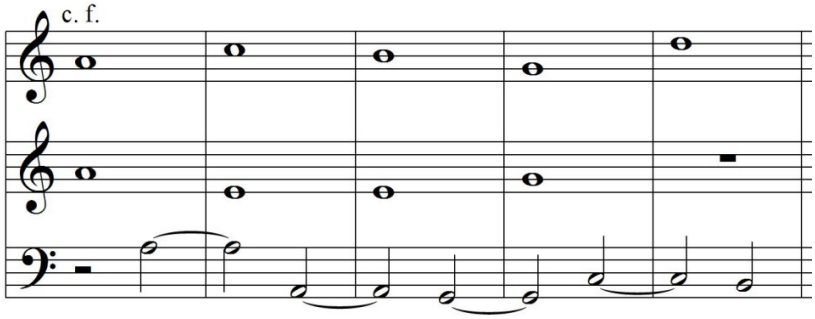
13.



The middle voice is more static so I can leap into contrary motion in the outer

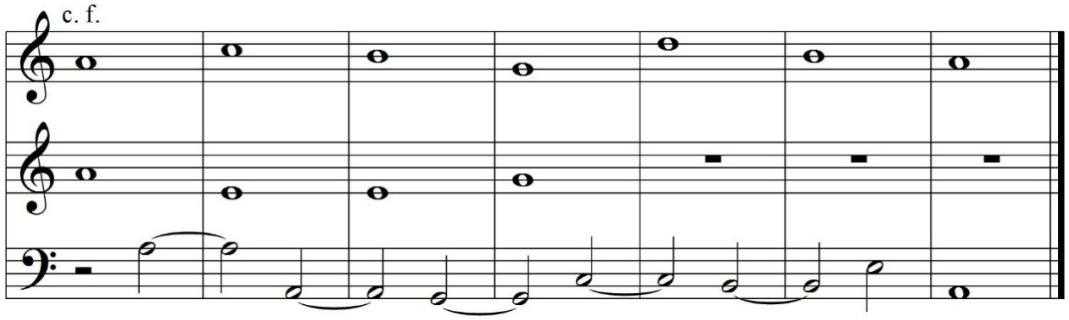
voices.

14.



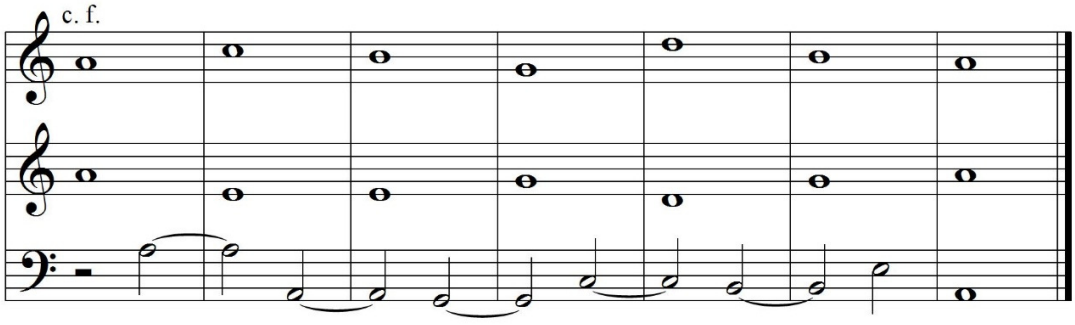
I continue to pay close attention to the outermost voices. When they are resolved, the middle voice doesn't present much of a difficulty.

15.



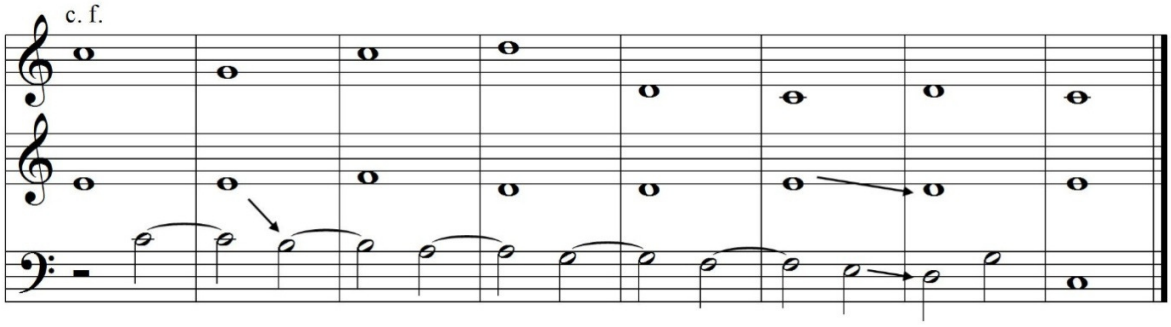
Last step:

16.

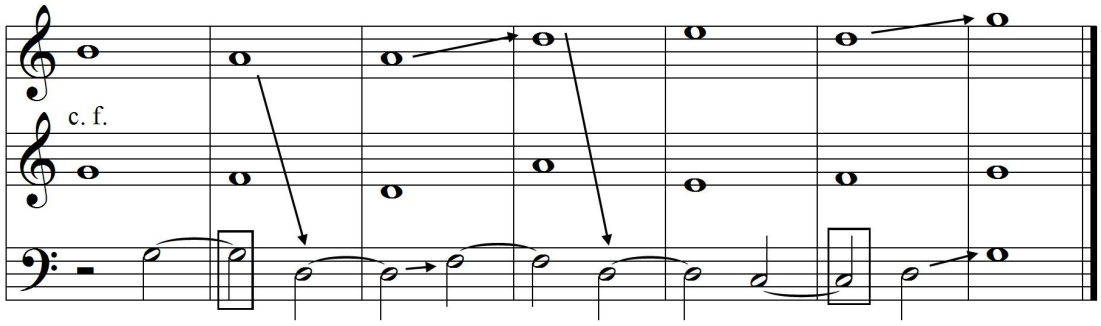


Errors

17.



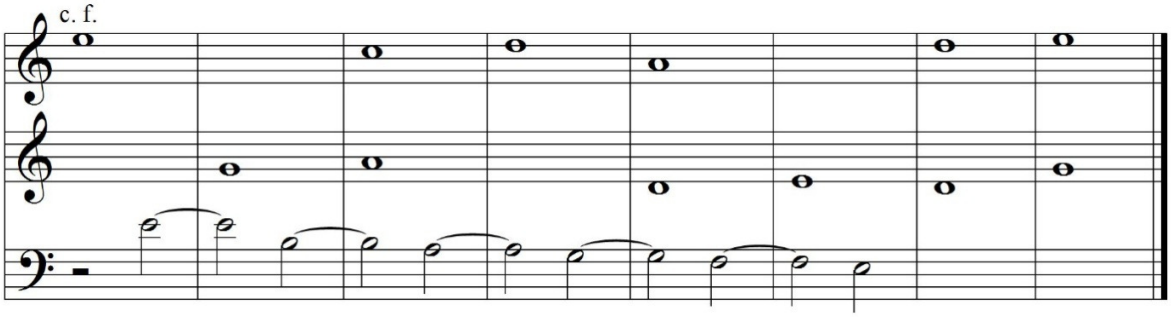
18.



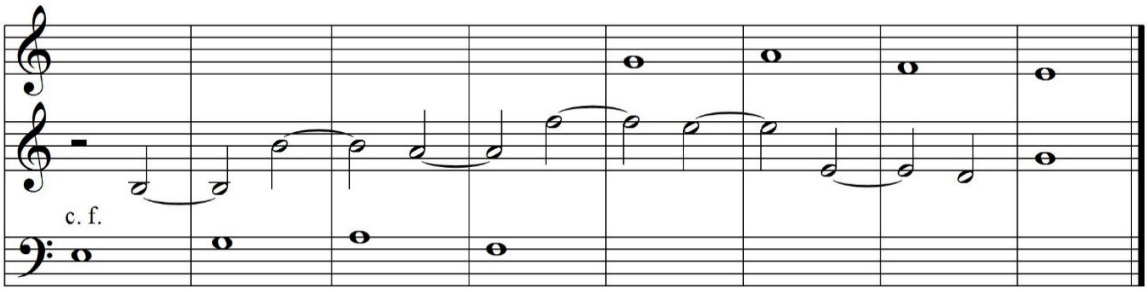
The arrows indicate hidden or parallel intervals and also a fourth with the lowest voice. The rectangle appears where I have leapt from a dissonance or have taken it upwards.

Exercises

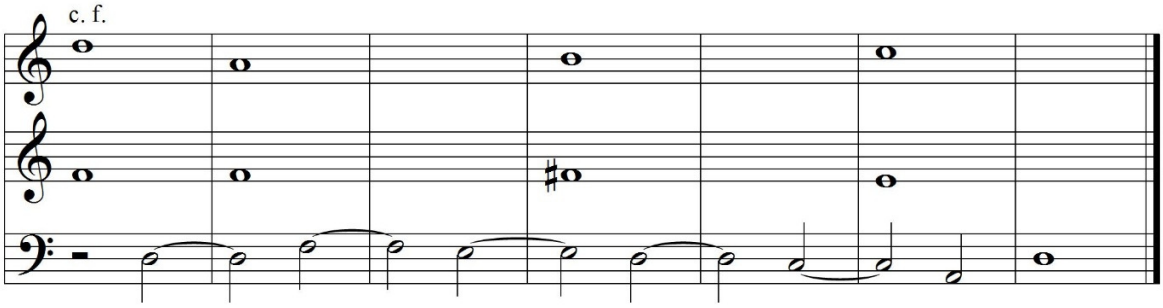
19.



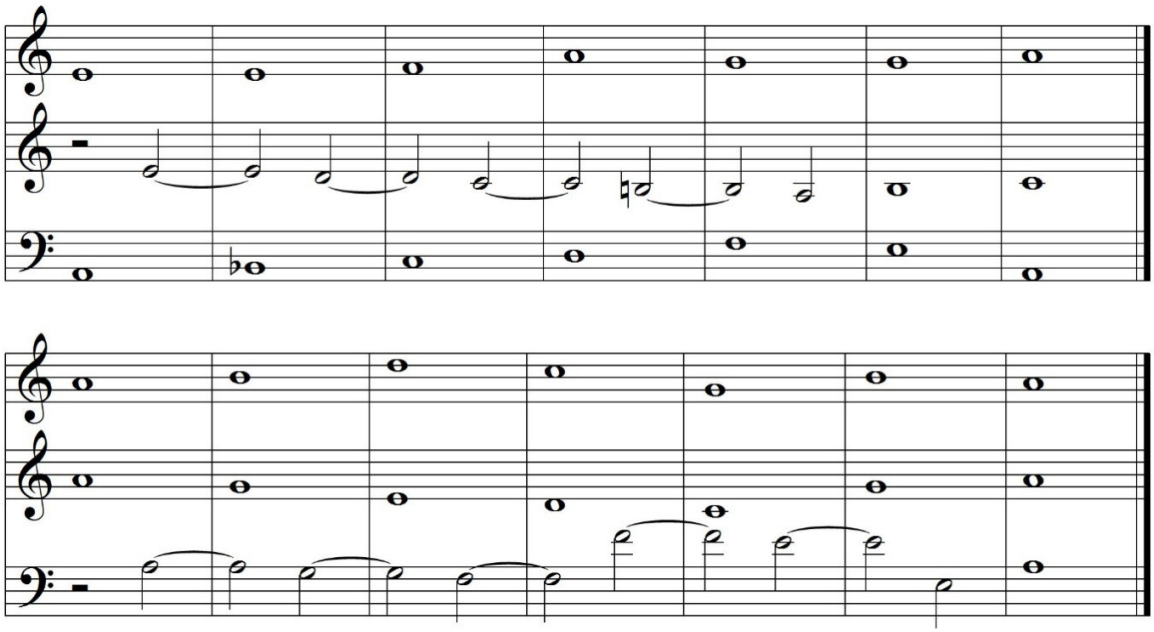
20.



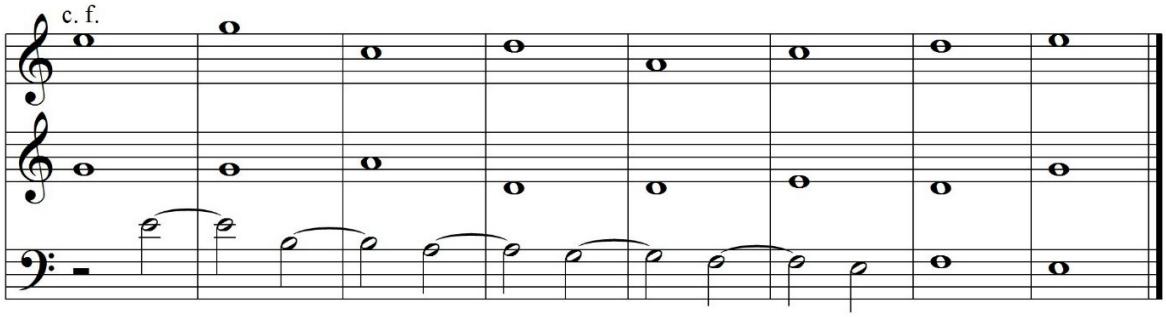
21. The ficta can sometimes be used. Here it is written,
to prevent a tritone between the upper and middle voice.



22. The following two examples have some relation to each other.
I am confident that they are correct.



23. Resolution of example 19.



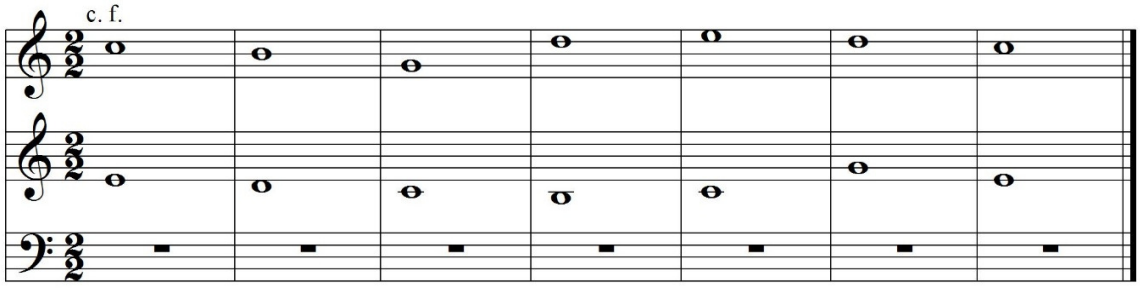
Regarding example 22. - maybe there are some exchanged voices?

Fifth Species

This is the easiest kind of three-voice polyphony. The presence of smaller note values as counterpoint gives multivariate solutions.

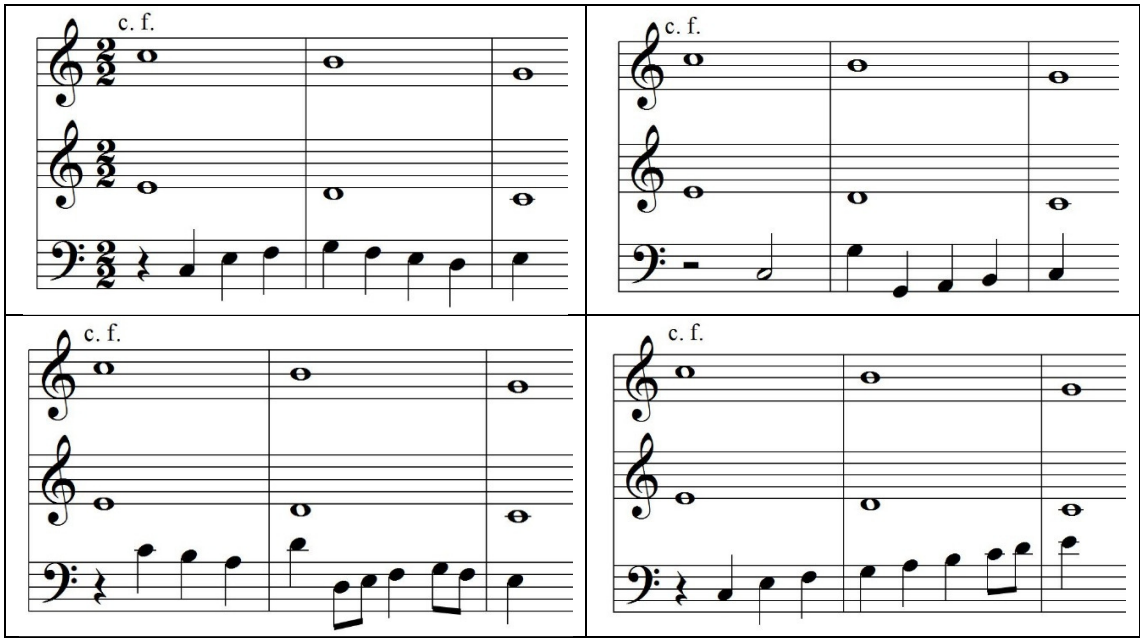
Perhaps the most interesting variant would be to write a florid melody in the lowest voice with the c.f. and another counterpoint already present. In the example shown below, these two components are already present.

1.



Here are some options to begin with:

2.

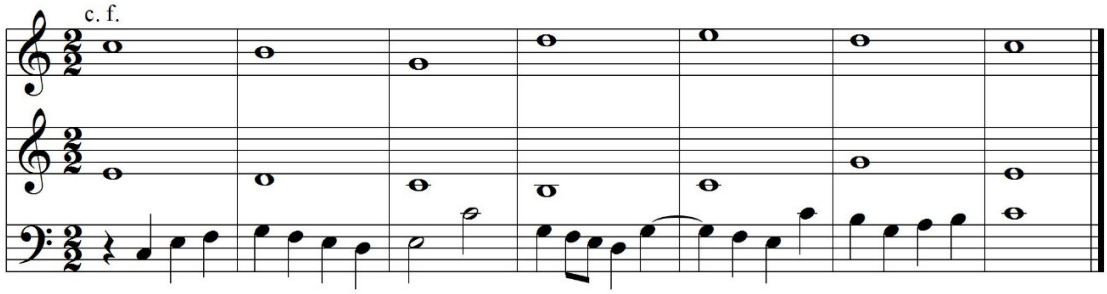


C and E are the only two variants for the first note in the lowest voice in measure 3[1].

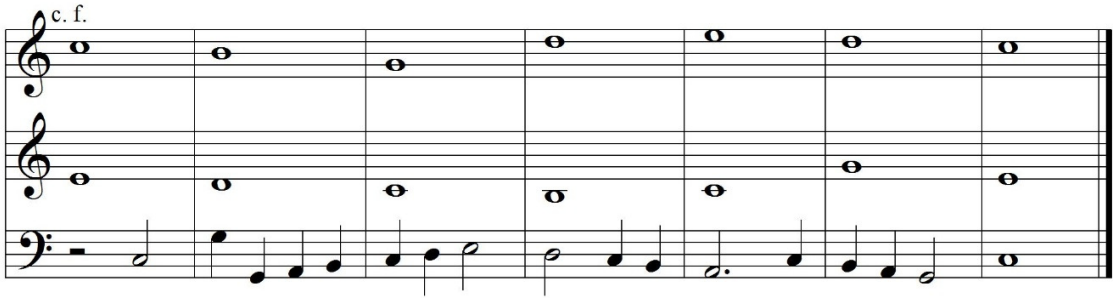
[1]In theory also G1 - a bit extreme, but possible.

Here is a complete solution to each variant:

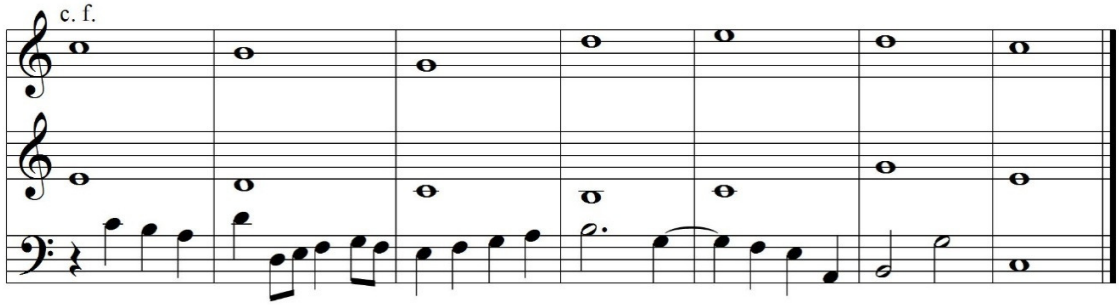
3.



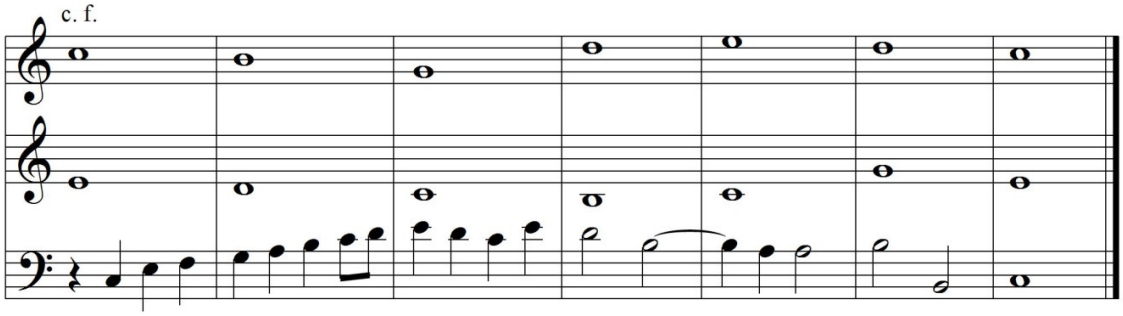
4.



5.

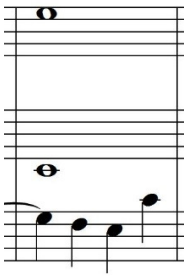


6.



Regarding example 3, measure 5:

7.

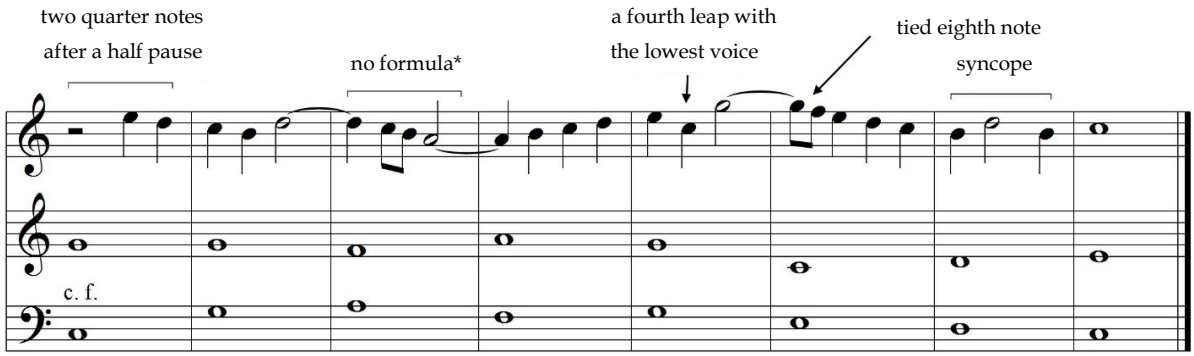


This is an interesting example. There are different meanings of the same note here. For example, G in the lowest voice is a suspended dissonance in relation to the middle voice, C. But G appears as a consonant in relation to E in the uppermost voice.

The F that follows below is consonant with C. But F appears as a transient dissonance to the note in the uppermost voice, E. So it happens that F also has two meanings, two different contexts at the same time. The functions of G and F are thus found to be interchanged depending on their relationship to the different voices. This is, of course, correct, and also very interesting.

Incorrect solutions

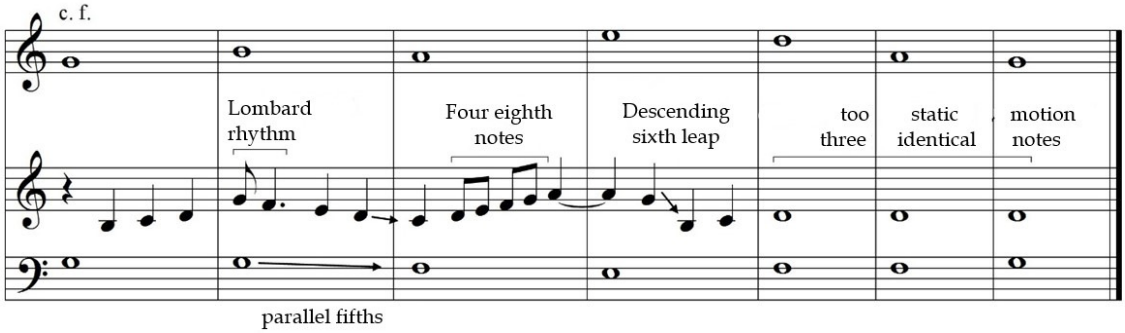
8.



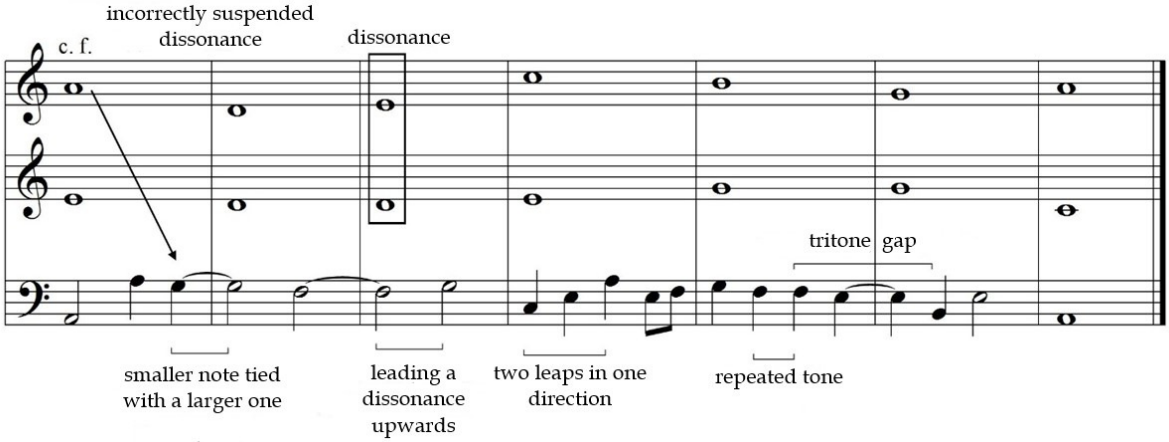
* The fact that there is no formula for fragmenting a dissonant syncope is not considered an error in the true sense of the word. However, the formal purpose of the fifth polyphonic species is to exercise these formulas.

I'm afraid there is another unacknowledged error here.

9.

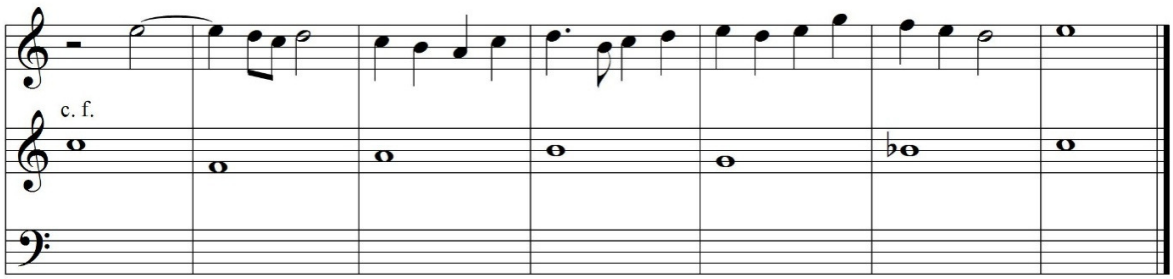


10.

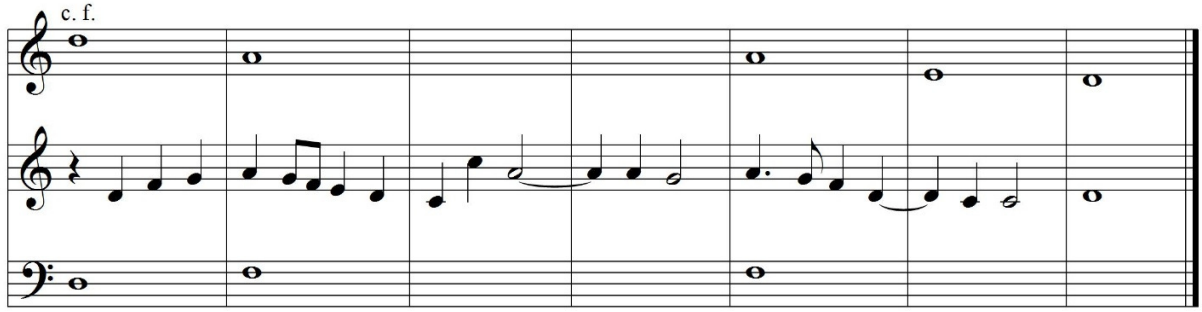


Exercises

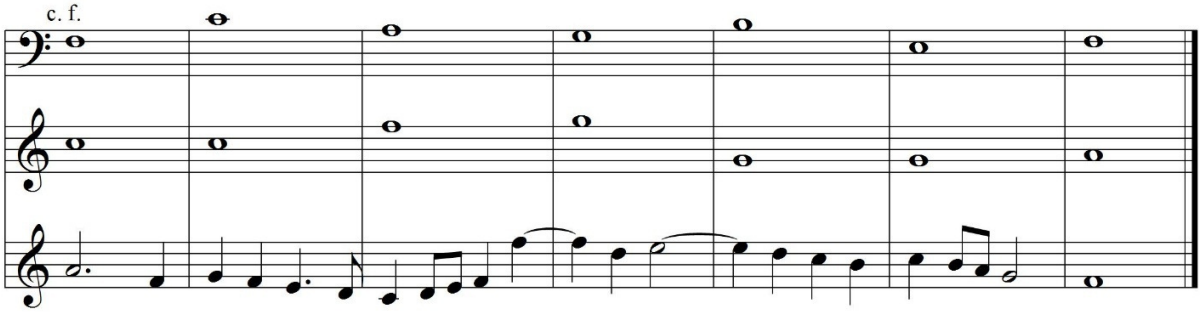
11.



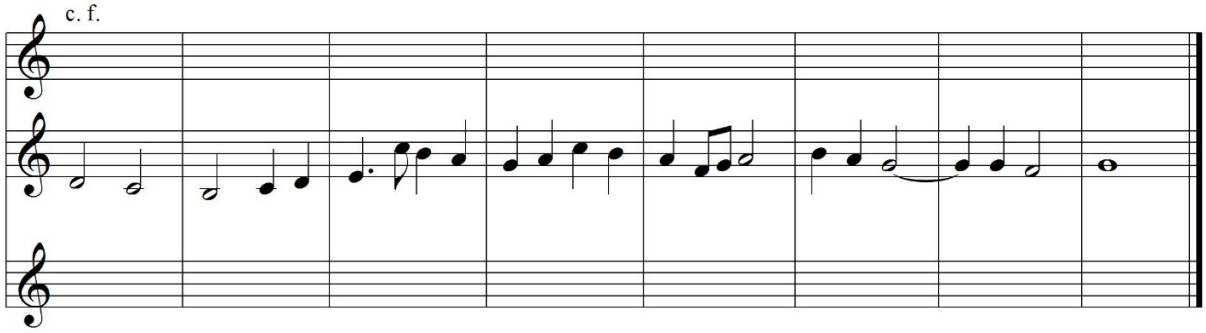
12.



13. The voices don't seem to be arranged properly.
Is the exercise correct?



14. Here the missing voices are written in words instead of notes.
But the instructions seem somewhat incomplete; it is not clear
which tone belongs to which voice and to which octave.



G G, D A, C C, E D, F G, B D, A G

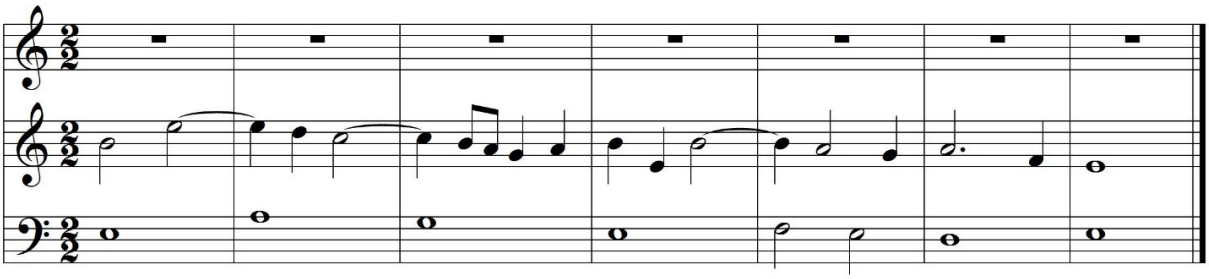
I wonder why at the end of this chapter I thought of two eighth notes on the downbeat?

Three Florid Melodies

Although it seems even more complicated, the more liberal rhythm in each voice further simplifies things. This arrangement has creative potential; we can try to write beautiful melodies and weave them together. It's not good to have the same rhythm, though - that's considered unstylish.

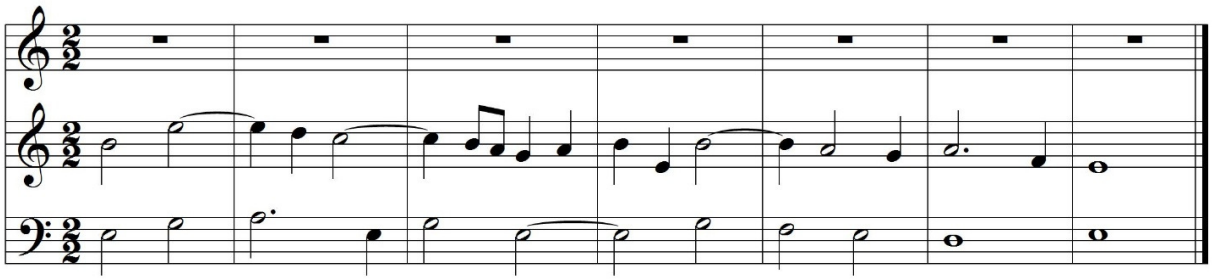
The more pragmatic solution is the first one - by adding the lowest voice as a first step. The predefined voice is the middle one. Although we have already practiced writing two florid melodies, in the example below I will first impose a scheme or outline; this is how I show the reader (and myself) what my consonant anchor points are:

1.



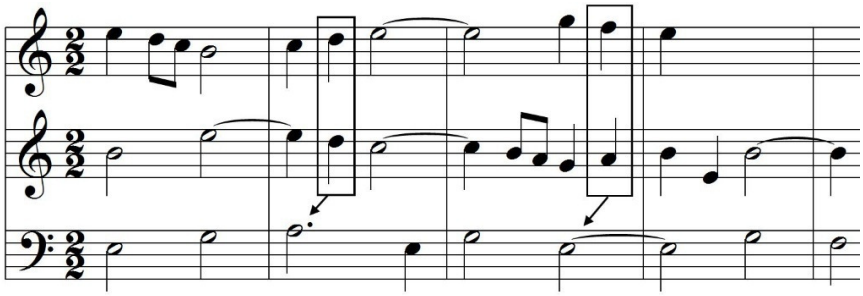
As a next step I could now get the lowest voice moving to some extent:

2.



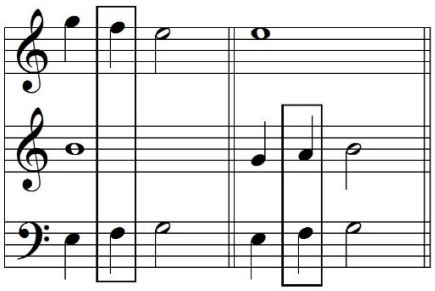
And the last step is the uppermost voice.

3.



Here in two places I have marked a correct legitimate phenomenon - transient dissonances appearing simultaneously in two voices. Below I show two more examples unrelated to the solution of our exercise,

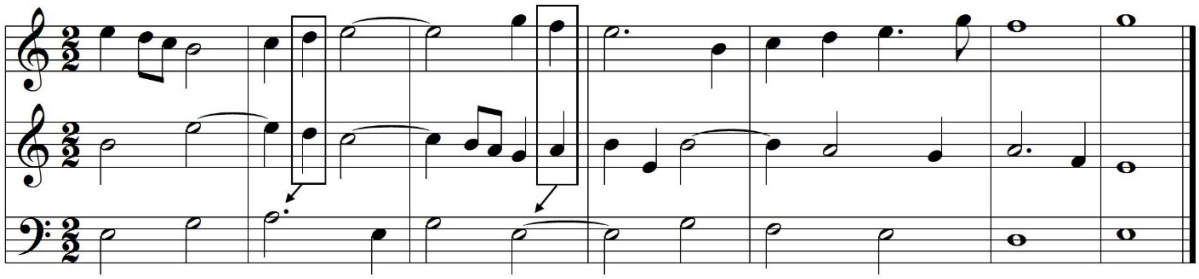
4.



where it is seen how this can happen in all voices.

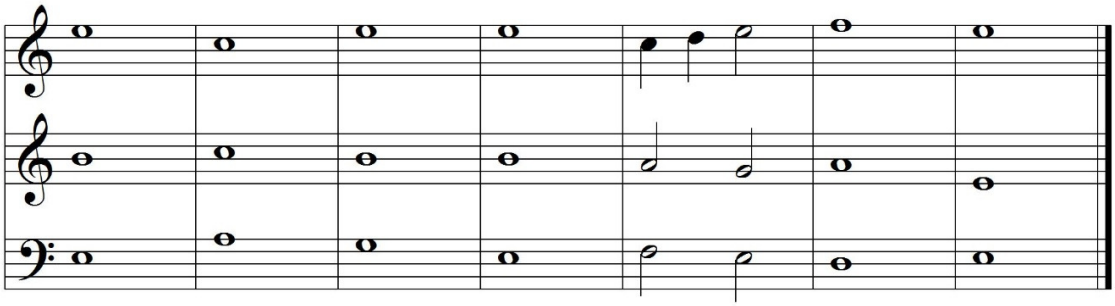
Then I proceed further to the end.

5.



I show the simplified consonant contour of the whole exercise:

6.



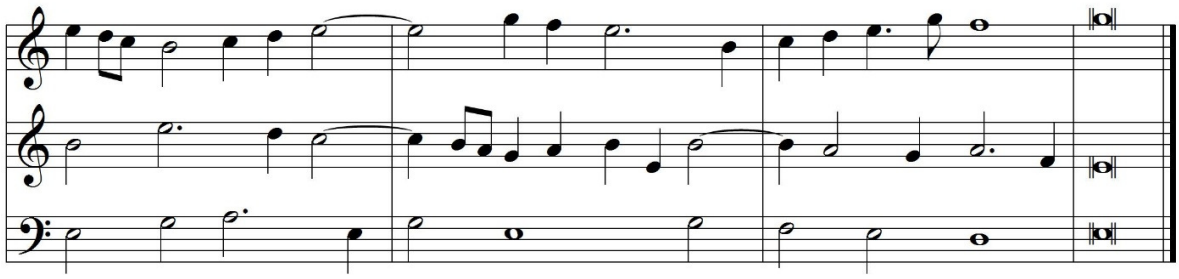
This is the logic and principle of the Renaissance style. My “concrete foundations” on any downbeat always remain in place. The only possible deviation on the downbeat is the suspended dissonance as realized in measure 3 (example 5) of the original solution:

7.

and all other dissonances are transient, that is, on a weak or relatively weak beat (and possible cambiata).

Writing or trying to figure out the whole contour in advance isn't really even necessary. It is enough to be aware that the downbeat of each measure is a consonant vertical with the possible exception of a suspended dissonance here and there. In fact, I could also present the completed solution like this:

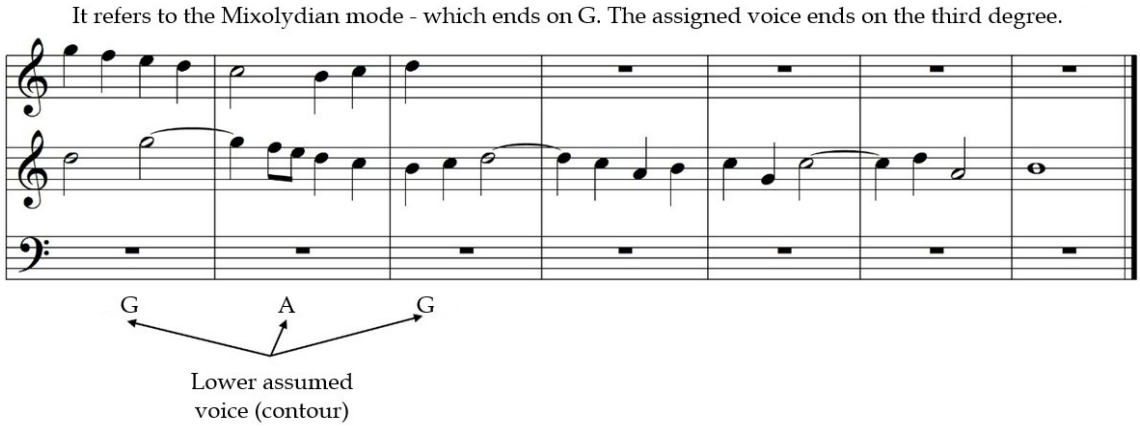
8.



And it will still be correct.

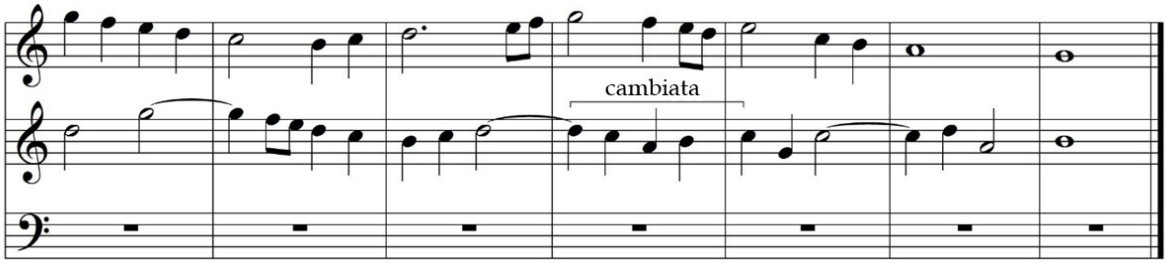
It starts to sound like a Renaissance writing. Below, I will attempt another solution - after the assigned voice in the middle, I will write the voice above:

9.



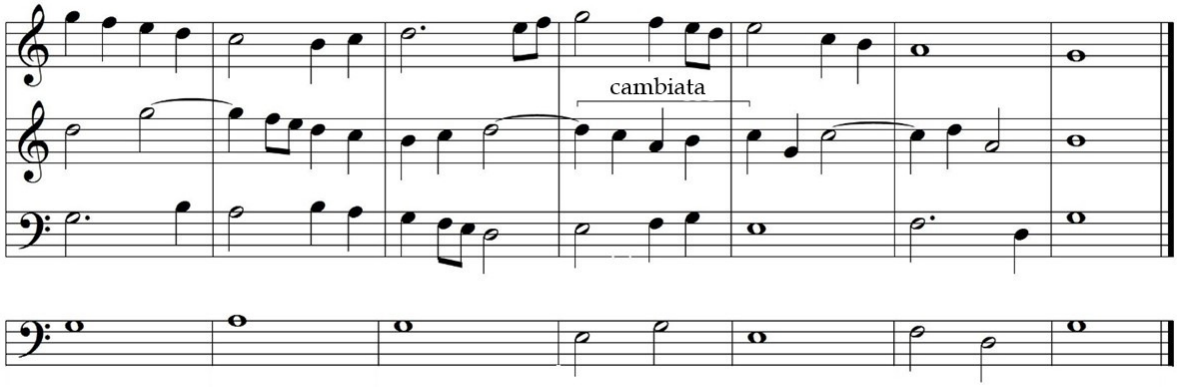
Further.

10.



Now I'll place the lower voice with a schematic contour written below it:

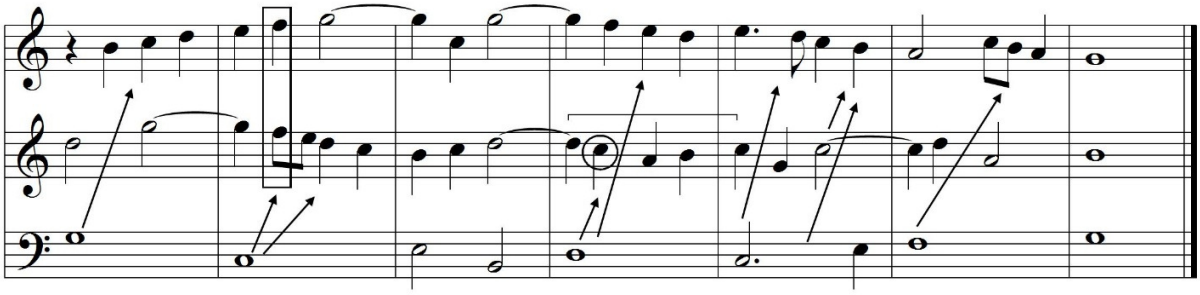
11.



This solution is quite smooth - in the upper voice there is only one leap.

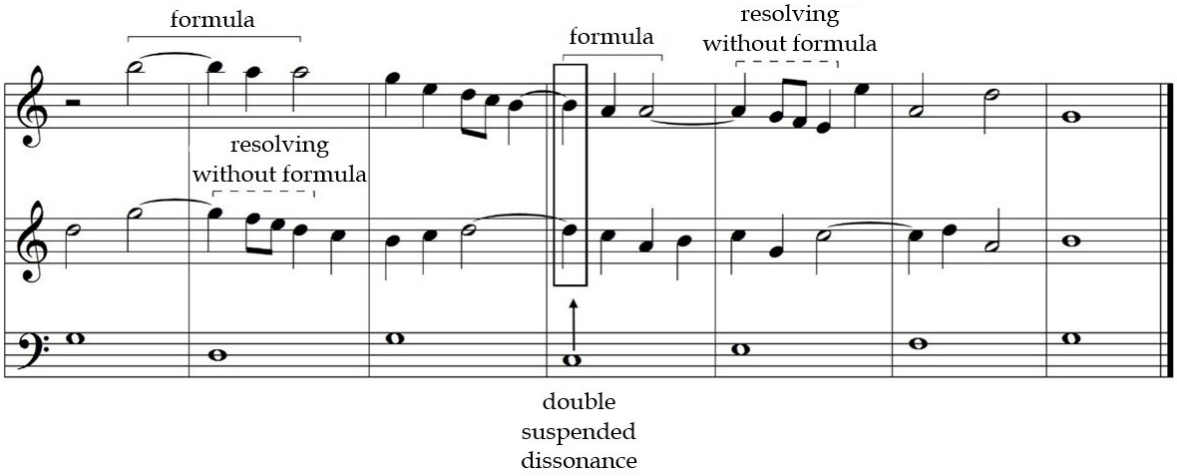
Following are some alternative solutions with explanations.

12.



In this decision, the lowest voice is deliberately a bit static. All arrows, except one, show the transient dissonances. One arrow indicates a tone enclosed by a circle. This is a leap from a dissonance with the lowest voice (seventh). This is possible because the leap is a component of the cambiata.

13.

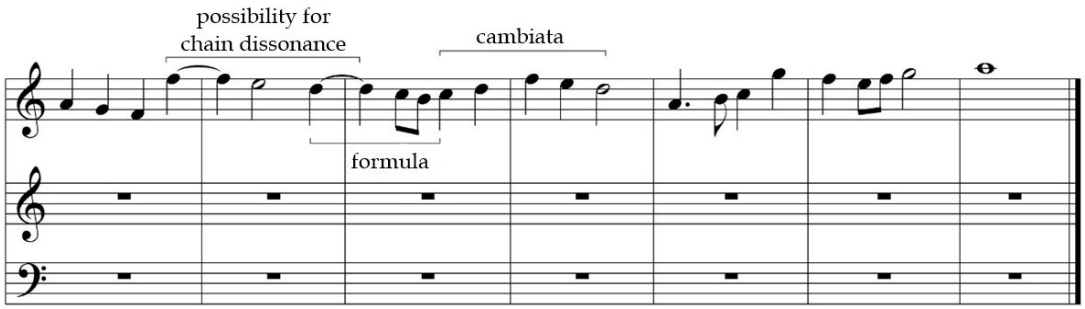


Here formulas including double suspended dissonance are used as well as non-formulaic movements. There are also several crossings of the upper and middle voices. The lowest line is again left deliberately a little unmoving[1].

[1]I did this in order not to overload the reader with too many notes to calculate. The lowest voice has many possible variants to move around. The examples thus written are not stylish, though correct; their purpose is rather to show the use of dissonant control between the lowest voice and the upper two. Further as an exercise they can be realized with more notes below, and the contour shown can serve as a reference.

The “construction from top to bottom” follows:

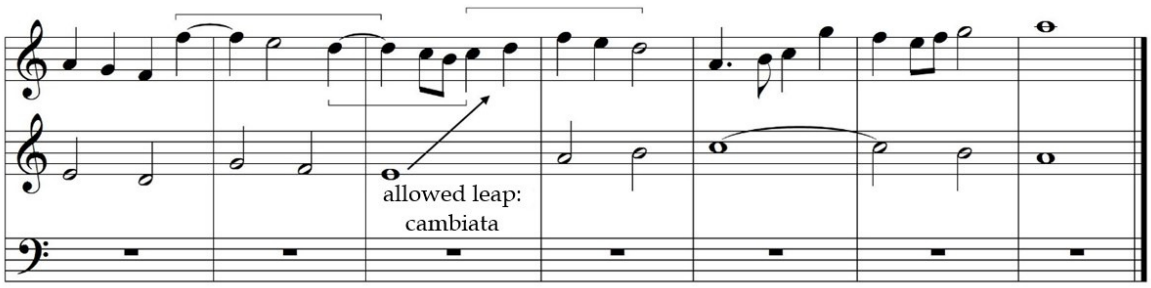
14[2].



I recognize elements in this line that I could use “by the book” - as I will show. But such an approach is not obligatory; other solutions are possible - this I also show shortly below.

Starting to write the middle voice, I deliberately place in it the material that in theory should belong to the lowest voice (without the first note). What would happen?

15.

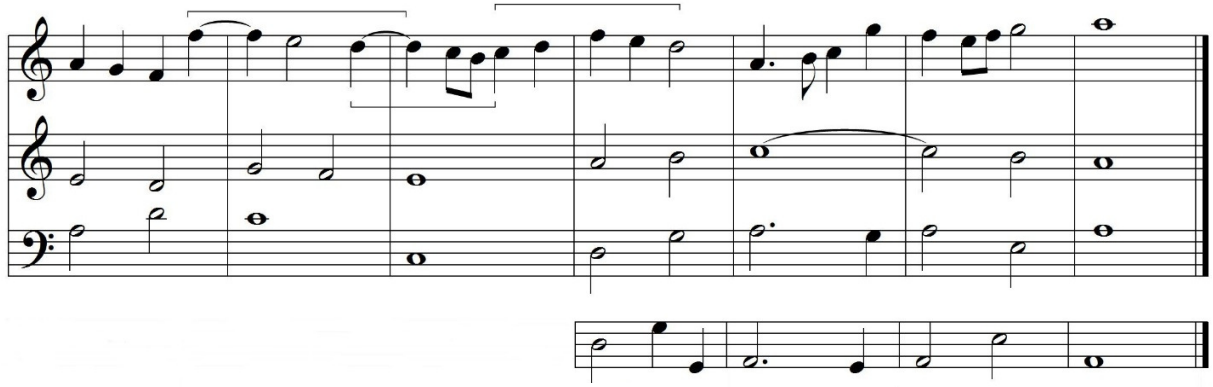


Done. Now I have a chain of suspended dissonances in measure 2. The formula in m. 3 is realized “by the book”. I wrote the leap from D in m. 3 deliberately - it is also allowed. The cambiata is resolved correctly with consonance on the first, third and fifth tones. In the penultimate measure there are seventh and second degrees that “enclose” the tonic at the end[3].

Now it's time for the lowest voice.

[2]This voice has been interpreted as written in the Aeolian mode with finalis (final tone) A. It can also be thought of as written in a Lydian mode, beginning and ending on the third degree (first degree F).
[3]Now the middle voice is more stationary in order to indicate the routine resolution of said components in the uppermost voice. A little later I can try to get it moving further. The lowest voice will also initially be more schematic.

16.



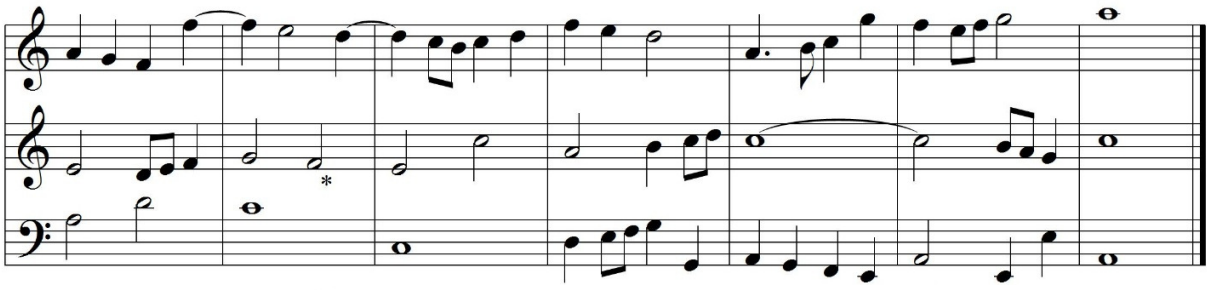
or:

(Here I propose to leap an octave downwards to the lowest voice in m. 4. Then I write the same notes. This suggestion is intended to avoid the hidden cadence octave between the penultimate and last measure.)

This resolution illustrates the following: even though the "bass" is in the middle voice, adding another voice below the middle is quite possible. There is one lateral dissonance with respect to the lowest voice in m. 3, the note B. I have already mentioned in the chapter on the fifth polyphonic species, the two-voice polyphony (the section on fragmenting a dissonant syncope), that at times this can be tolerated.

Now, as promised, I'm moving the middle and bottom voices as far as I can.

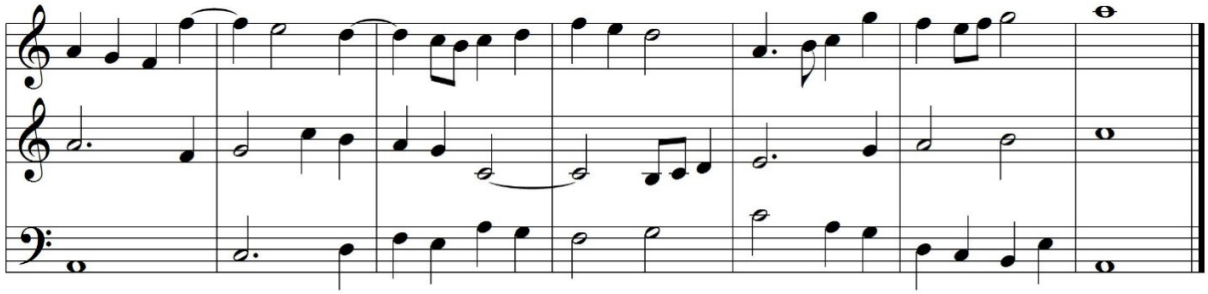
17.



* F is a transient dissonance to C in the lowest voice.

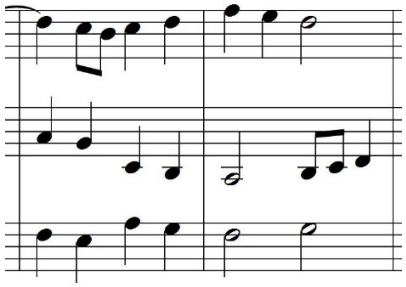
And below I show a different solution without any traditional approaches.

18.



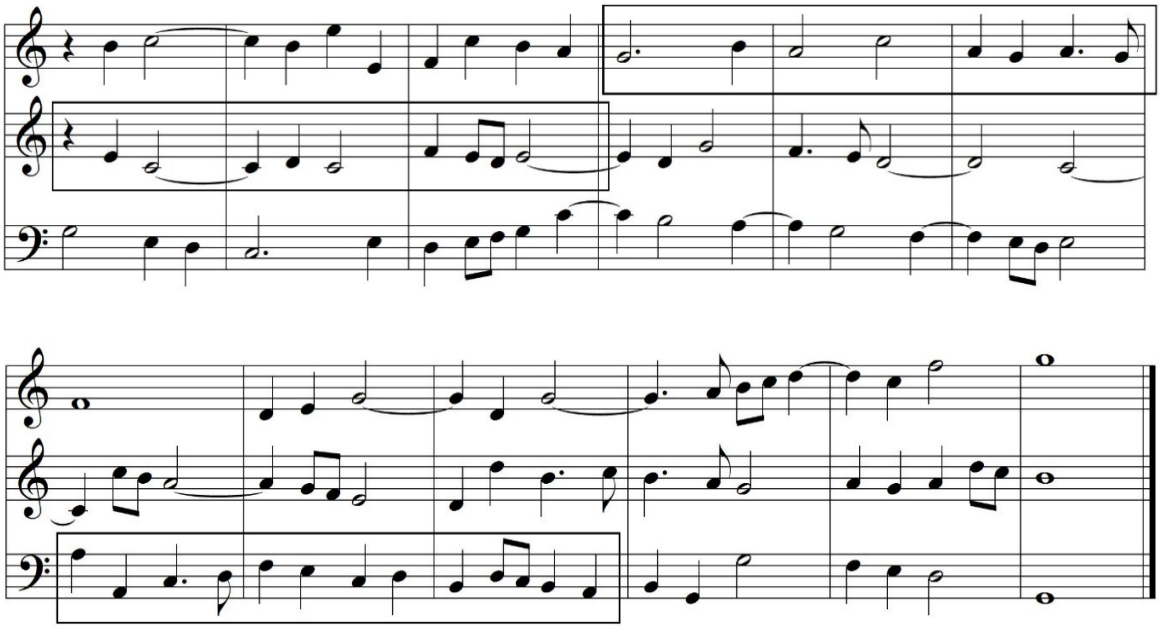
However, I have leapt from D at the end of the third measure - I have used the cambiata. But this is not obligatory - such a variant is also possible (m. 3-4):

19.

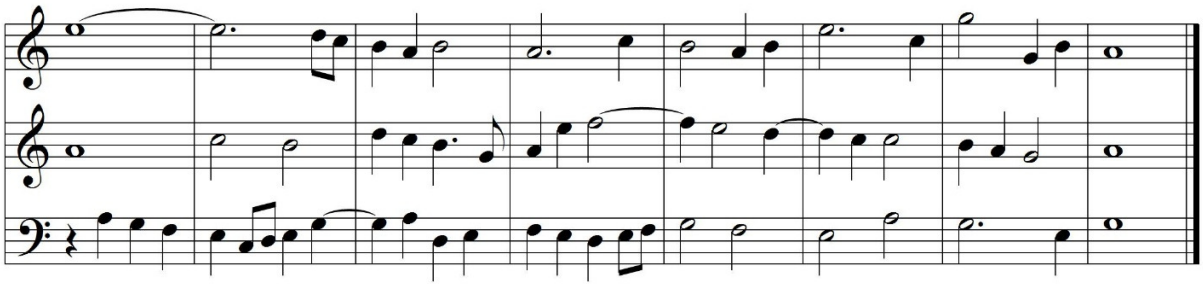


Exercises

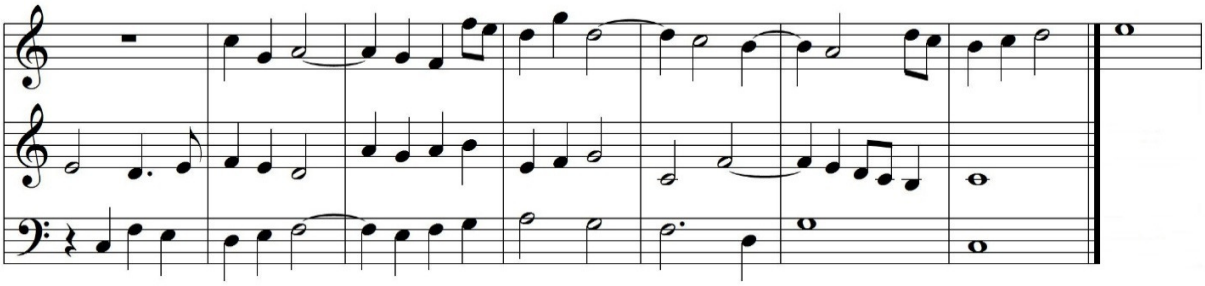
20. I believe that if the passages indicated are corrected, the world would become a more beautiful place.



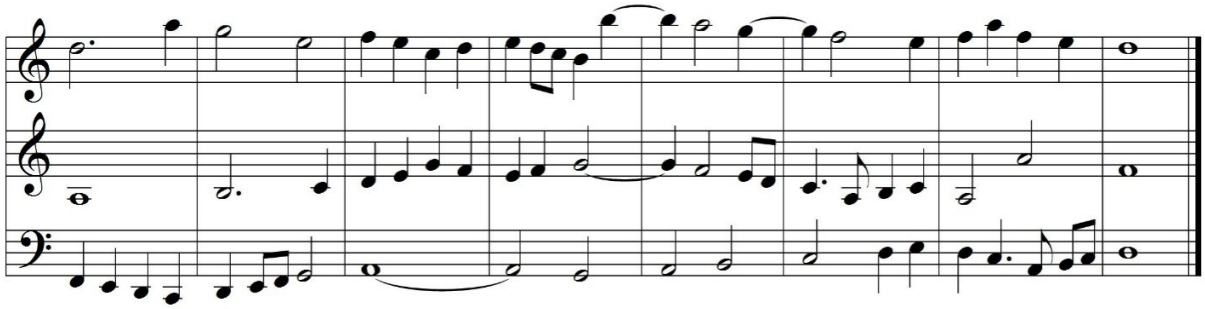
21. There seems to be (I hope) only one error here



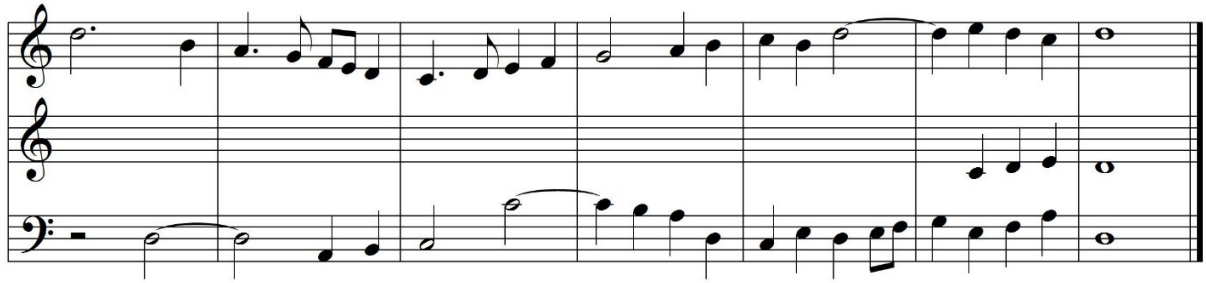
22. Something is wrong. But if it wasn't, would the exercise be correct?



23.There is also something wrong, but if it wasn't, the exercise would be correct.



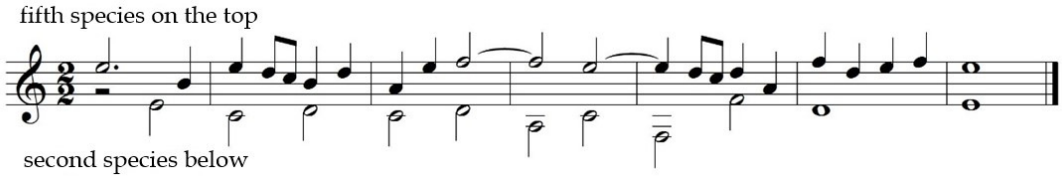
24.



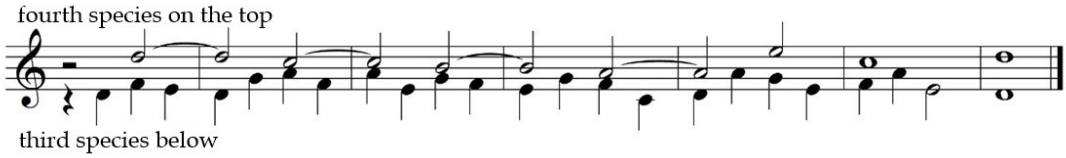
Contrast Triple Counterpoint

If every voice could be different, we are entering a large and interesting realm. There can be six possibilities in a voice (cantus firmus in whole notes and first to fifth polyphonic species). Then two voices would have 6x6 possibilities=36. Below I show two possible variants[1]:

1.

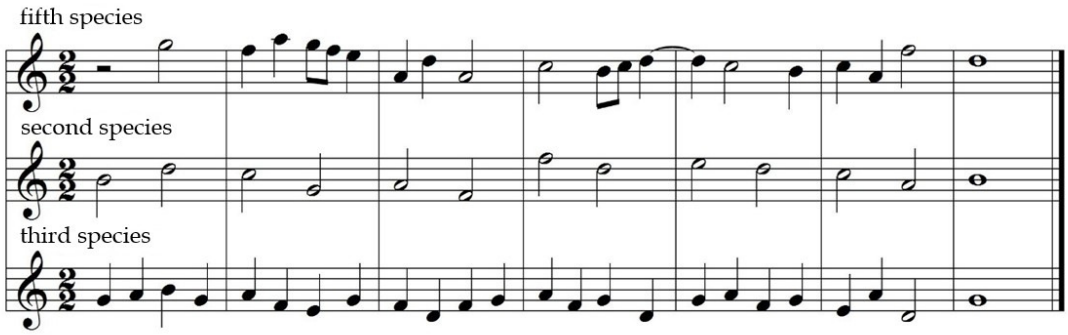


2.



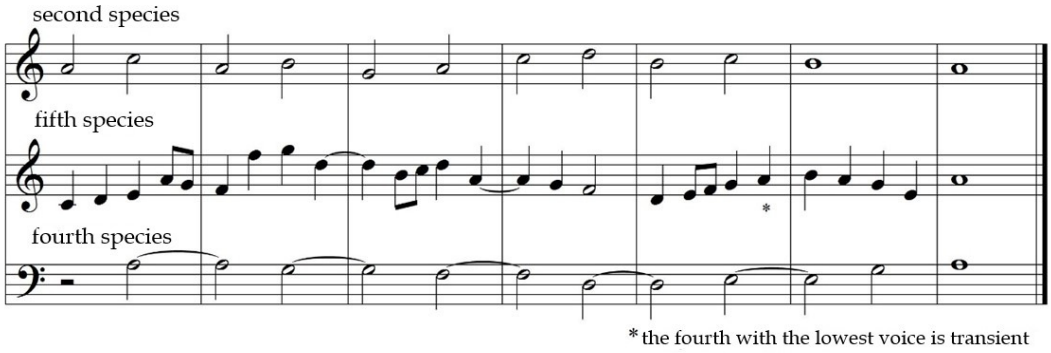
And if the voices are three, then the variants are 6x6x6 = 216, i.e. six raised to the power of three – 6^3. The equation is derived as follows: the base 6 is the number of variants, and the exponent ^{3} – the number of voices. Two examples are shown:

3.



[1]It is not clear here which voice has been written first and which one was constructed later on the basis of the first. As I write a little further down in the text, the second polyphonic species (half notes) and the fourth (syncopated chain) are the most difficult in polyphony. So it would be logical to consider that they served as the main melody, i.e. they were written first. Conversely, to write a florid melody in fifth polyphonic species and build a voice to it in first or second species would be much more difficult. It might also be impossible. Therefore, when writing a contrast multi-voice polyphony, the teacher must be sure that the solutions to such problems are theoretically possible.

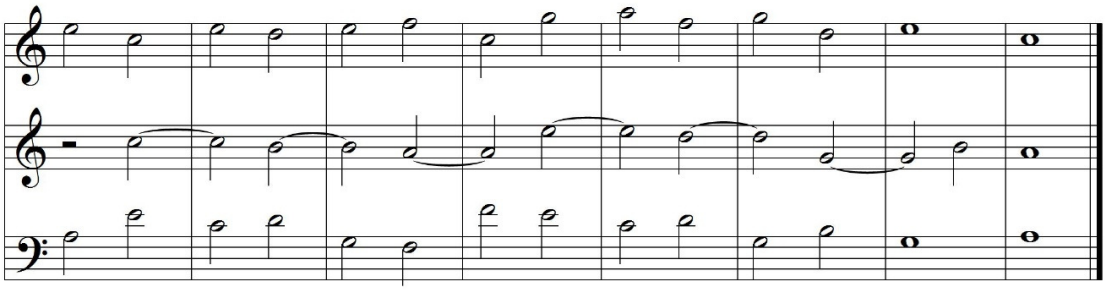
4.



In the teaching tradition it is not customary to practice all the variants - for a four-voice polyphony the number would be 1296 (6x6x6x6, or 6^4). But Fux and I are in agreement: practicing contrast three-voice polyphony is very useful, as well as considerably more difficult. More complex problems arise whose resolution develops our combinative thinking. As far as I know, there is no living music of historical practice written in this way. So these exercises remain purely technical, synthetic.

Following the rules outlined so far, based on personal experience I could state that the most difficult combination of variants is the one that includes a second and a fourth polyphonic species, especially if they are together – for example 2:2:2, or 4:4:4, or 2:4:2:

5.



Not easier is the combination of the first, second and fourth species.

As I have written before, there are no new rules. The writing strategy is still basically the same - it's best to outline the lowest vote first. But if we plan for our solution to include a first, second, or fourth species, I recommend that they should be next in order.

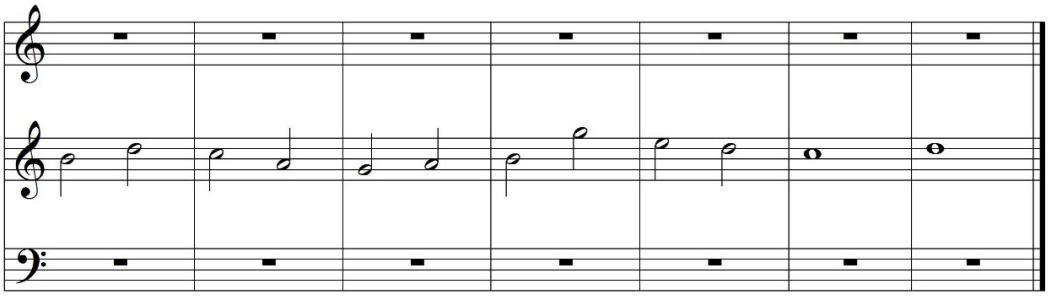
Example

If in the course of writing I encounter a problem, I will not hide it - that is, I will not demonstrate a problem already solved. I write the exercise in real time[2].

My exercise would have: fourth species in the lowest voice, second in the middle, and third at the top. In the course of work, problems may arise both horizontally and vertically, so I reserve the theoretical right to change any voice (including "retrospectively"). In solving this particular exercise, I assume that there is no "fixed" cantus firmus.

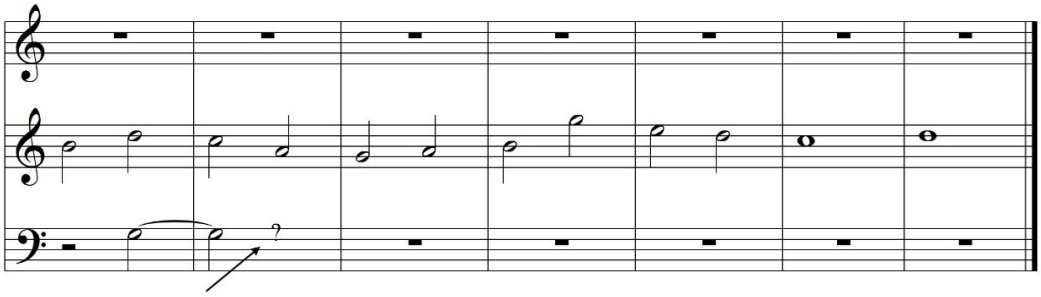
First I write the second species in the middle voice.

1.



As I begin to write the fourth species below, I immediately run into a problem:

2.



According to the rules of the fourth species I can only start from the second beat after a half pause. Since this is the lowest voice, I can only start on the first degree of the mode and I have decided that this will be G. The tone C in the second measure in the

[2]Some teachers allow lateral dissonances in a three-voice polyphony. In this chapter I don't show examples of such solutions. I am not forbidding lateral dissonances in principle, but I consider it more important to show that they are not absolutely necessary in a triple counterpoint, that is, their inclusion would be for reasons of color or style, but not an absolute necessity.

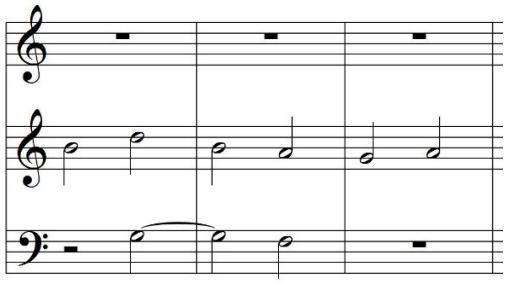
middle voice should be resolve stepwise downwards, but it hasn't - the previously set second species leaps from C to A.

Conclusion is very simple and obvious - I have given myself impossible conditions.

The way out of the problem contains two variants: either to change the second species in the second measure, or to do the same with the fourth species, eliminating the tie and writing a different tone on the downbeat of the second measure. I decide not to use this variant, because from the beginning of the exercise I would end up writing two voices in the second polyphonic species.

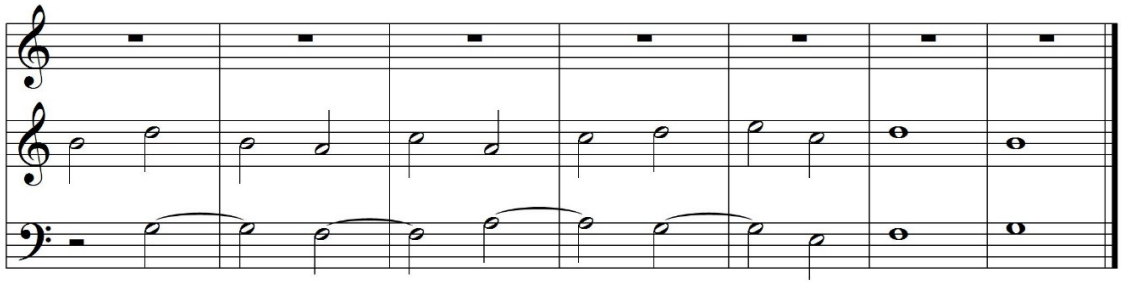
I choose the first variant - I change the second measure of the middle voice:

3.



The second measure is now fine, but in the third again there is no solution. If I tie F, then the dissonance on the downbeat will be resolved upwards. Again I have to change the middle voice. That way the lower voice can continue to work normally with the one above it. The fourth measure is problematic on exactly the same principle. Therefore, after changing the third and fourth measures of the middle voice I reach a situation where I can fully complete the lower voice:

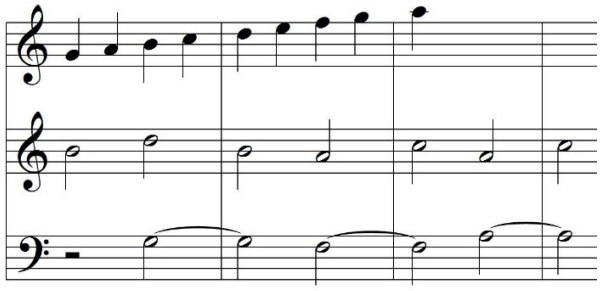
4.



I deliberately place B in the last measure of the middle voice - third degree (what would happen?).

Now it's time for the upper voice. I find, to my utter amazement, that I can write a stepwise ascending line. I didn't expect it, and I am astonished that it is possible:

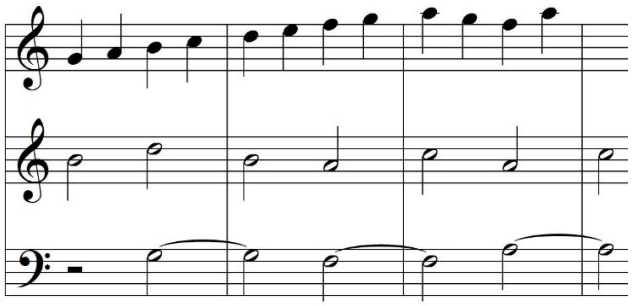
5.



This is generally rare - an unexpected strike. Checking the relations between the three voices, I see no error anywhere. My thinking is conservative - I decided to write an ascending line simply as an experiment, and this experiment has proved unexpectedly successful.

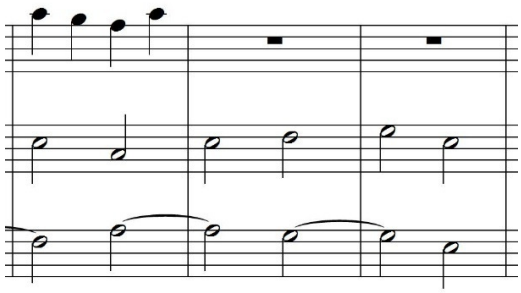
On the third measure in the upper voice I “turn” - again without any particular problem:

6.



The fourth measure follows. Here I show the exercise from it – m. 3 – 5.

7.

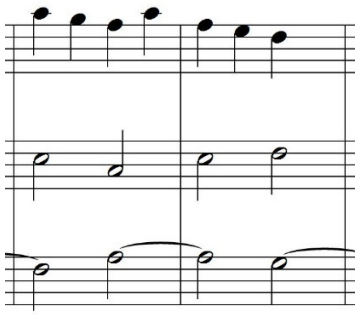


At the beginning of m. 4 in the upper voice:

I can't write A (of the second octave), that way the tone would be repeated.
I can't write G, it would form a seventh with the lowest voice.
I can write F, the fourth with the middle voice is allowed.
I can also write E.
I can't write D, it would form a dissonance with both lower voices.
C also doesn't work- that would be a sixth leap downwards on the horizontal.
On the same logic B would be a seventh leap downwards and not the only potential trouble.
I can write A1.

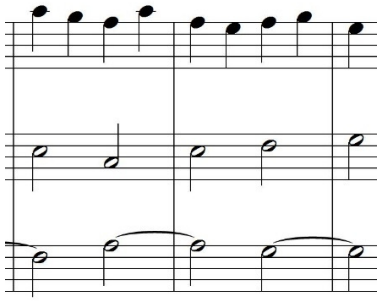
I choose F, and immediately after it - E; they are both consonant with the other two voices. Can I then move to D?

8.



Unfortunately, it results in parallel fifths with the lowest voice. I delete the D and write F again in its place. Can I then move on to G?

9.



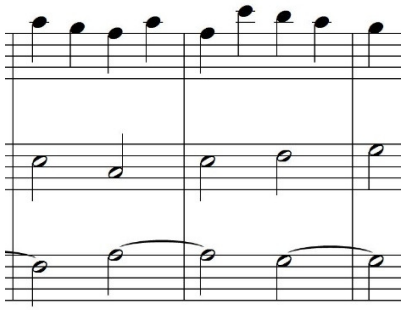
The tone F is appropriate because it forms a transient dissonance on a relatively strong beat (hard transition) with the lowest voice and doesn't dissonate with the

middle voice. But on the fourth quarter note G it formed a fourth with the middle voice, then it leapt from that fourth. Is that allowed? I call this attempt Variant 1.

Again, the answer is ambiguous. This is more about a maximalist approach and less about a stylistic one.

After a few experiments, I find that I have to erase all the notes in the measure except F.

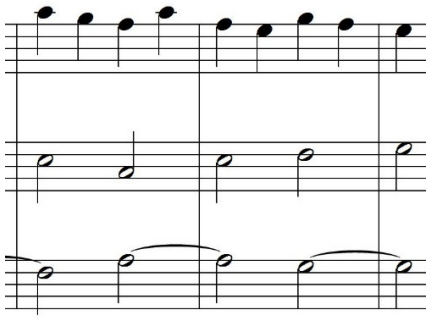
10.



There are no more problems on the vertical; but I have paid for it by going a little too high, up to C3. Once again, some compromise. This is my Variant 2.

Variant 3: instead of leaping from the fourth, as in Variant 1, I leap *into* it[3]:

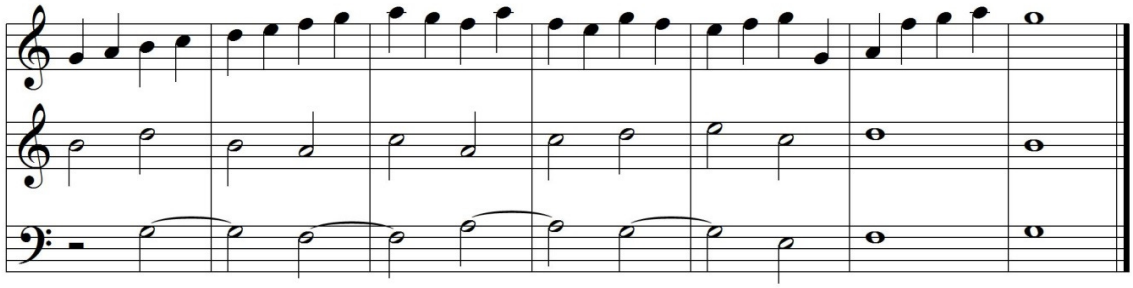
11.



Which variant is the best? Again, I am trying to think pragmatically. The rules say it's allowed to have a fourth anywhere as long as it's not with the lowest voice. Nowhere is it written that one can't leap into it. I choose that variant and continue further.

[3]I already did this once, writing F on the downbeat in measure 4.

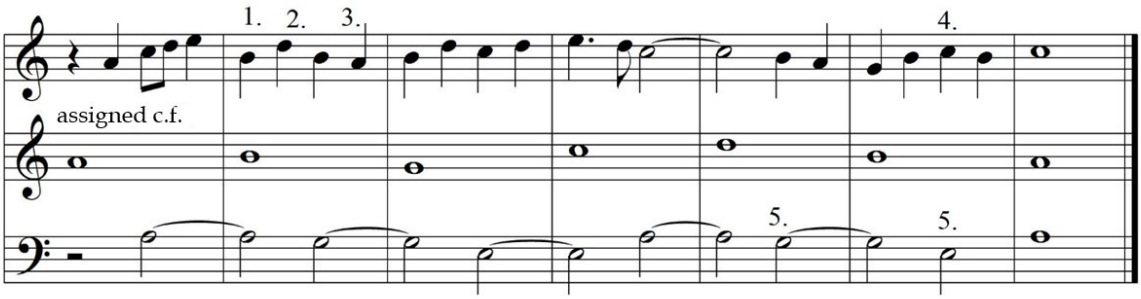
12.



In measure 5 from E onwards I discover that now the work is easy until the end. I have briefly crossed the upper and middle voices. B in the last measure gives me no trouble. And G in the upper voice is a hard transition (fine).

In the example below I show some minor errors.

13.



Tone 1. leaps in dissonance with the lowest voice
T. 2. has leapt into a fourth with the lowest voice
T. 3. is a lateral dissonance with the two lower voices. In general, lateral dissonances are harder to notice
T. 4. is a lateral dissonance towards the middle voice
Parallel fifths between middle and lowest voice on notes 5.

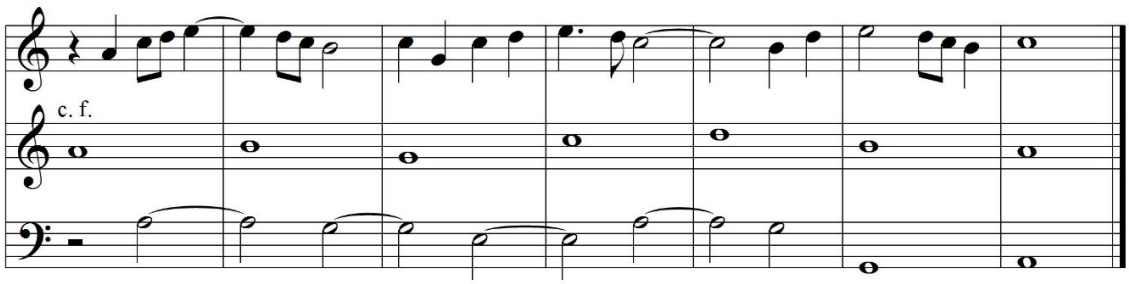
The half note C in m. 5, tied over from the previous measure, forms a suspended dissonance with the two lower voices. This is normal, it occurs frequently.

I deliberately ignore the rule that forbids the use of a tie in both voices at the same time, because there should be ties on almost every measure in the fourth species. And this rule is generally not relevant for more than two voices. That's really my only

consideration, my only argument. I make no mention of the fact that I have done this to demonstrate the double suspended dissonance - my modesty doesn't allow it.

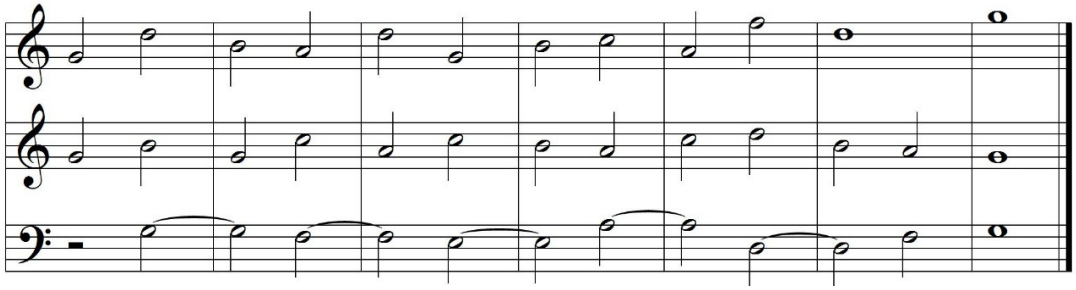
Below I show the same example, solved (hopefully) correctly.

14.

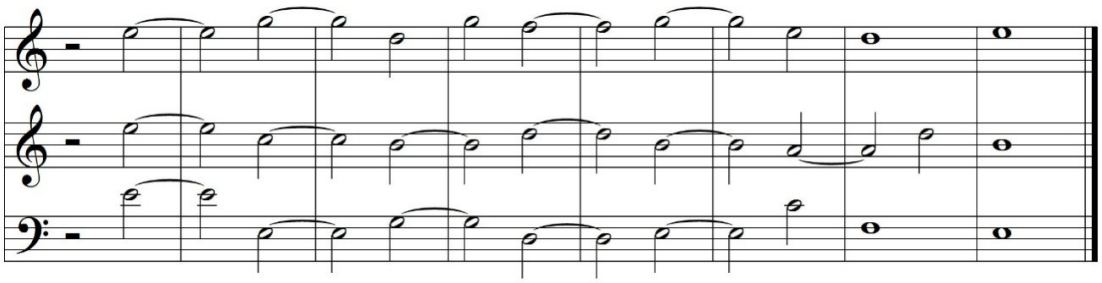


A few more examples:

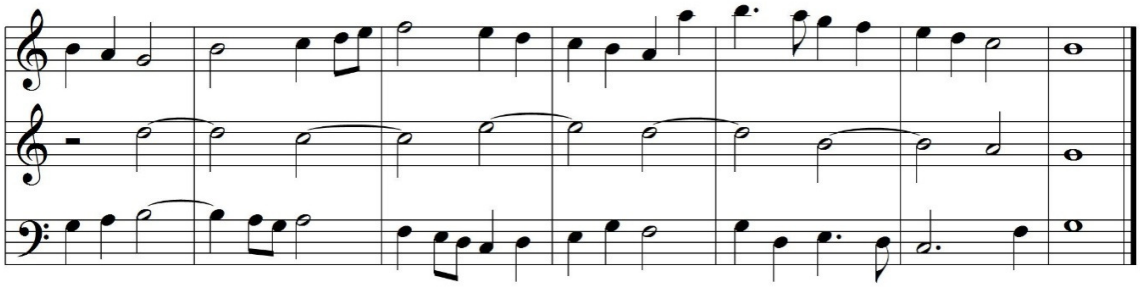
15.



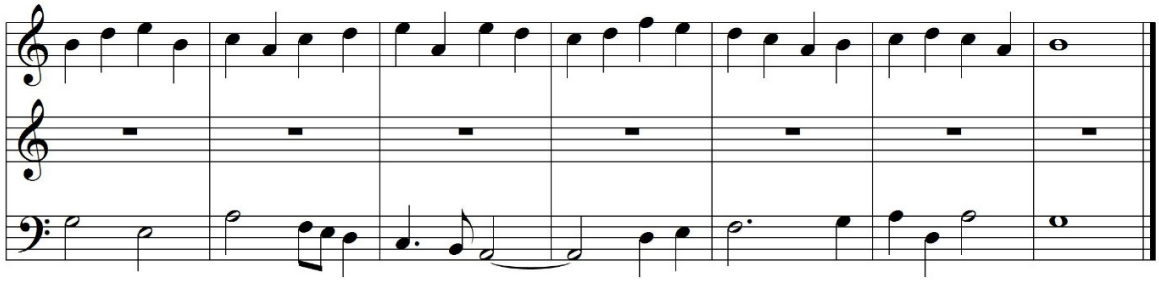
16.



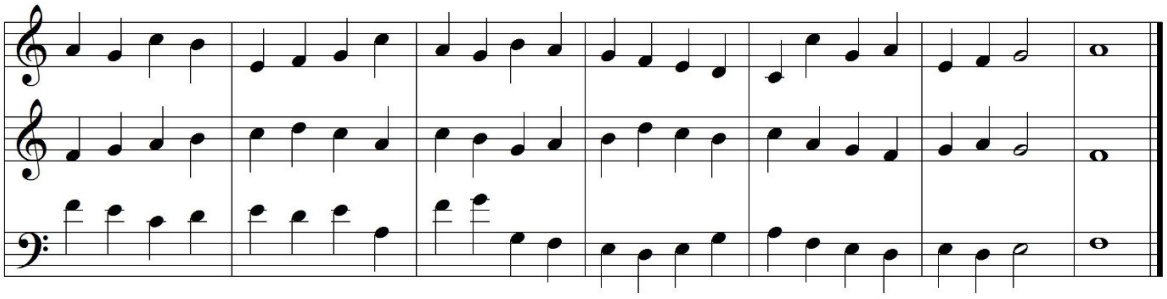
17.



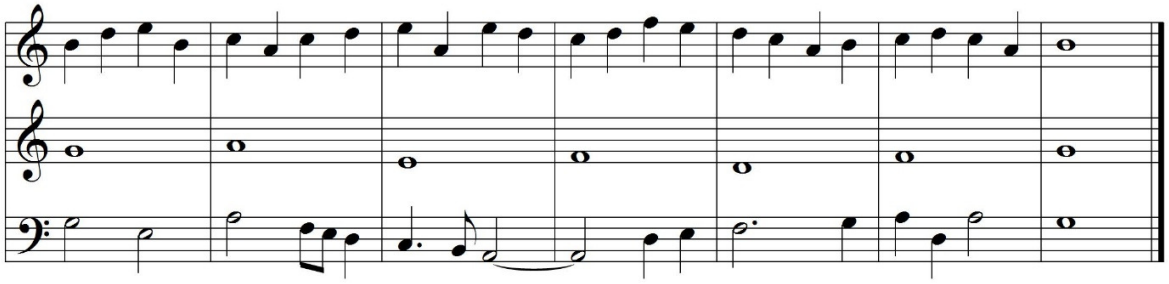
18.In the middle voice a “cantus firmus” in whole notes can be written.



19.



20. Solution of example 18.

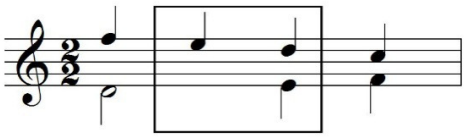


Additional Options

Before we move on to the "true" Renaissance style, I present a few options for treating dissonances a little more freely. These are intended for the learning style, but can no doubt be used in "living" practice as well. This is precisely the reason for their existence; there are such historical precedents, but they are relatively rare. Personally, I would only use them if it gets "too hot" - if I see no other options for a solution, i.e. in exercises for 3, 4 or more voices. This includes the use of lateral dissonances.

I. Two transient dissonances that form two consecutive dissonant verticals:

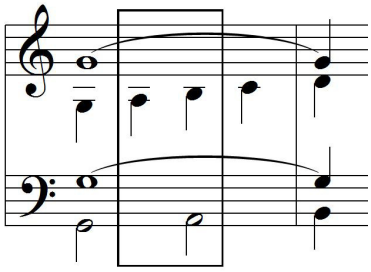
1.



The first interval is a ninth (D-E), the second - seventh (E-D).

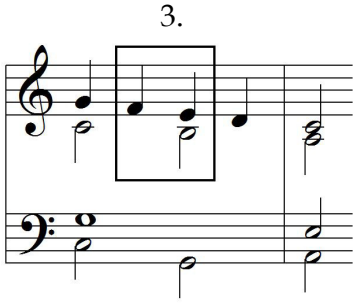
Or parallel ninths:

2.



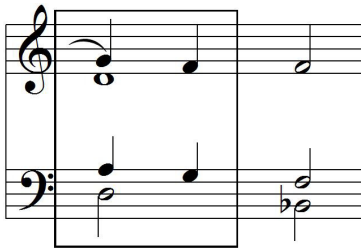
Or parallel fourths:

3.



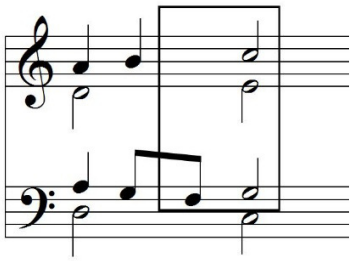
Variant-formula for fragmentation with simultaneous transient dissonance; result - parallel sevenths:

4.



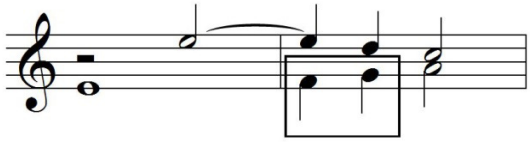
Or parallel fourths:

5.



II. Resolution of a held dissonance (naturally downwards) with a contrary motion in the lower voice:

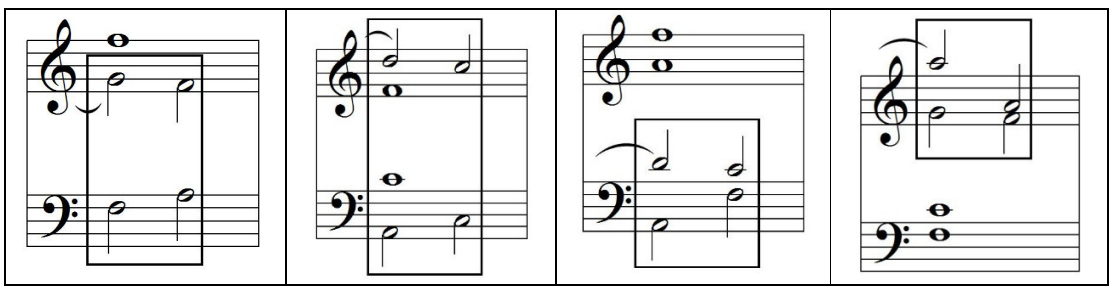
6.



The lower voice may or may not be the one next to it - it could be a tenor or a bass, for example. This phenomenon occurs more often.

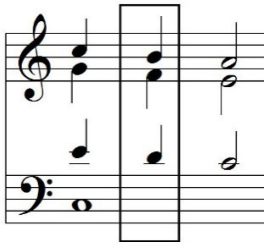
Another method of resolving a held dissonance with an upward motion in another voice, but using a leap:

7.



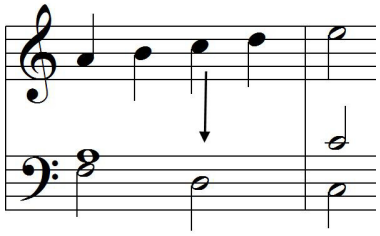
III. Three (or more) transient dissonances simultaneously:

8.



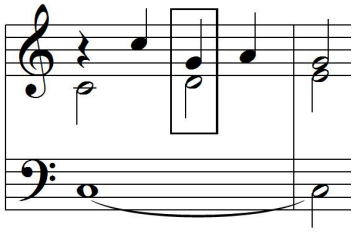
A rare case of lateral dissonance combined with a leap in dissonance:

9.



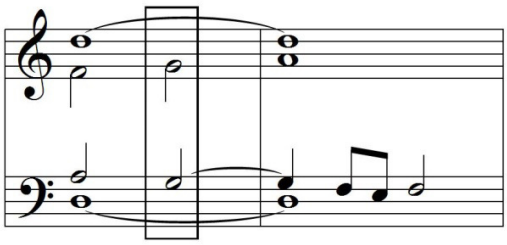
IV. Also rare: a leap in dissonance in internal voices in the presence of a transient dissonance:

10.



V. A certain delay of the transient dissonance[1]:

11.



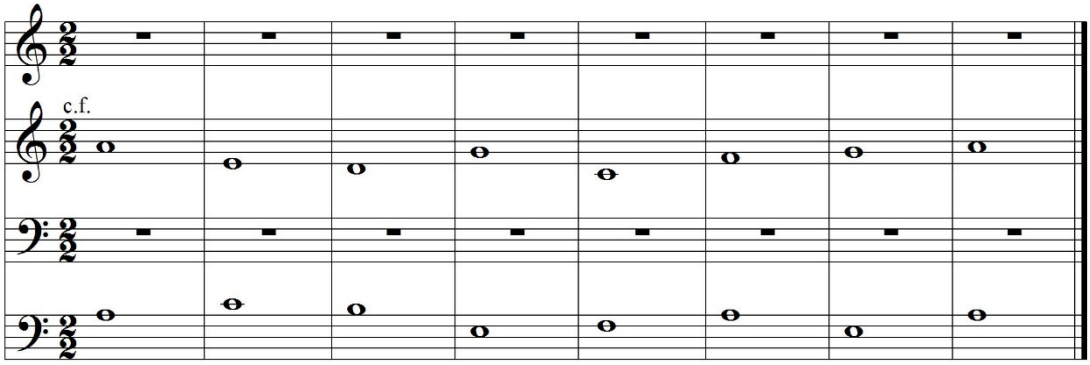
[1] Or explained in another way: in the tenor, the transient dissonance is suspended with a held dissonance in the following measure. Dissonance in suspension with a dissonance.

First Species

There are no new rules for four-voice polyphony. All principles remain the same. Now, except for three possible "chord" tones on the vertical, one of them can be doubled. Naturally, not all tones have to appear on the vertical, nor are there laws for doubling them - this is not the science of harmony. We are not yet speaking of chords in the true sense, nor of tonal functions.

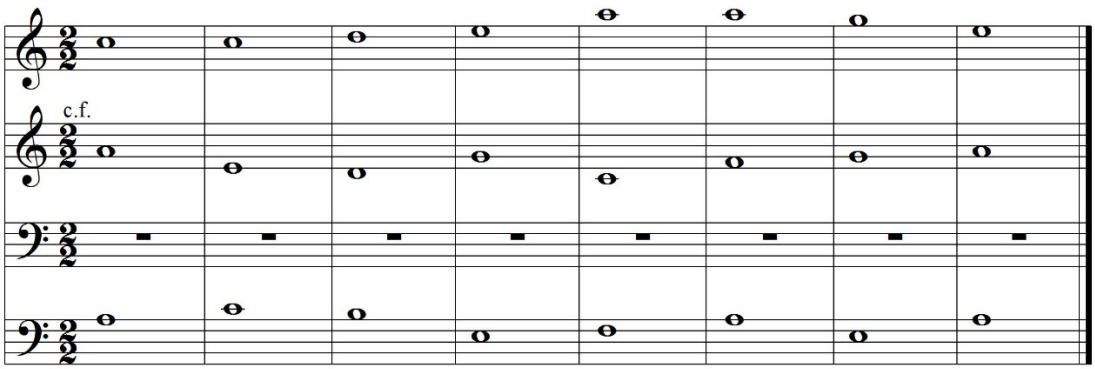
After the cantus firmus I lay the first counterpoint in the bass.

1.

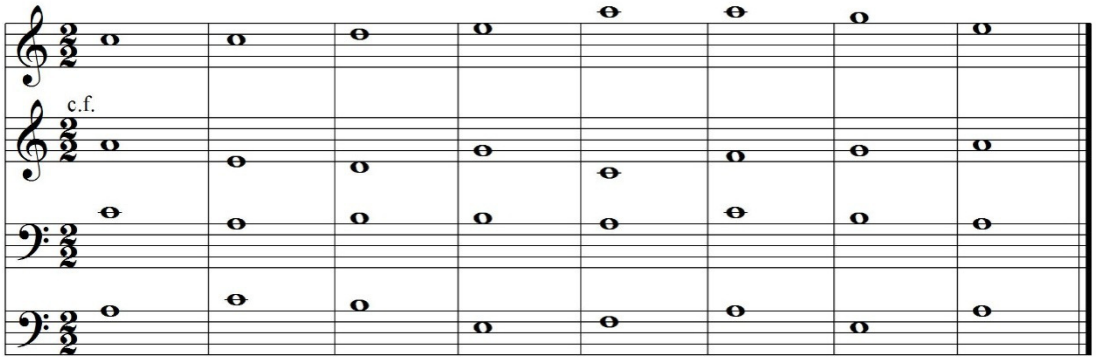


The soprano follows.

2.

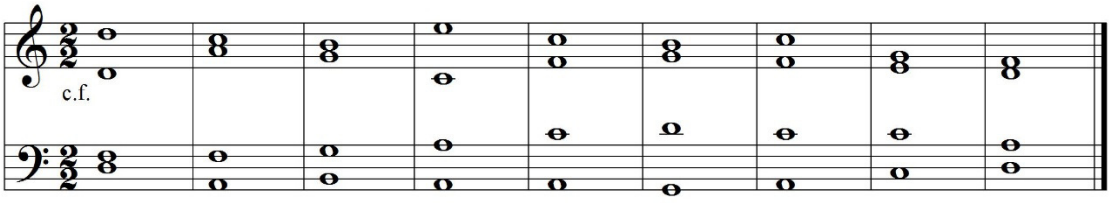


3. Final solution

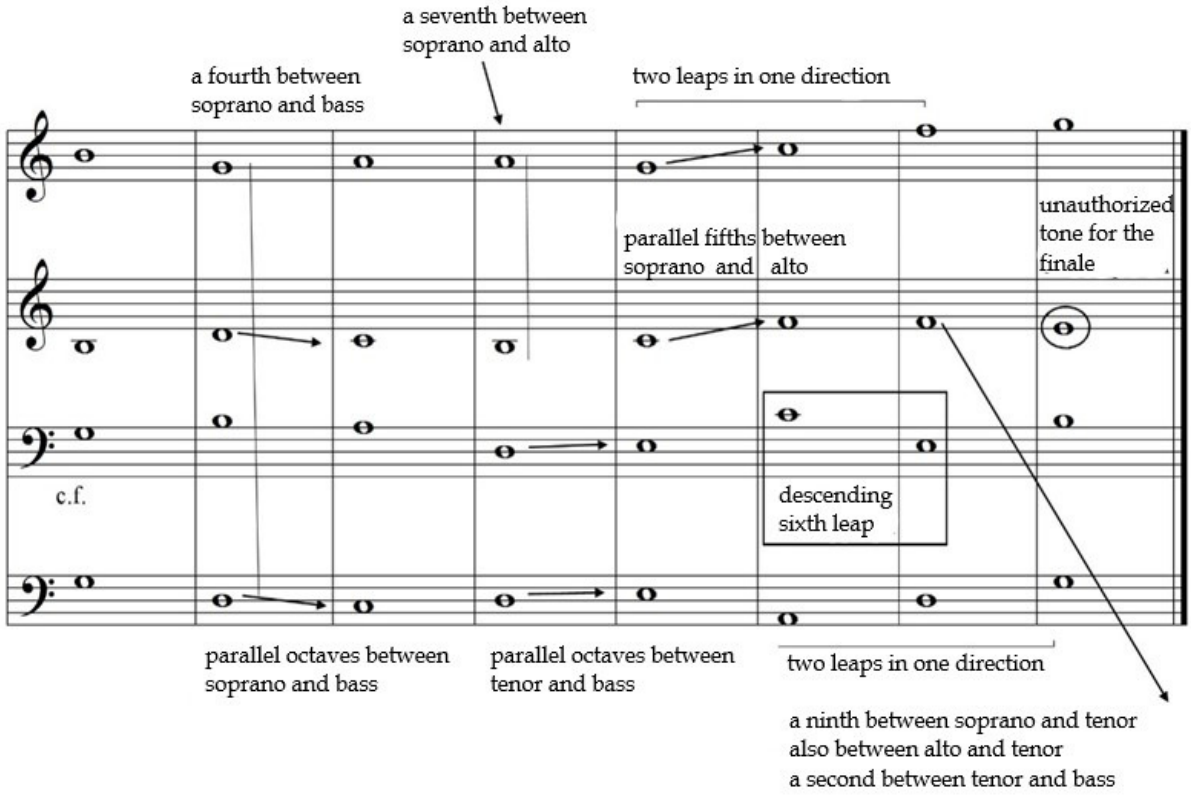


A four-voice polyphony can be written in either two or three staffs - in this example there are no crossed voices (c.f. in the alto):

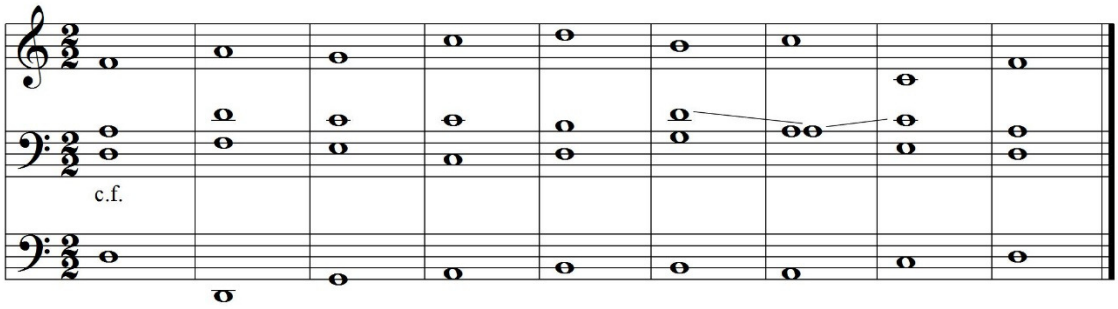
4.



5. Incorrect example

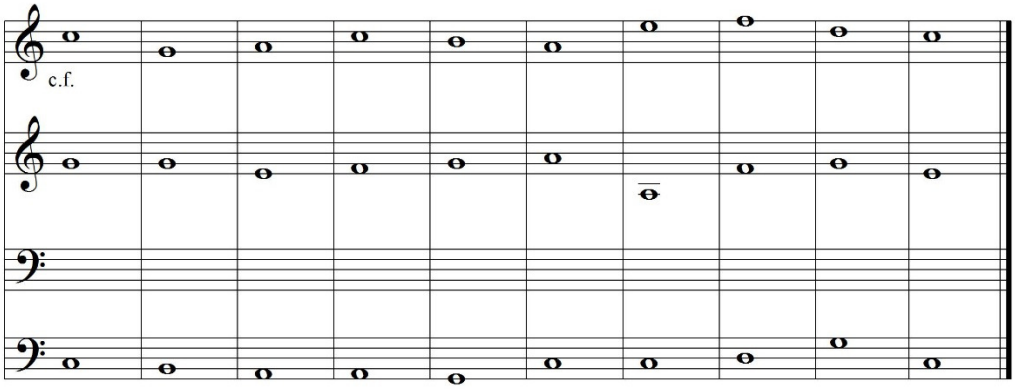


6. Example for analysis (no crossed voices)

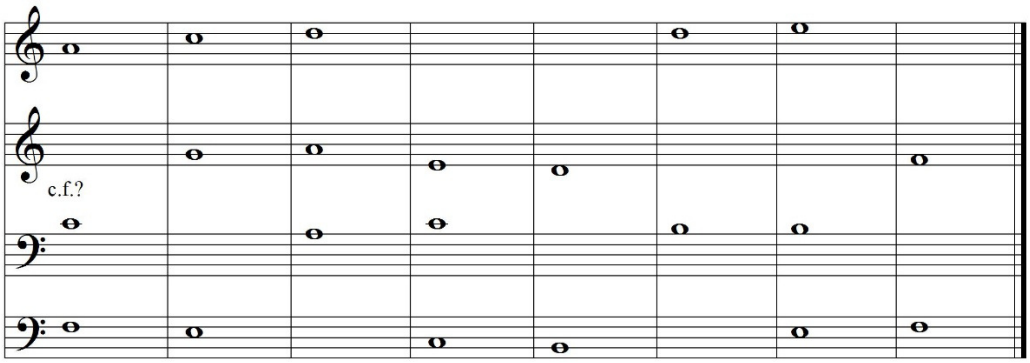


Exercises

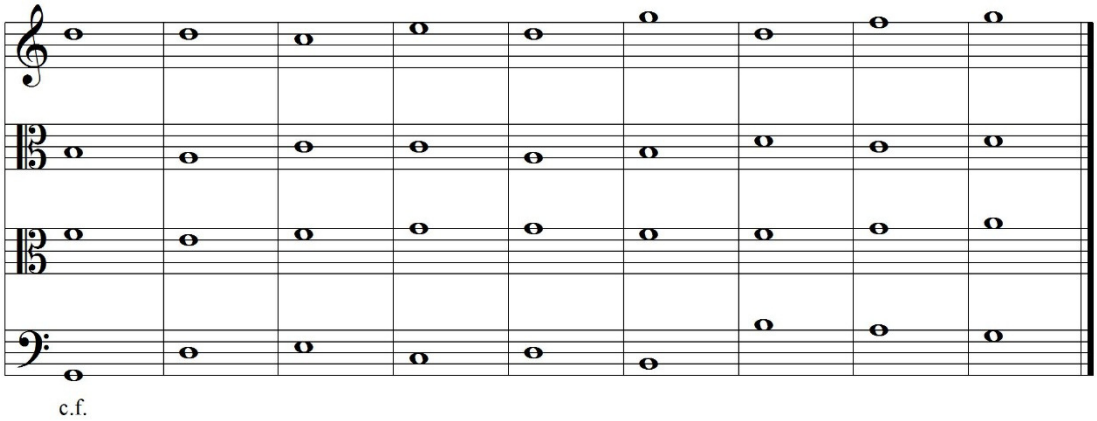
7.



8.



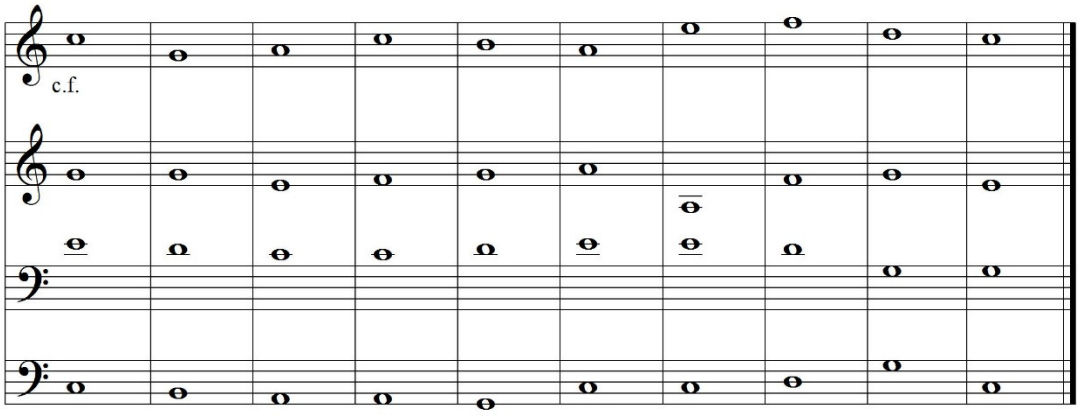
9.



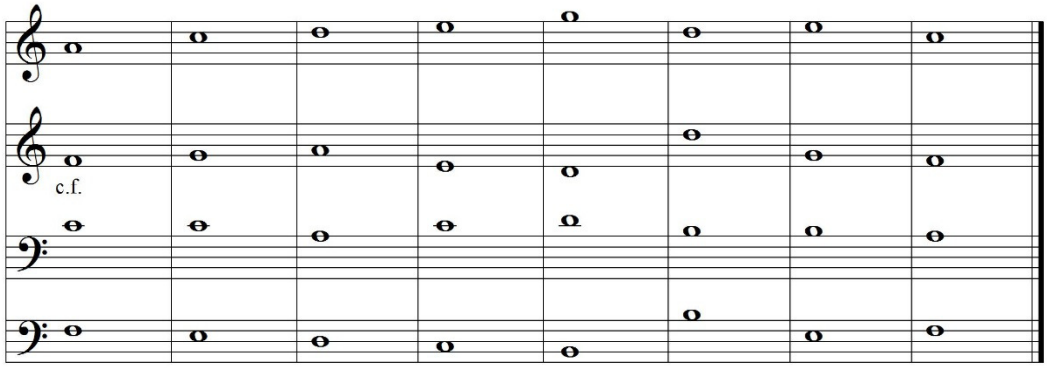
All notes must stay in place - there is one wrong clef.

On the next page there are answers to exercises 7 and 8.

10.



11.



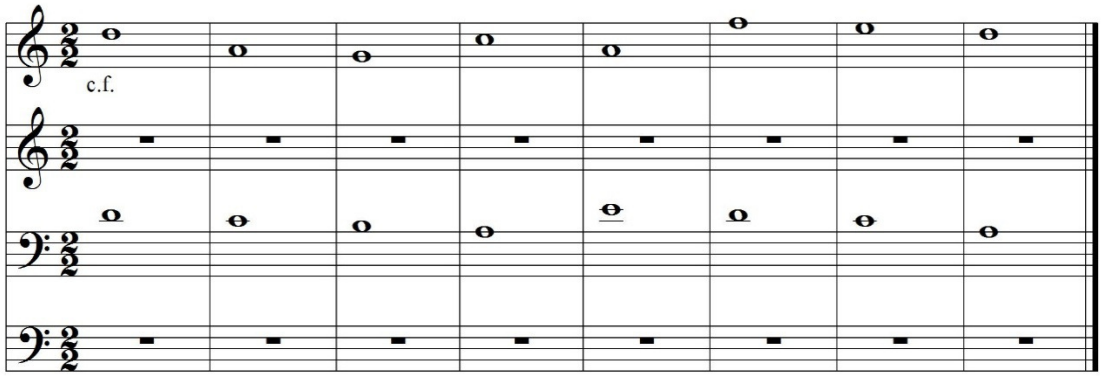
Second Species

If I put all voices in their places, and the part in half notes - last, that would represent the most complicated way of solving. It would be more appropriate after the c.f. to go either to the half notes, or to write one voice in whole notes and then the half notes.

The second and fourth species are some of the most difficult and at the same time the most important exercises, without, of course, counting the contrasting four-voice polyphony (which is extremely difficult, but not that important).

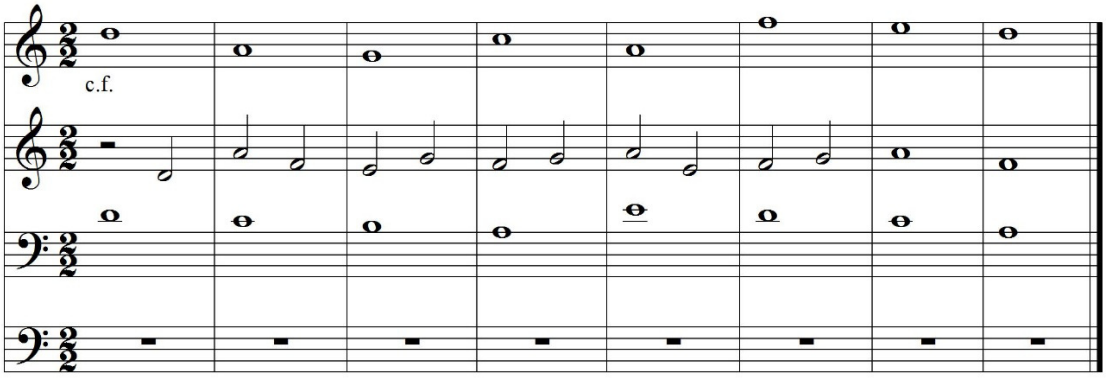
After the cantus firmus in the soprano I place a counterpoint in the tenor:

1.



The alto in half notes follows.

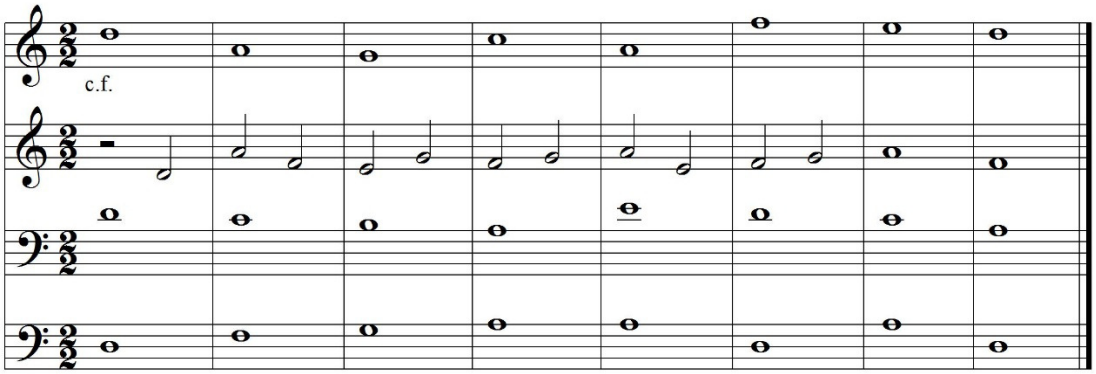
2.



It is possible that the problem will come out after adding the bass; it is also possible that it will not. I don't know yet: I'm writing in real time. But I do know that if the part in half notes leaps, the bass will be able to move a little more easily stepwise. And vice versa: if there are two or more voices, which move stepwise in general, the next part will have extra difficulties.

Putting the bass last (especially in half notes) makes the task even trickier, as does the very principle of writing from top to bottom.

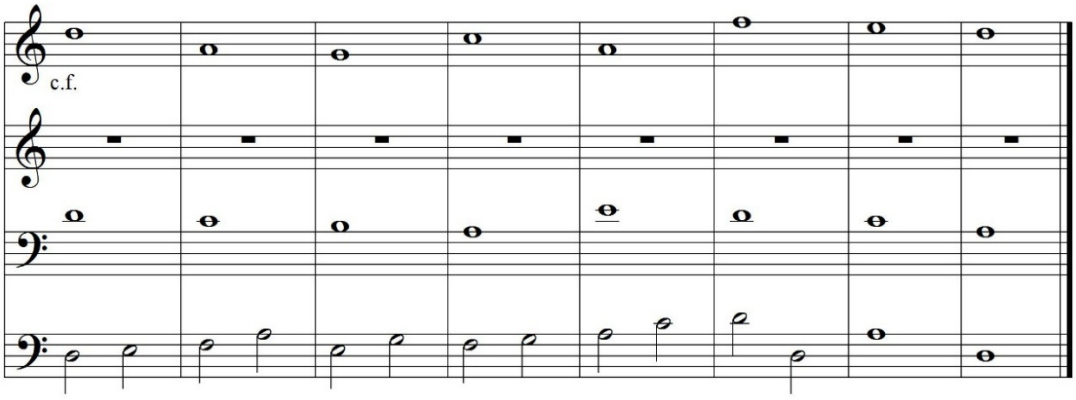
3.



It came out right from the first time - without any problems.

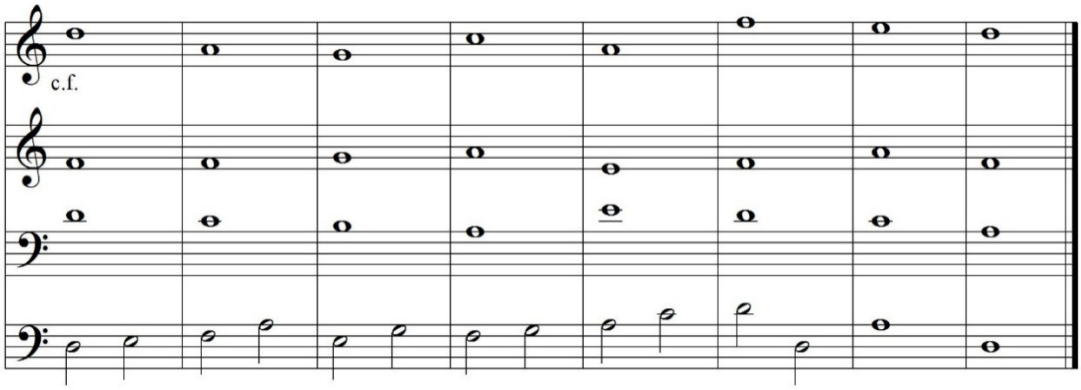
Now I return to Example 1. The half notes will go into the bass and the alto will be last.

4.



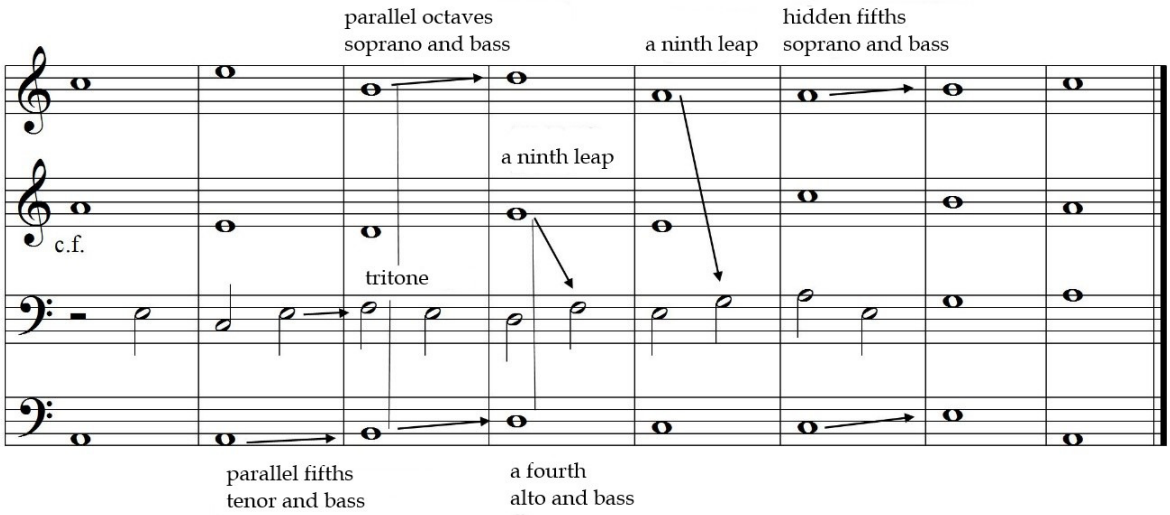
This is more difficult. But it didn't turn out to be impossible. I could change the tenor (not the soprano) if I needed to, but it didn't turn out to be necessary. Will the alto be possible?

5.

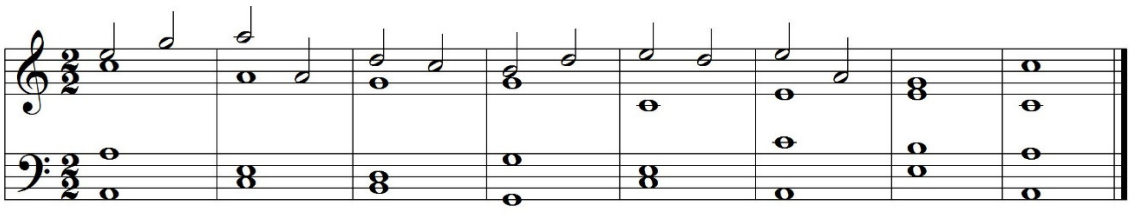


It worked out easier than the previous step. A common problem is the half note on the second beat - it can sometimes leap into dissonance with the other voices.

6. Incorrect example

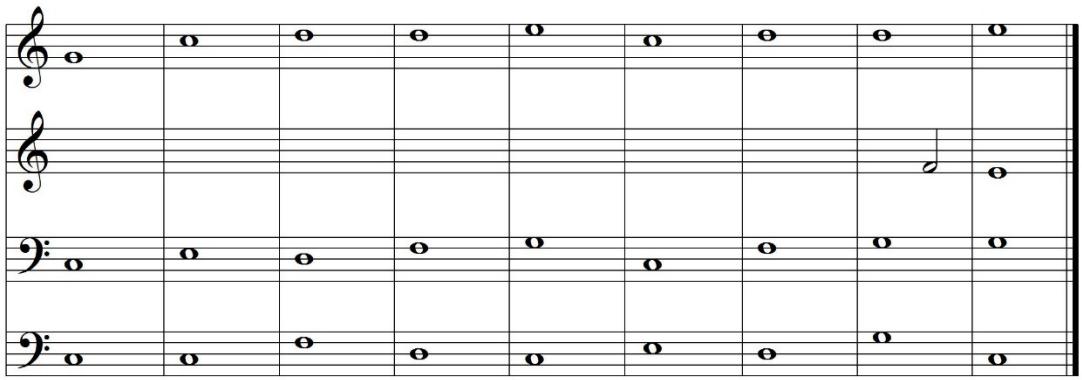


7. Example for analysis - no crossed voices in the parts with whole notes



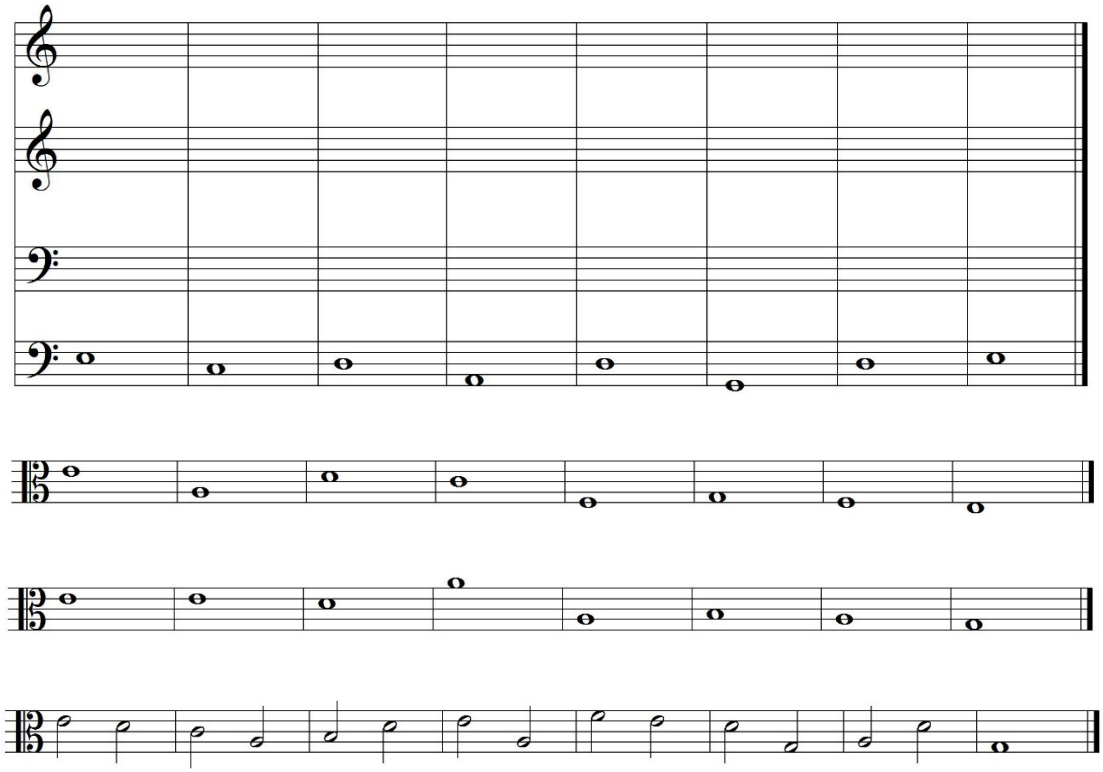
Exercises

8.

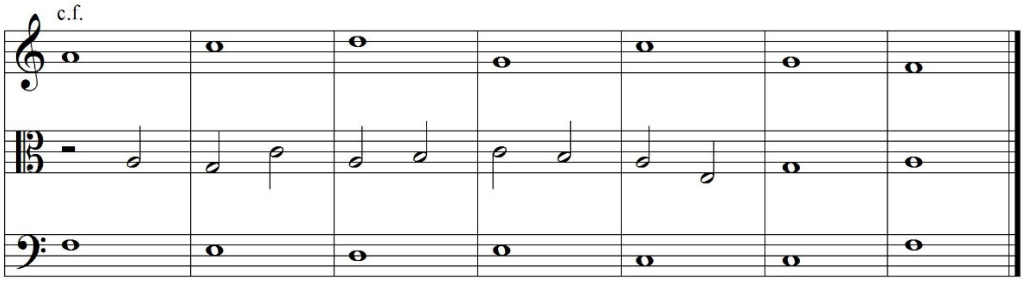


In the following example, only the bass is in place, and the other three voices are all written in alto clef for some reason. In what order should they be written? Note: in order to confuse the reader, I wrote an abnormally low soprano - in fact, this assignment seems to be for a bass, a tenor, and two alto voices.

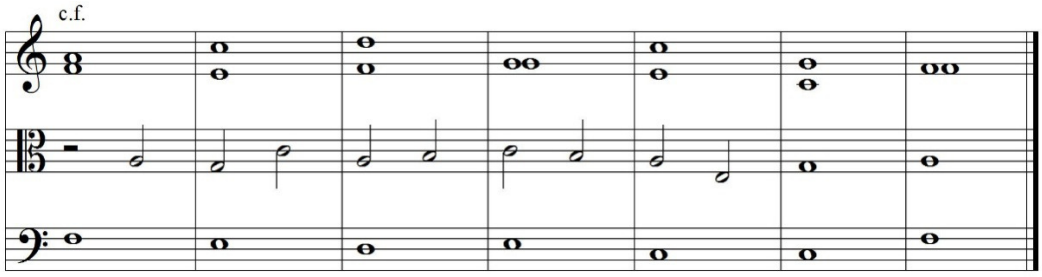
9.



10. The alto is missing here - it could in theory be crossed with the other voices. It can be written on the uppermost staff.



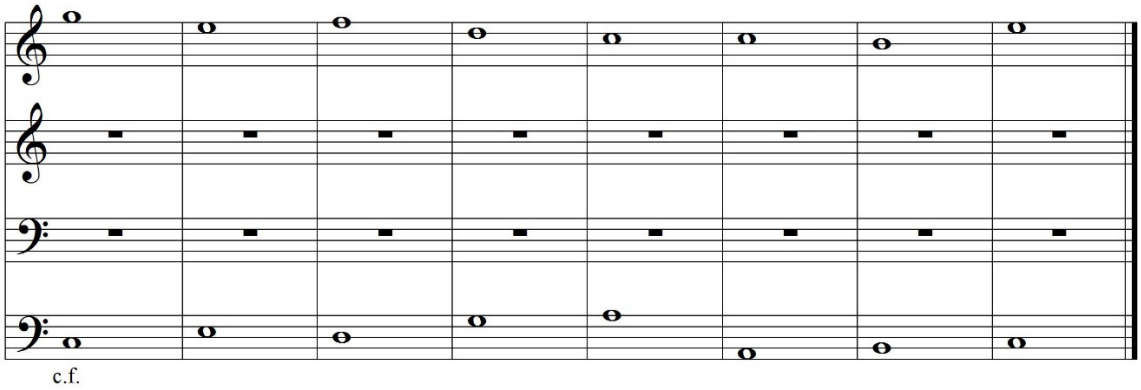
11. And here I tried to solve the exercise. Did I do it right?
There are no crossed voices in my solution.



Third Species

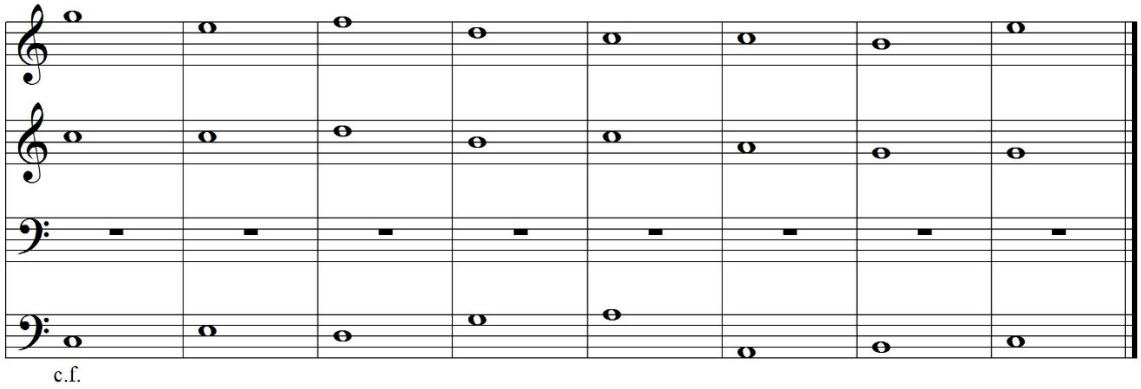
This time I'm going to experiment - I'll insert the part in quarter notes last.

1.

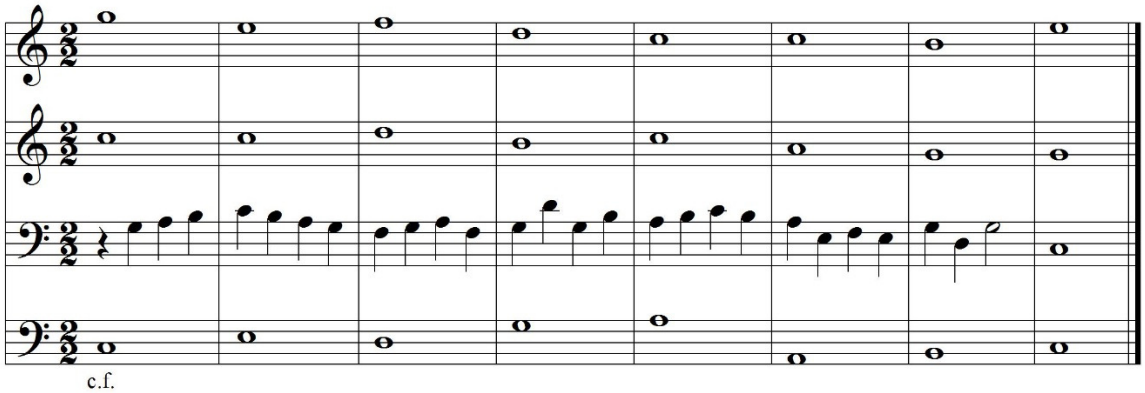


Further:

2.

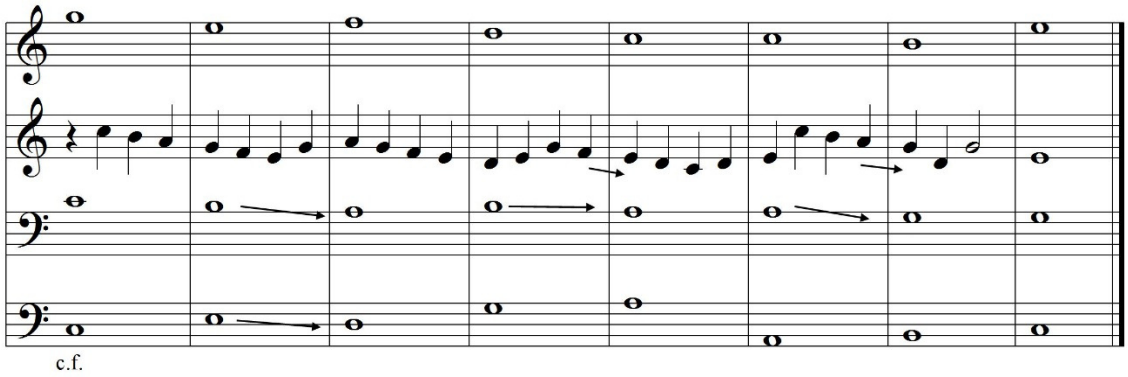


3.



It is not particularly difficult. The following is an incorrect example.

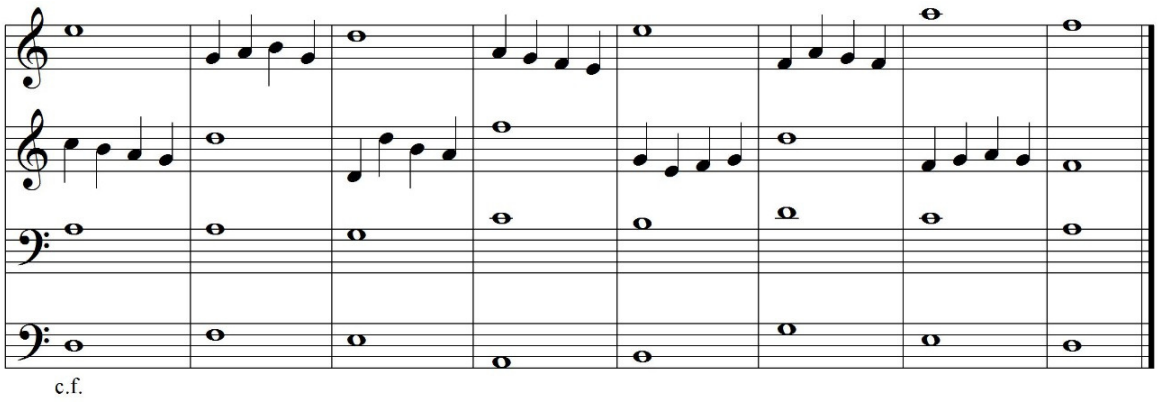
4. (Cambiata in m. 4)



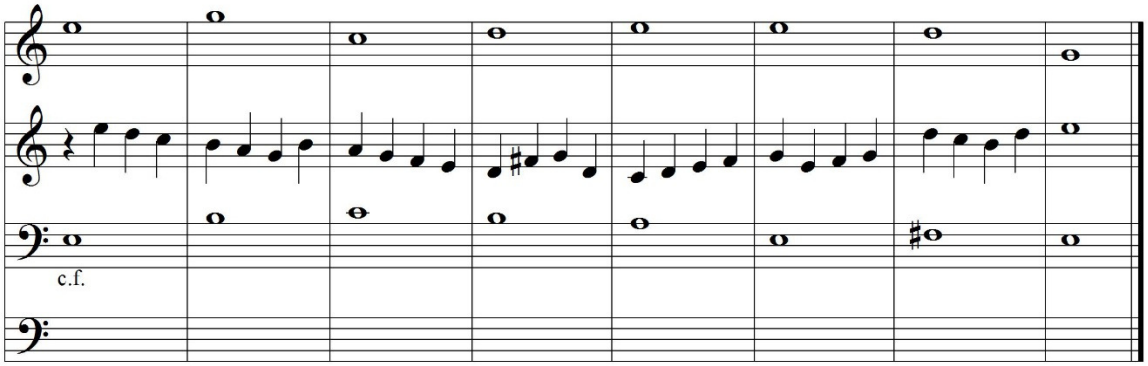
Exercises

The soprano and alto are both wrong, but they are mistaken in pairs - there is a correct note in the alto under each note of the soprano. The last measure is correct.

5.

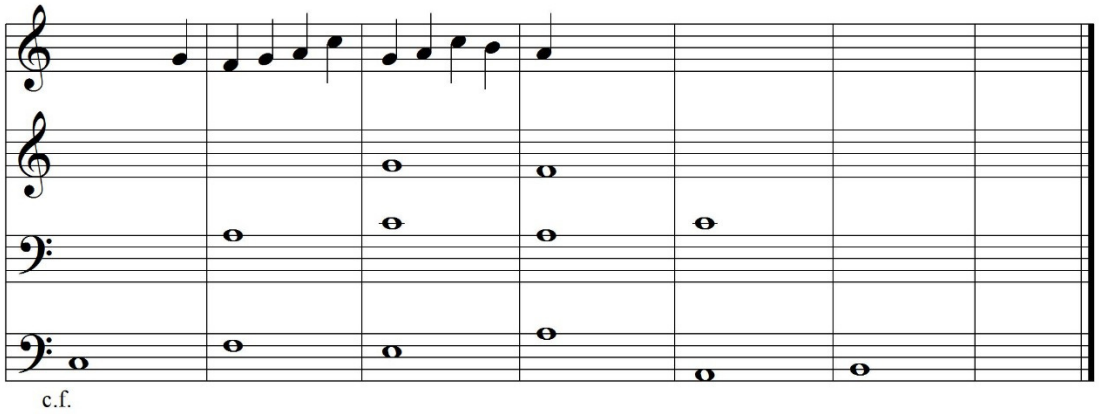


6.

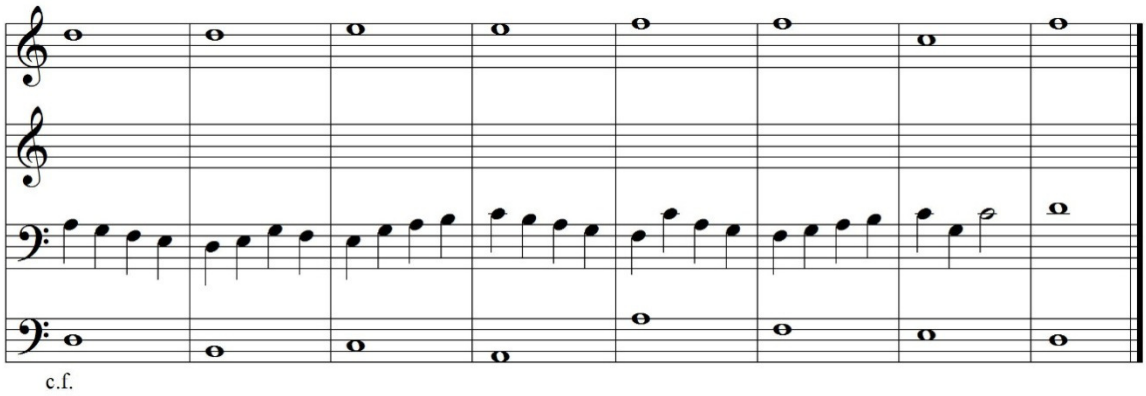


(If there is a F sharp somewhere, I wonder what other note there might be?)

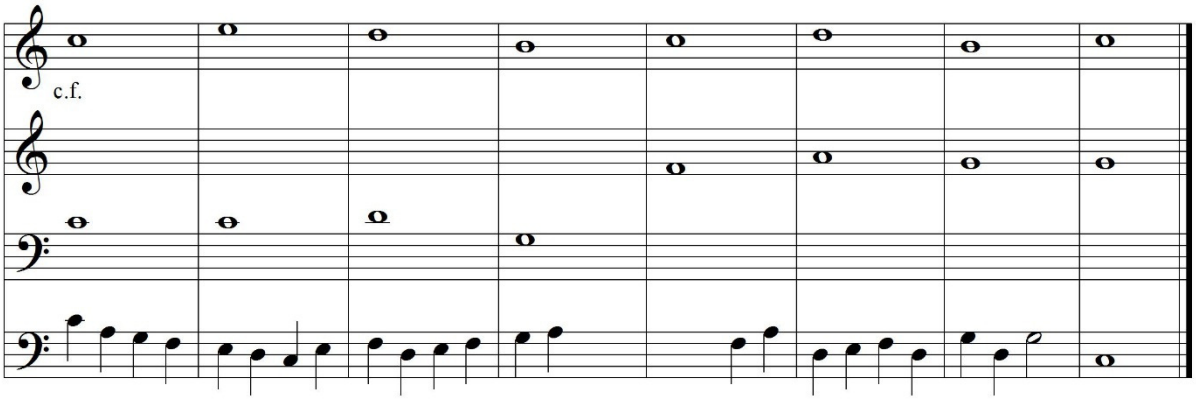
7.



8.

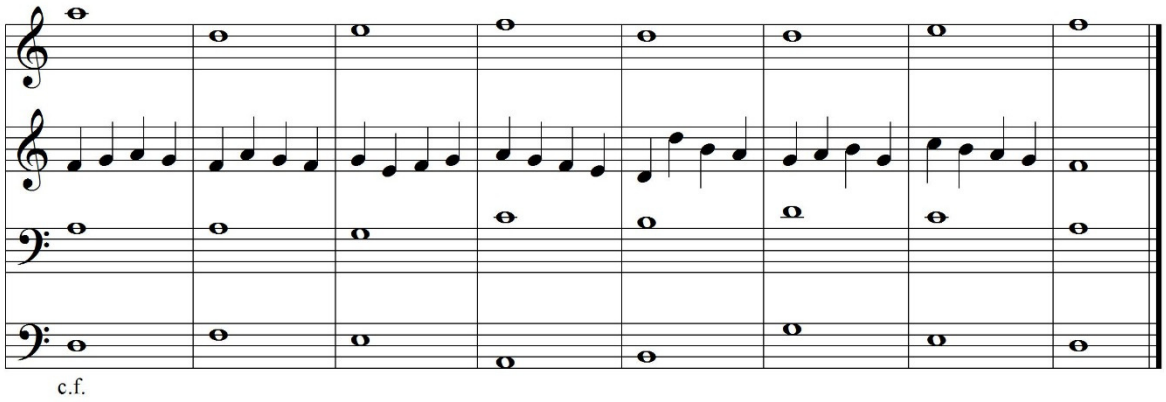


9.

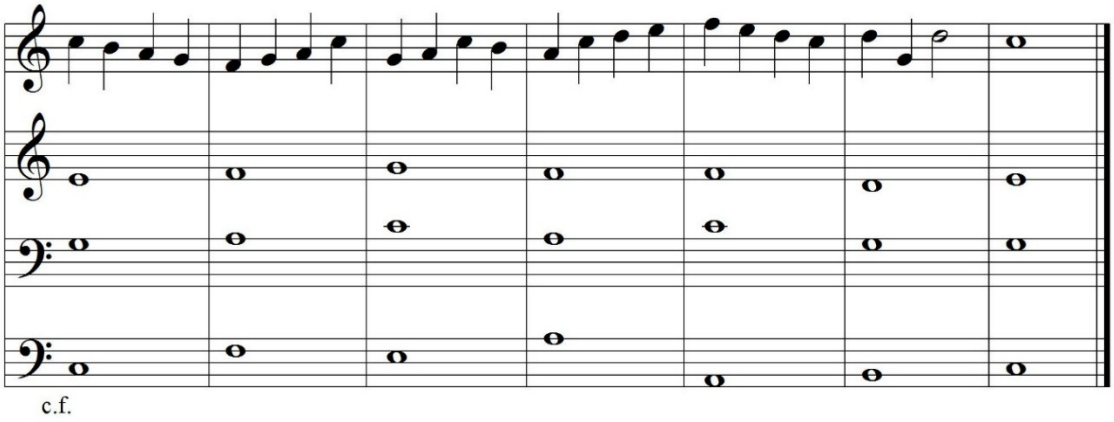


Two solutions follow.

10. Solution for example 5



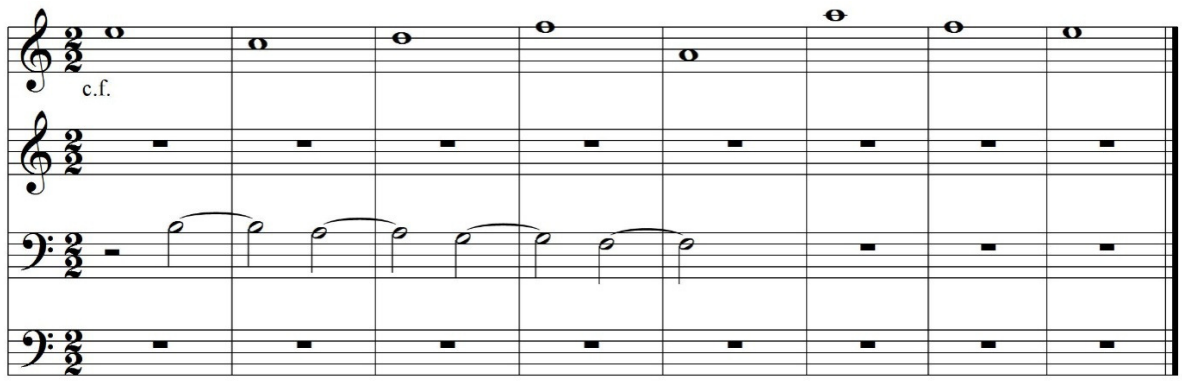
11. Solution for example 7



Fourth Species

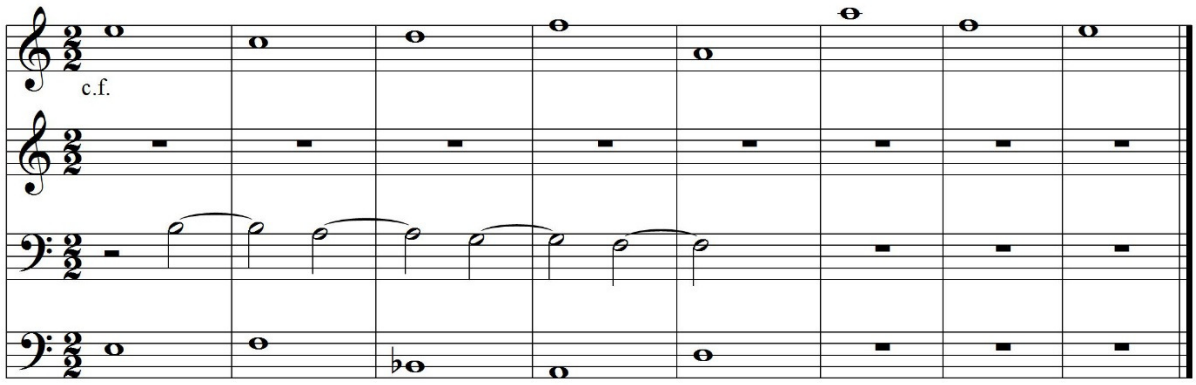
This type of exercise is a bit more complicated. My first approach is to add the fourth species right after the cantus firmus[1].

1.



As you can see, if I have chosen to start the counterpoint from the fifth degree, I have no choice until m. 5 except to descend stepwise. For now, I won't go any further - first I'd like to clarify whether I could add the bass in the first five measures.

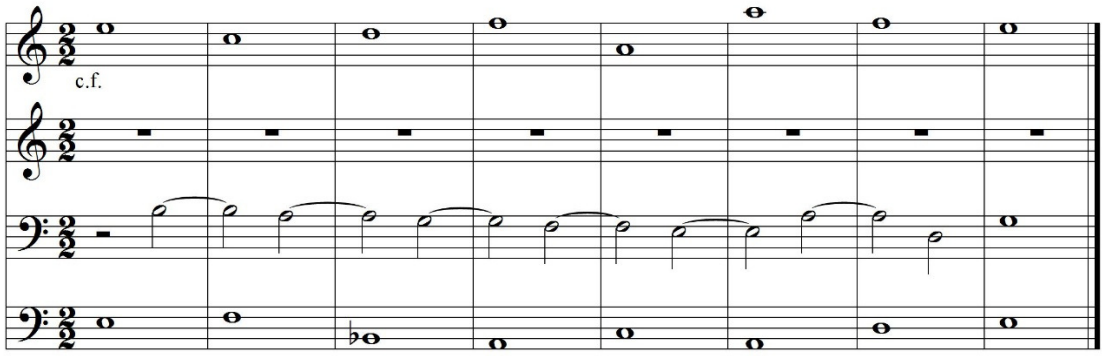
2.



This is what seems to me to be the only possible variant. I'm obliged to add either B fla*t* in the bass in m. 4, or F sharp in m. 2. The possibilities are limited in this mode; my experiments have shown that if the counterpoint in the fourth species starts at the first or third degree, there is still a solution, but it is not easy. For now I will continue with the solution already started.

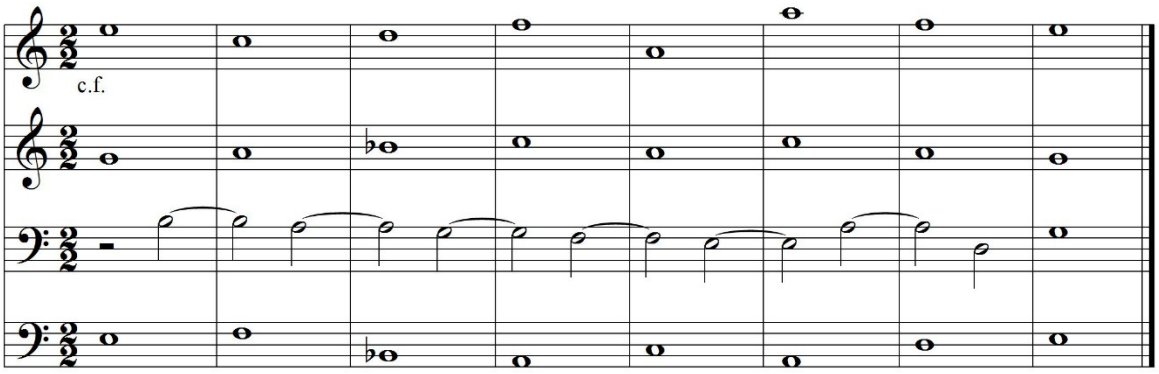
[1]Having treated it as a one-time exception, I allowed myself a sixth leap downwards in c.f.

3.



This also seems to me to be one of the few possible variants, if at this stage I insist to avoid breaking of a tie.

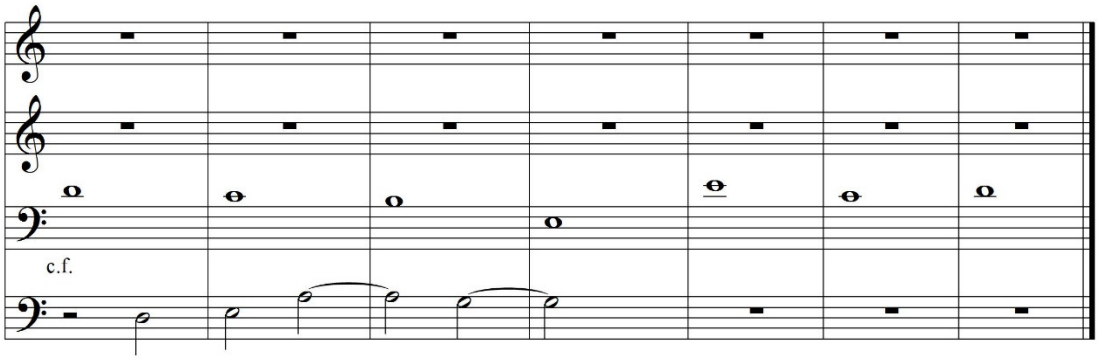
4.



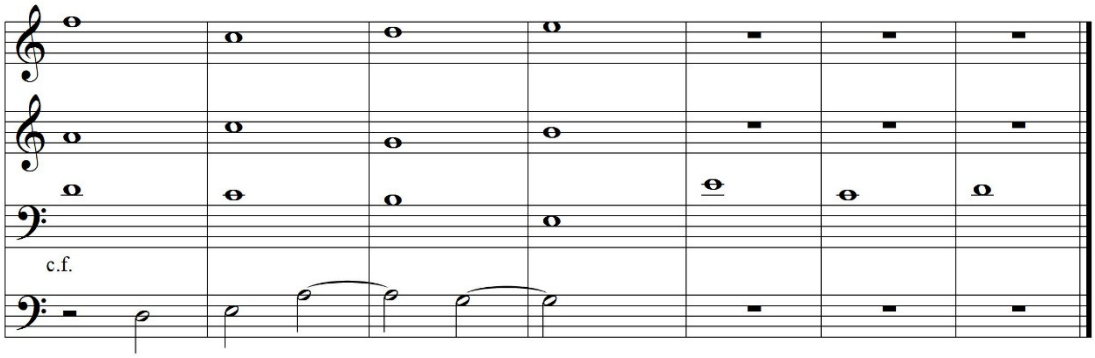
Still it worked out.

In the following example, the c.f. starts at the first degree and descends, so there must be a break of the tie in the bass. Again, I write it only partially and then explore the possibilities of adding other voices.

5.

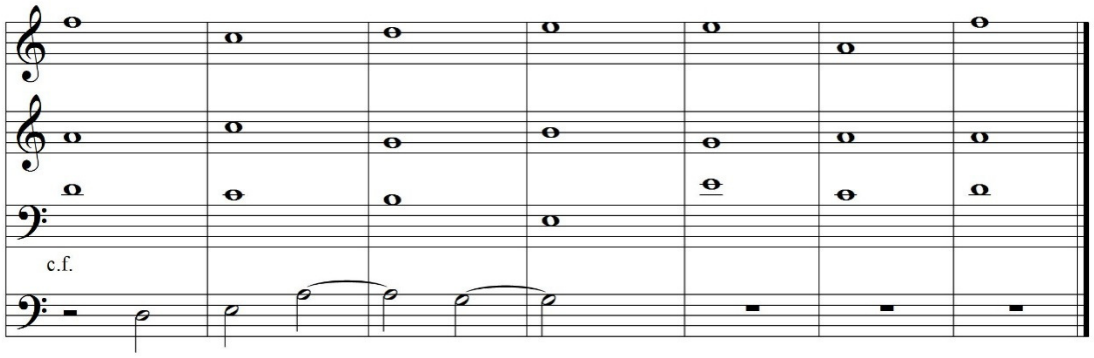


6.



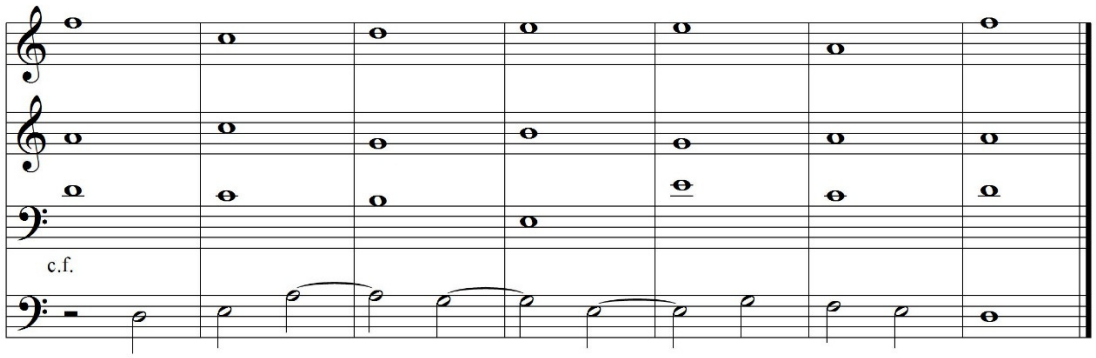
I observe the same phenomenon: the variants are few, but they exist.

7.



At this stage, I do not yet know if there is a solution.

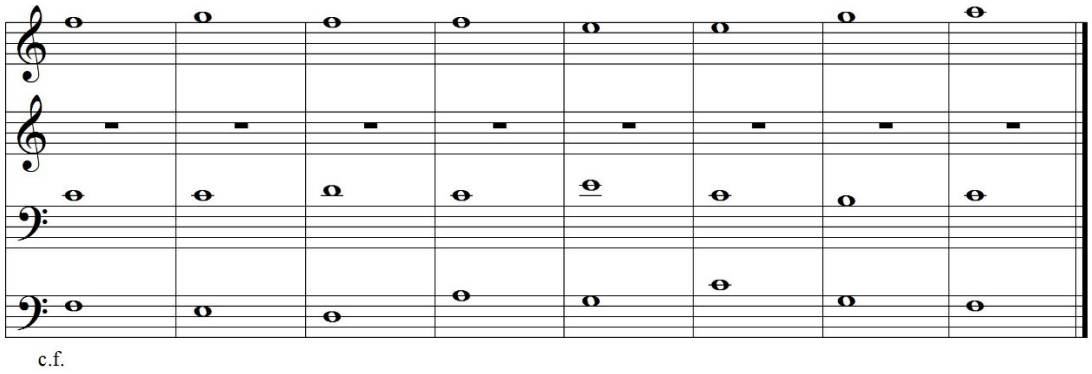
8.



With one more break of the tie, the task is completed. I come to one conclusion: if there is a unison between two (or even three) of the inner voices, it simplifies the syncopated voice.

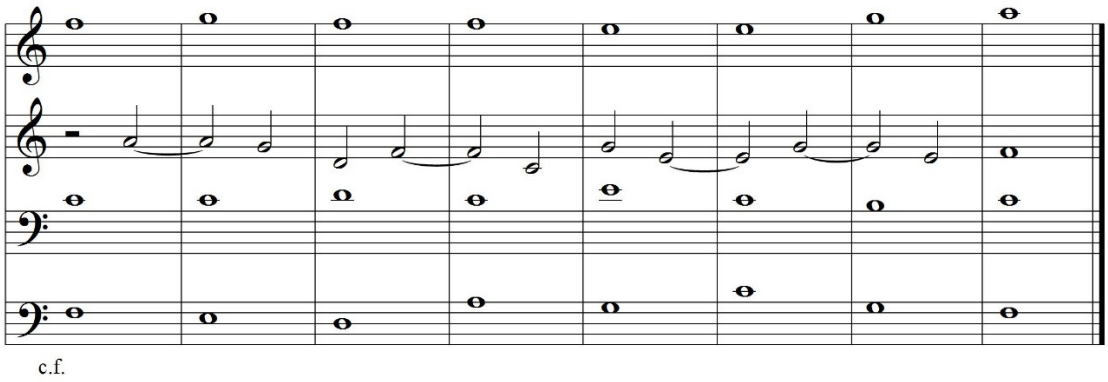
Last experiment - I place the fourth species last.

9.



Preparing the alto, I tried to move in a mutually contrary motion, repeating notes and allowing unisons here and there. It doesn't have to be the latter, an octave can be used; if one of the three chord tones is missing, i.e. there are only two (or one) tone, that would make my work easier. But I still have to put it to the test.

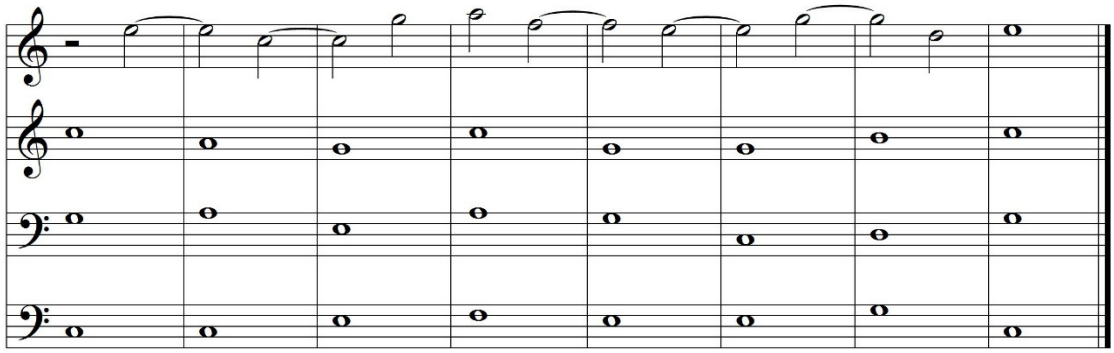
10.



Significantly more difficult. In m. 5, 6 and 7 I can only leap between E and G. There is a crossing of voices and again two tie breaks. But it turns out to be possible - as long as I have the patience to look for the correct variant.

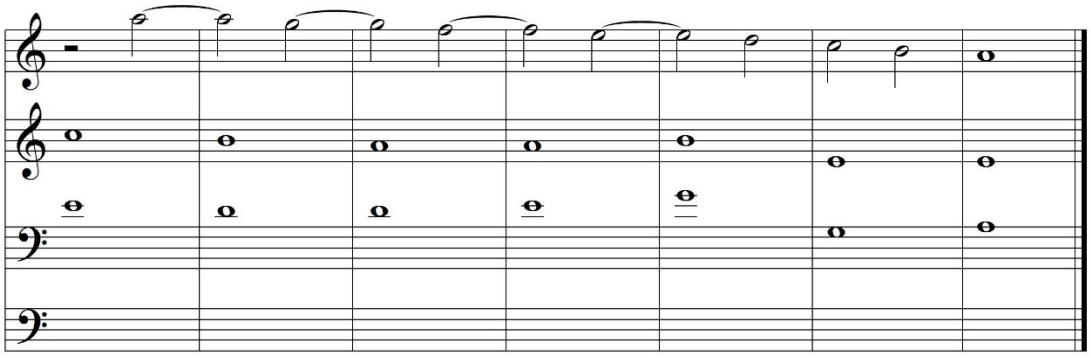
In the following example there are two errors. Or maybe there are more?

11.

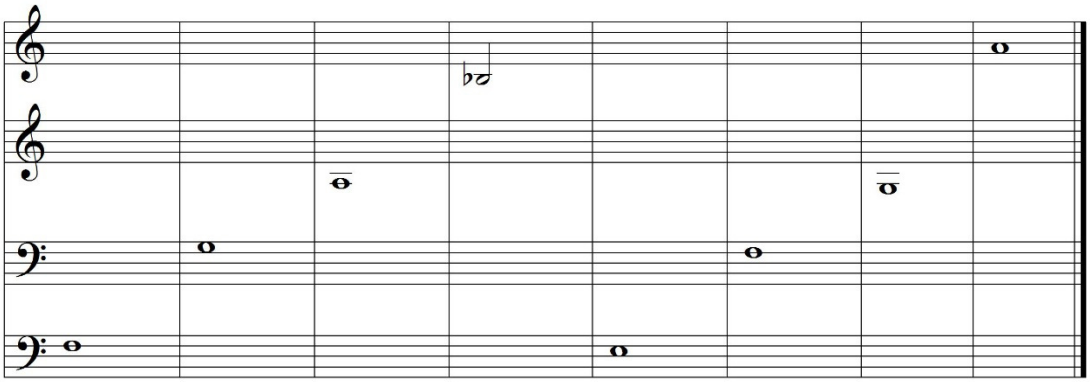


Exercises

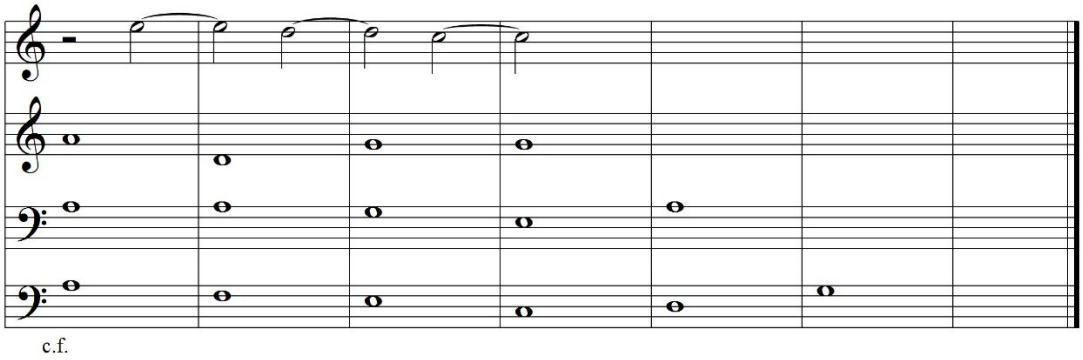
12. There is no cantus firmus in this exercise



13. In this - too (according to my solution)

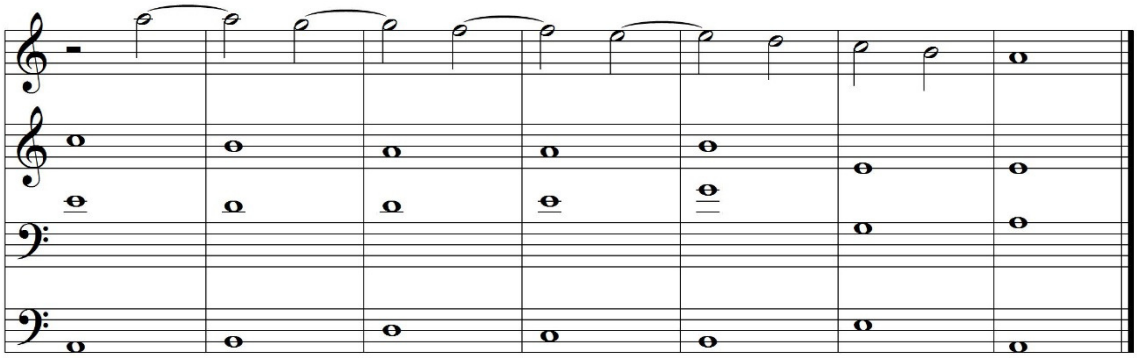


14.

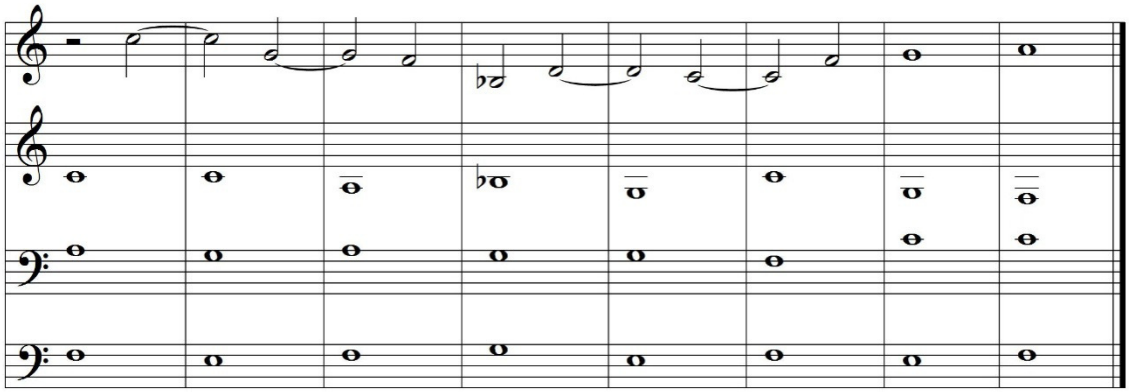


Following are the solutions to all three exercises.

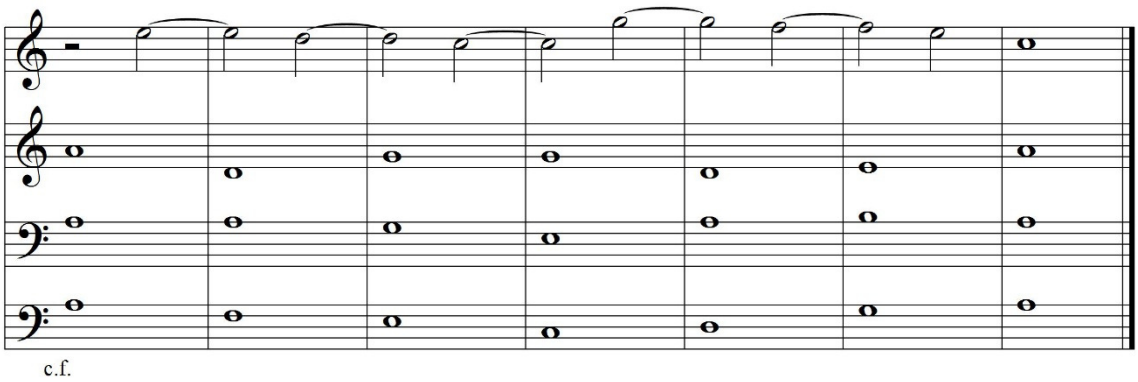
15.



16.



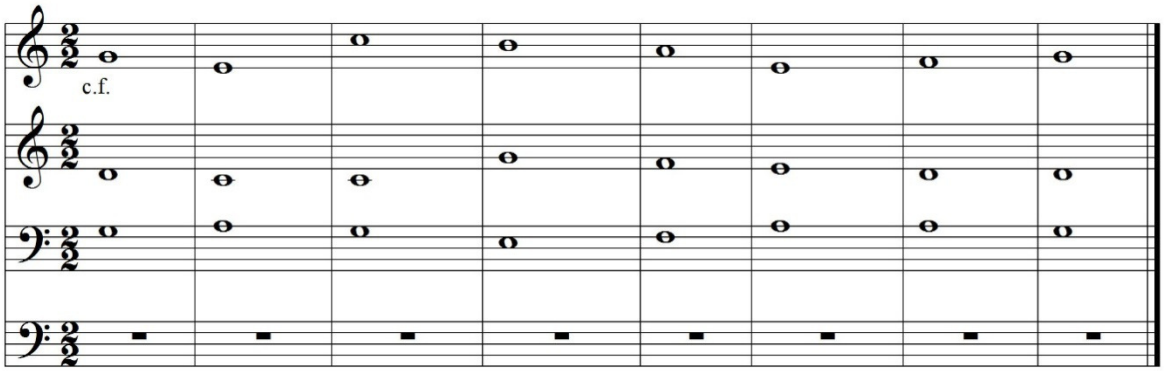
17.



Fifth Species

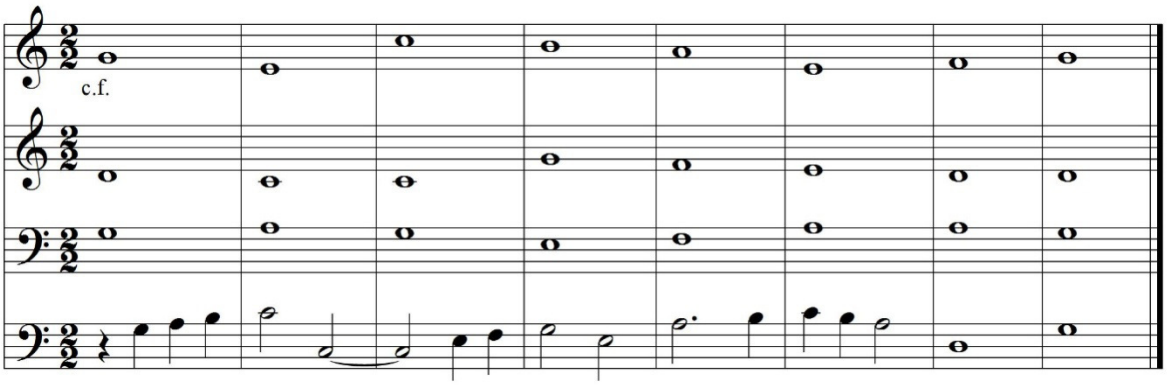
This time for the sake of the experiment I will place the florid melody last.

1.



I write without planning ahead. Here's what resulted:

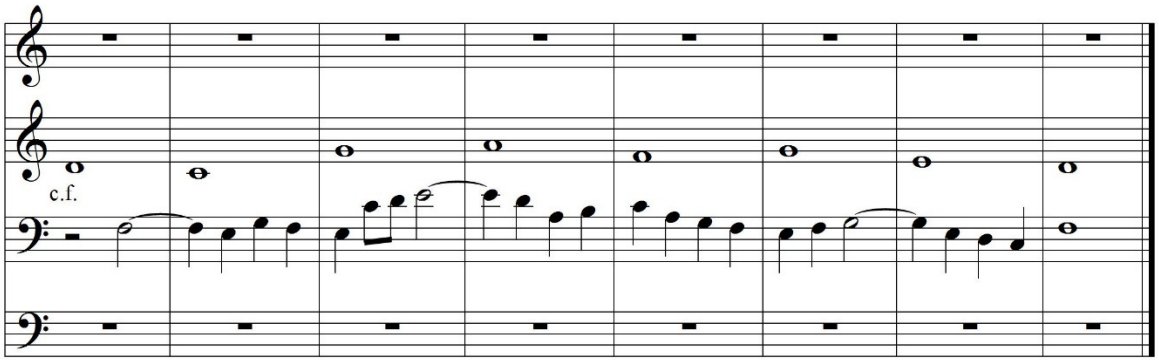
2.



Even though I write from top to bottom without much thought, a paradox of sorts occurs - placing the fifth species last in the bass proved easier. The reason for this is that the last voice can now conform to all the others, whereas if it had been placed second or third in order, it might have led to conflicts in laying the last counterpoint in whole notes.

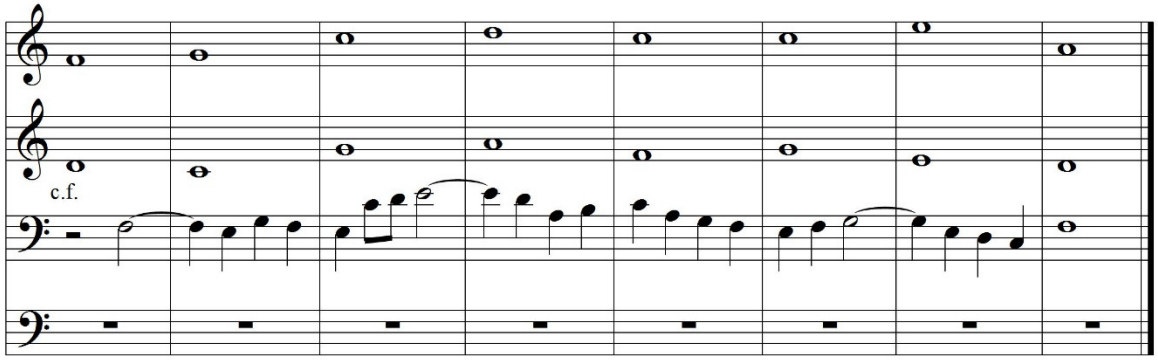
I'll try this right away too.

3.

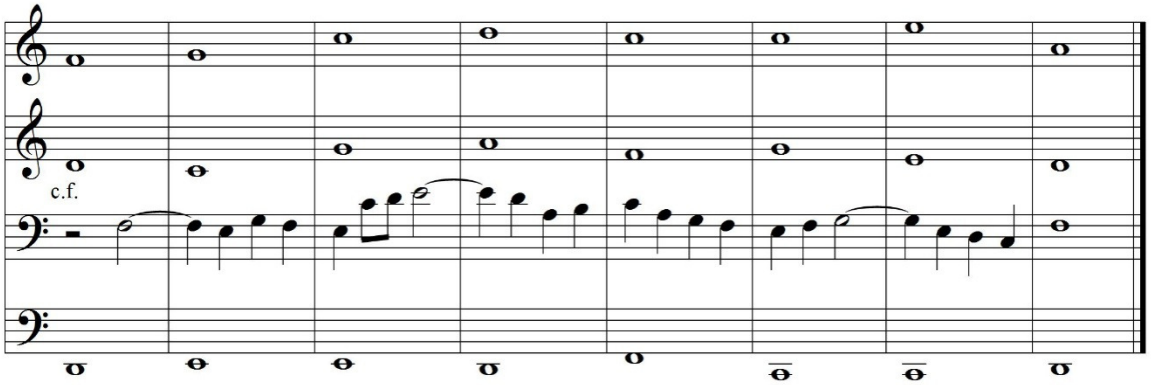


The next step would be to write the bass. But I place the soprano first:

4.



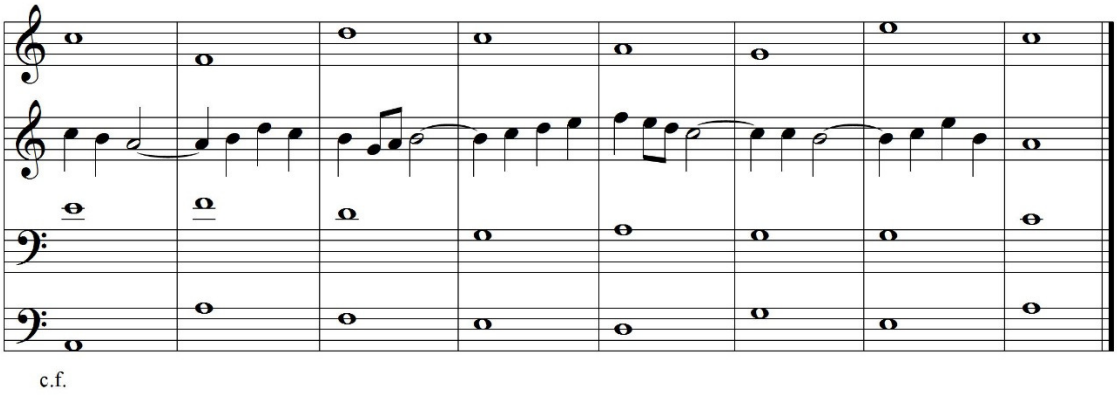
5.



The exercise worked out without any particular problem, because even on the previous step I had some idea of what my bass would be like[1]. This is done simply by calculating the vertical of each measure. For example, in m. 1 the bass has to be D; the same applies to the last measure. In the second measure, the bass must be E or C. The same applies to measure 3. In m. 4 the bass should be either F or D, and so on.

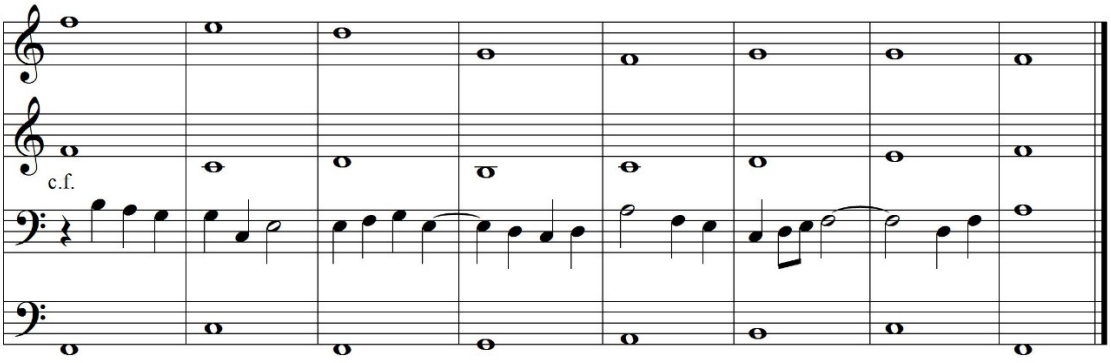
[1]This bass is a little low for performance. It could be written an octave higher - except on the second measure. There the bass and soprano clash unpleasantly - a faux second results.

6. Incorrect example - without showing the errors.

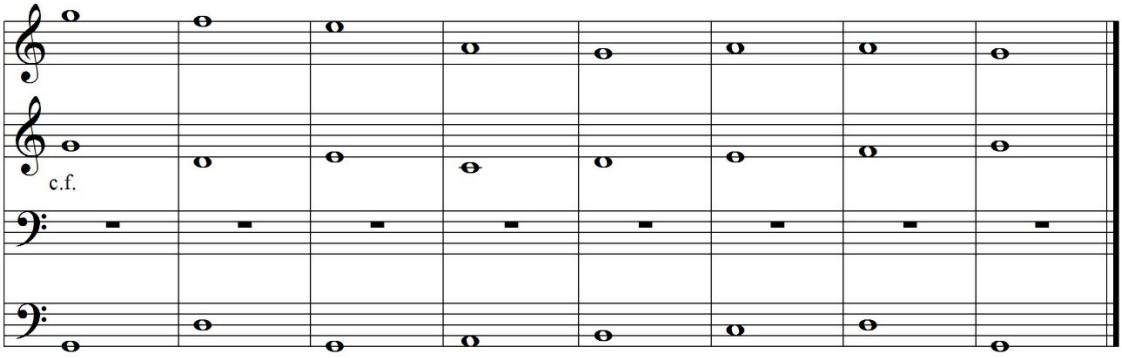


Exercises

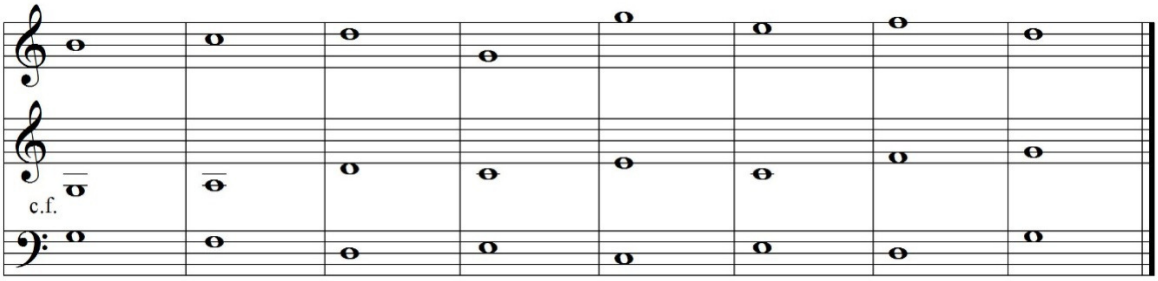
7. Some measures of the florid melody are transposed



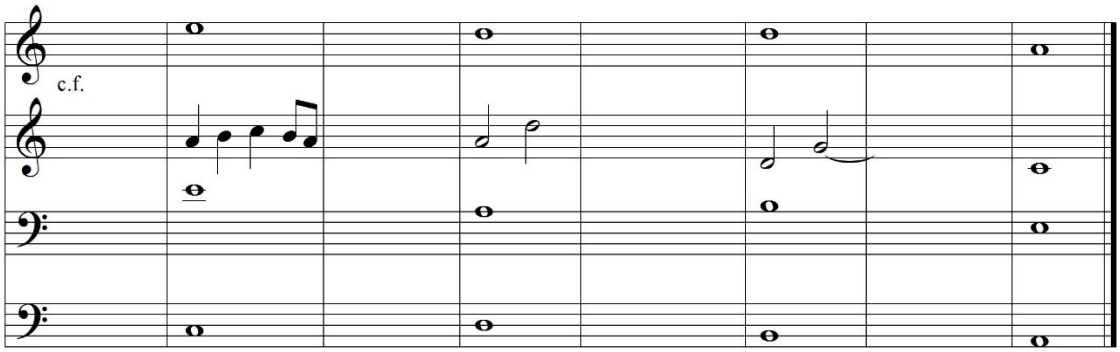
8.



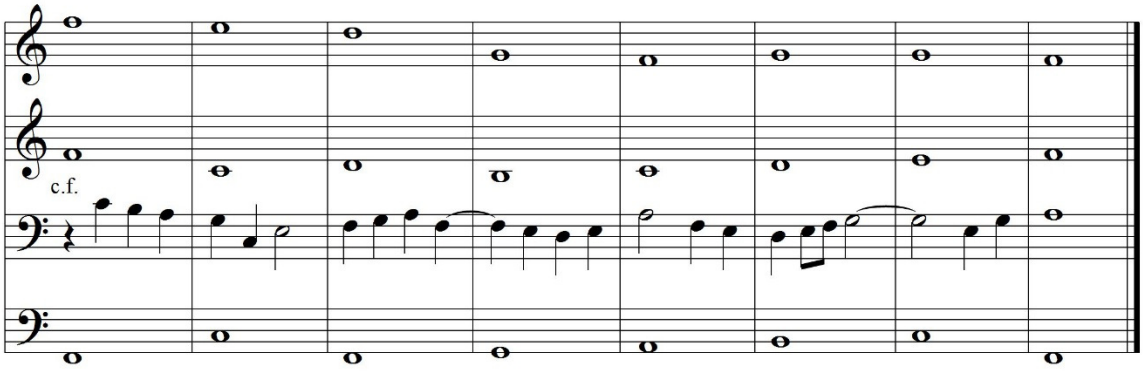
9. Can you write the fifth species under the soprano?



10.



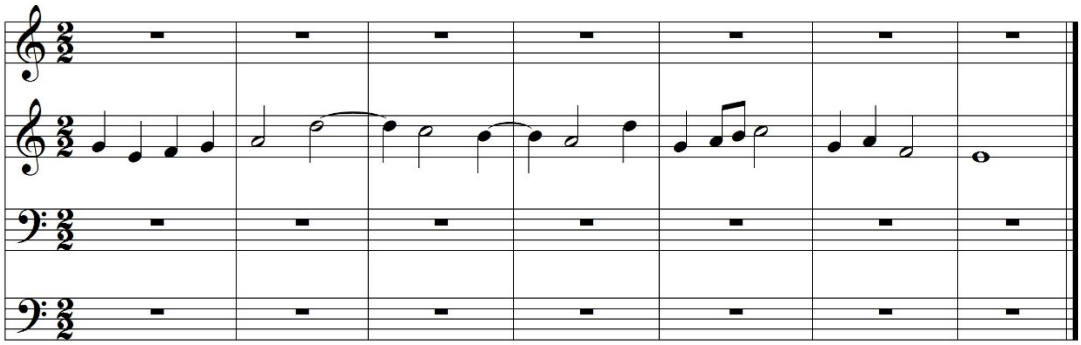
11. Solution of example 7



Four Florid Melodies

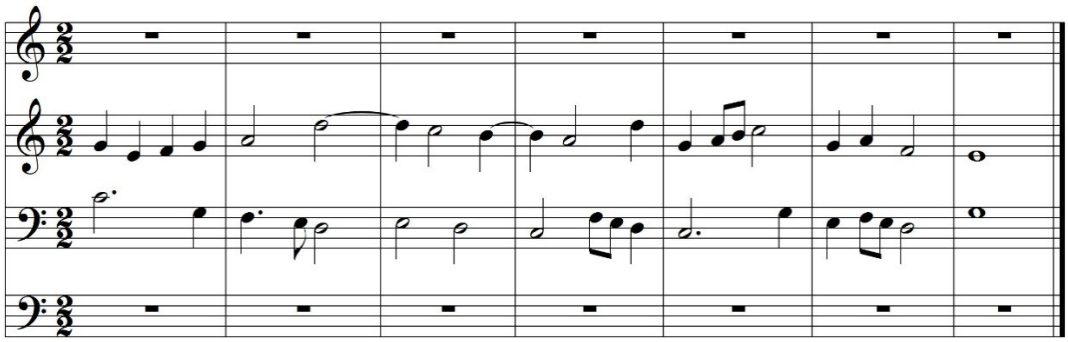
Complementary (quarter note) rhythm is important in this mode; the principle is to write independent melodies.

1.



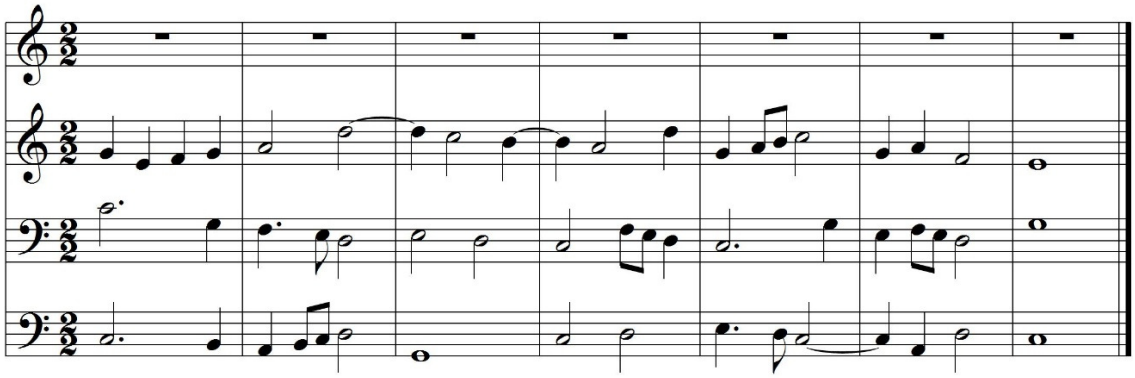
This melody is thought to be in the Ionic mode (from C). There is a "hint" of chain syncope which can be realized in m. 3 and 4.

2.

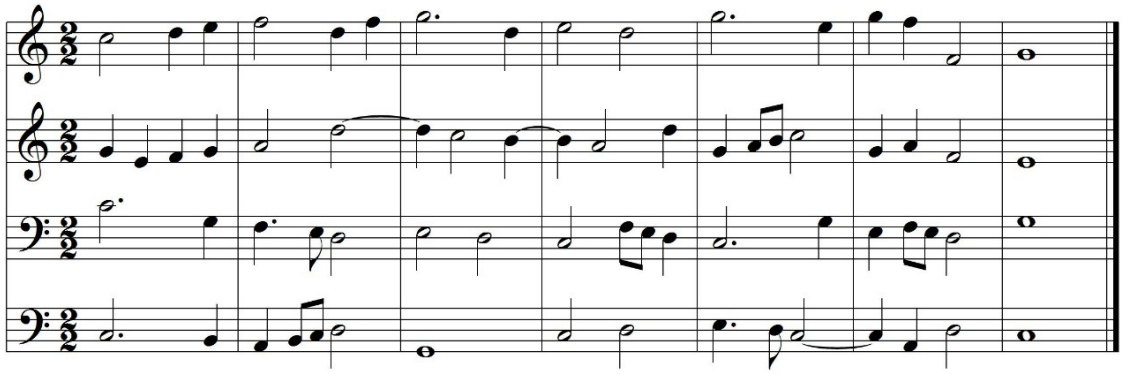


Here this chain syncope has already been realized. Since the bass has to have C in the last measure, I decide to have a G in the tenor and E in the alto.

3.



4.

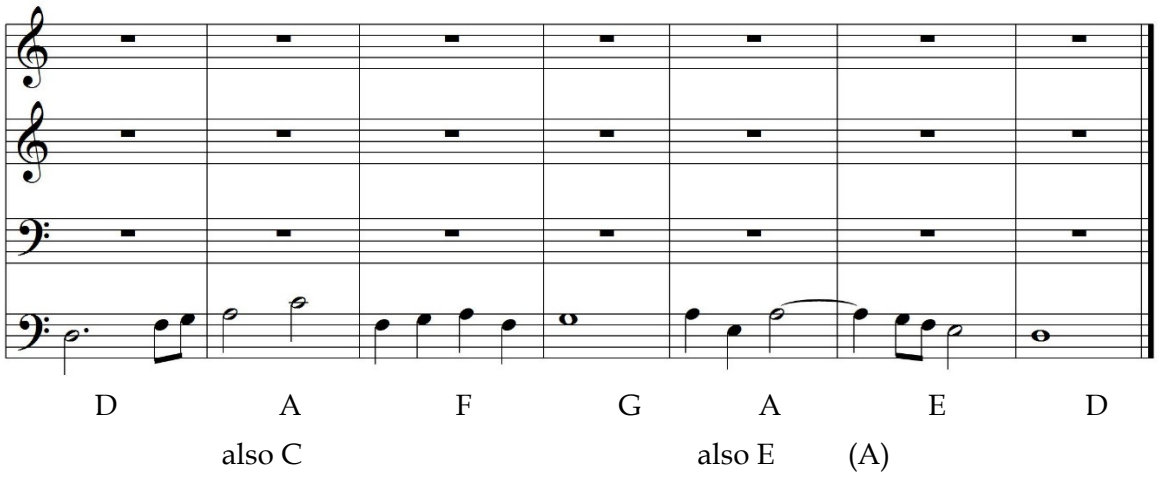


The penultimate measure of the alto has complicated things a bit, and that's why I need some "equilibristics" in the soprano's penultimate measure. As in two-voice and three-voice polyphony, if the other voices are moving, it is possible to write a whole note sometimes - as in m. 3 in the bass. This exercise was not written entirely in real time. I had to spend some time with its penultimate measure. Also, the original first note in the tenor in m. 4 was E. I have replaced it with C.

There are many possible variants for solution in this mode, and it's basically easy - as long as I can see those variants. Vertical planning continues to be very important - especially targeting the "concrete foundation" - the bass. It would be illogical to place it last.

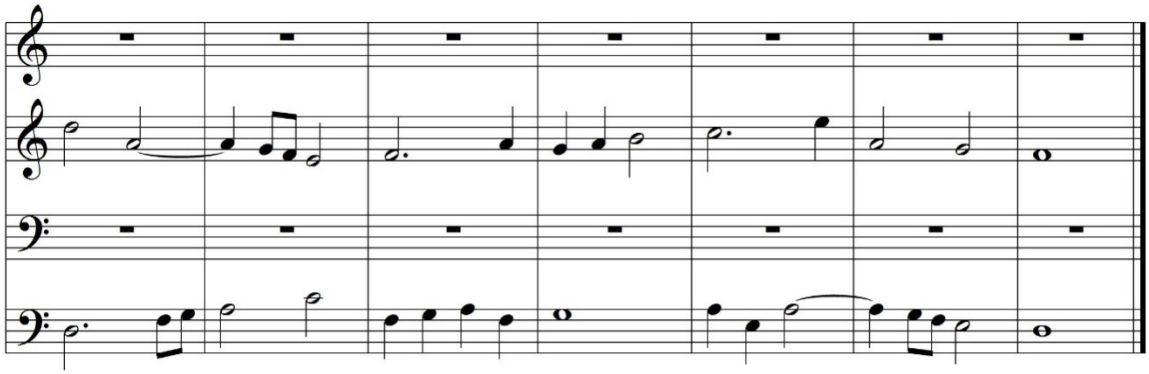
Again, an experiment - I will try to write a real-time exercise where the bass is marked and therefore is marked its potential vertical.

5.



Alternatively, an A in the third measure for a quarter note can be used. The next voice I will take care of will be the alto.

6.

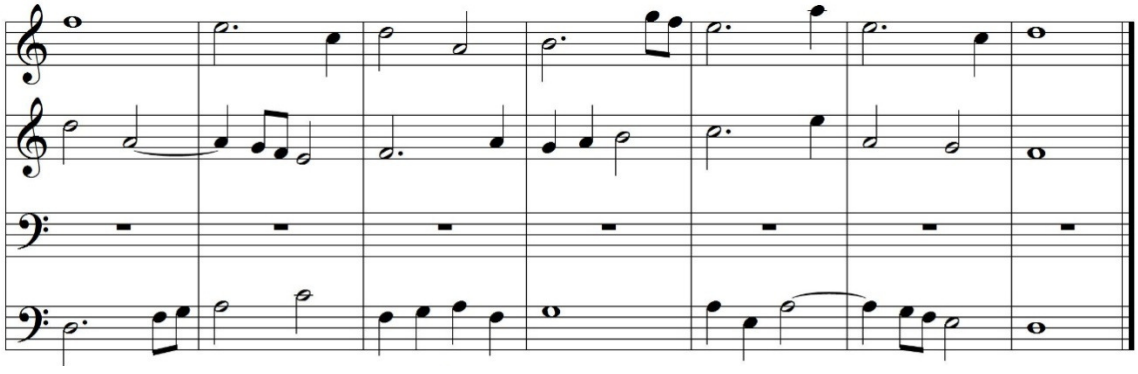


D A F G A E D

also C also E (A)

The soprano follows.

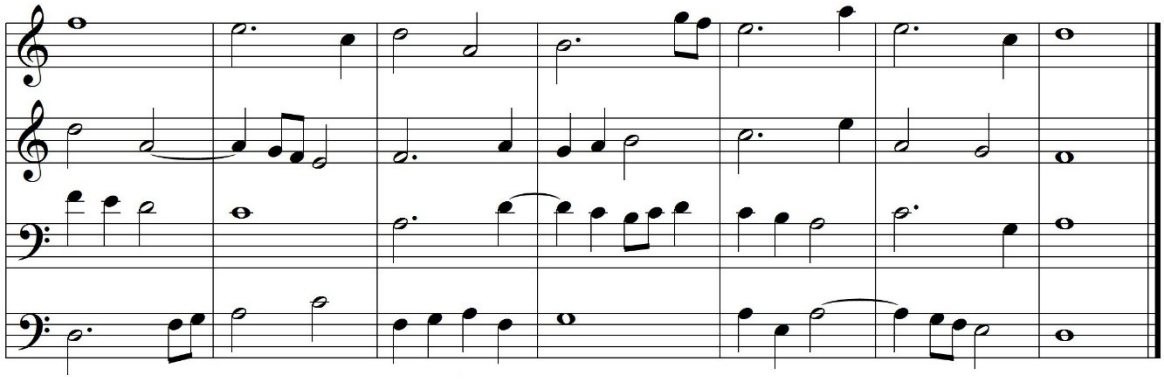
7.



D A F G A E D

also C also E (A)

8.

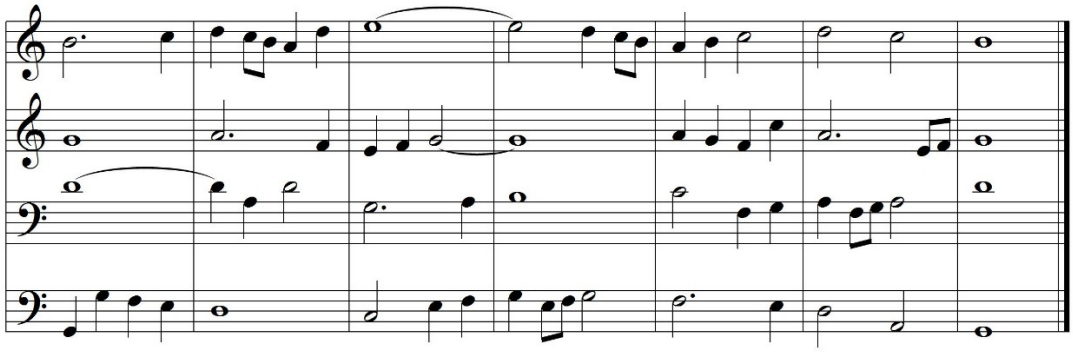


D A F G A E D

also C also E (A)

Planning in advance is helping me, without a doubt.

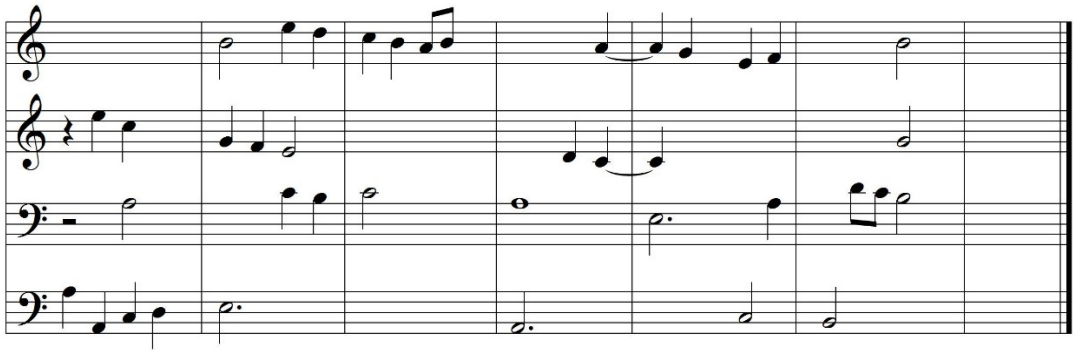
9. Incorrect example



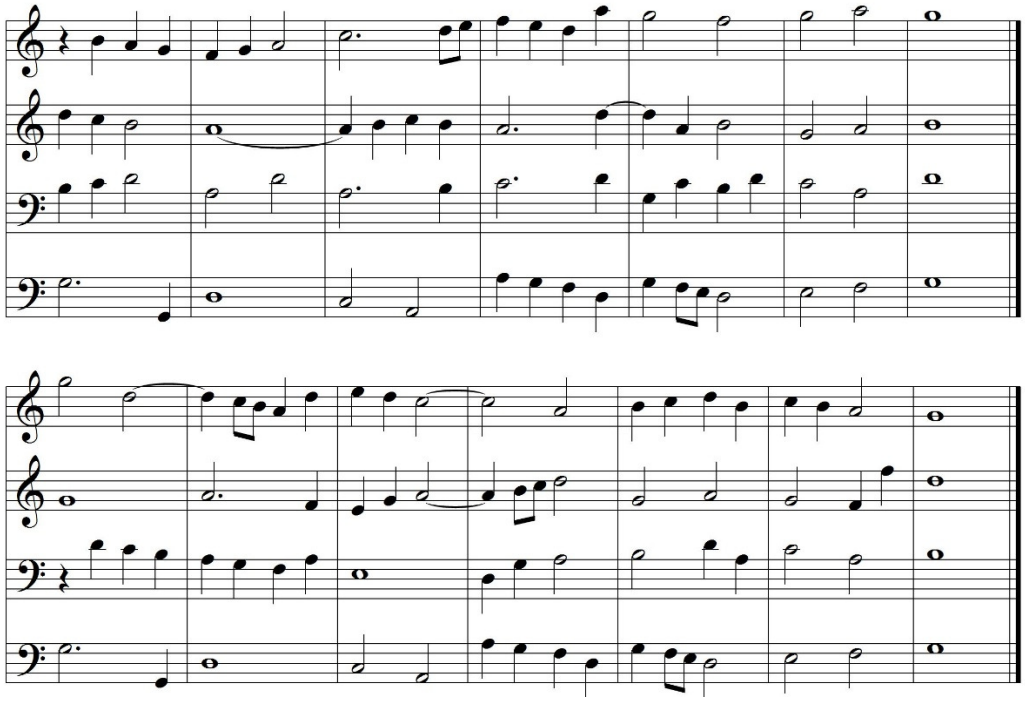
Am I mistaken, or is there really only one error?

Exercises

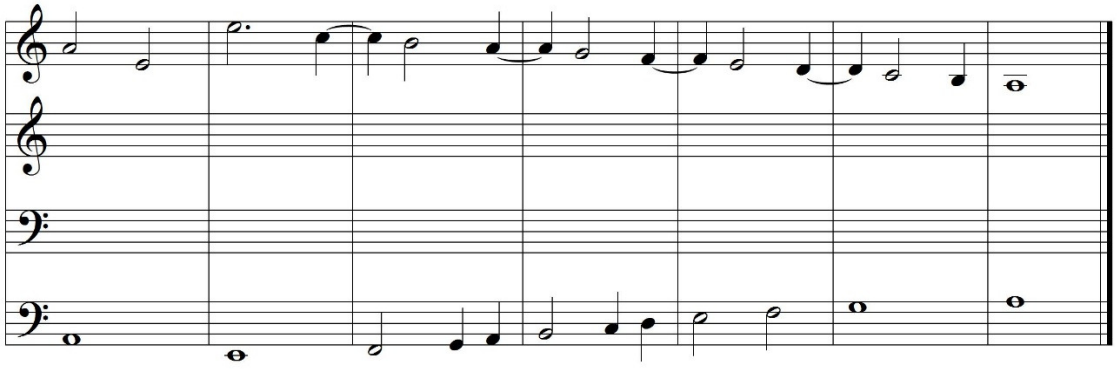
10.



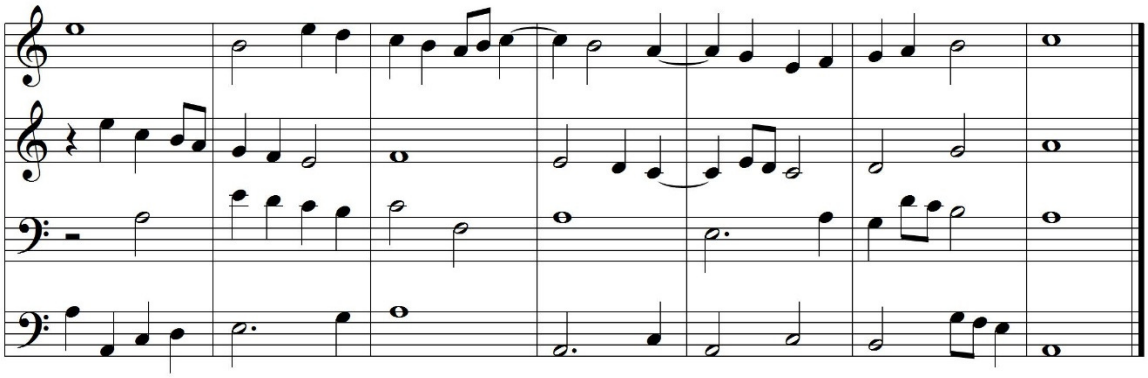
11. For both exercises to be correct, something must be taken from the one and placed into the other. The bass is identical.



12.



13. Solution of example 10



Regarding example 11:
?Selpm axeow tehtf osonar posehte gnahcx eIfinep pahlliwtahw

“Mystery creates wonder, and wonder is the basis of man’s desire to understand.”

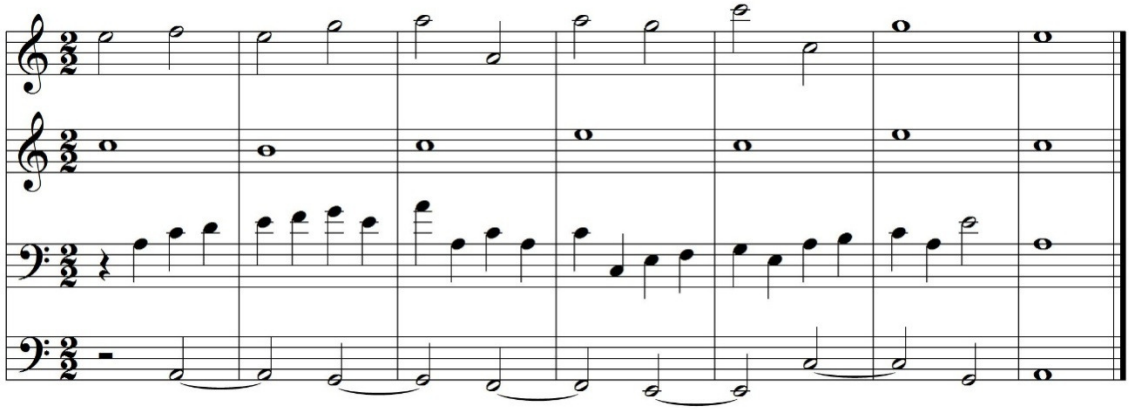
Neil Armstrong

Contrast Four-Voiced Polyphony. Polyphony in More Voices.

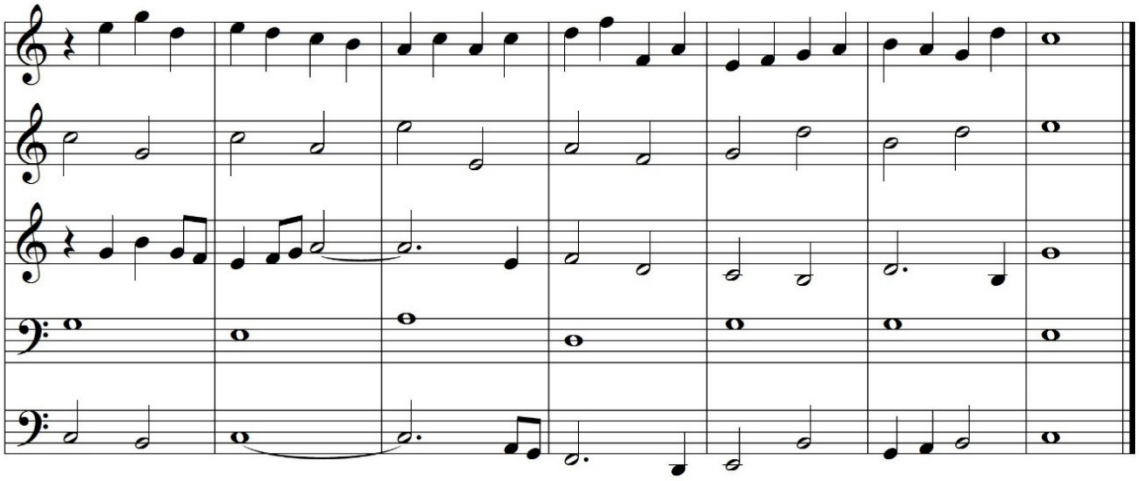
Traditionally, the contrast four-voice/quadruple polyphony and the contrast counterpoint are not practiced as part of the standard curriculum. As I mentioned in the chapter on contrast three-voice polyphony there are many variations in four-voices (1296); furthermore, it is very difficult. Here I merely present some basic principles and show three examples. All the rules and solution techniques remain the same[1].

1. It is a good idea to begin with the Fux's species in larger note values - first, second and fourth.
2. The vertical planning (bass) must be taken into consideration.
3. If the bass is being left last, it will make things considerably more difficult.

1[2].



2.

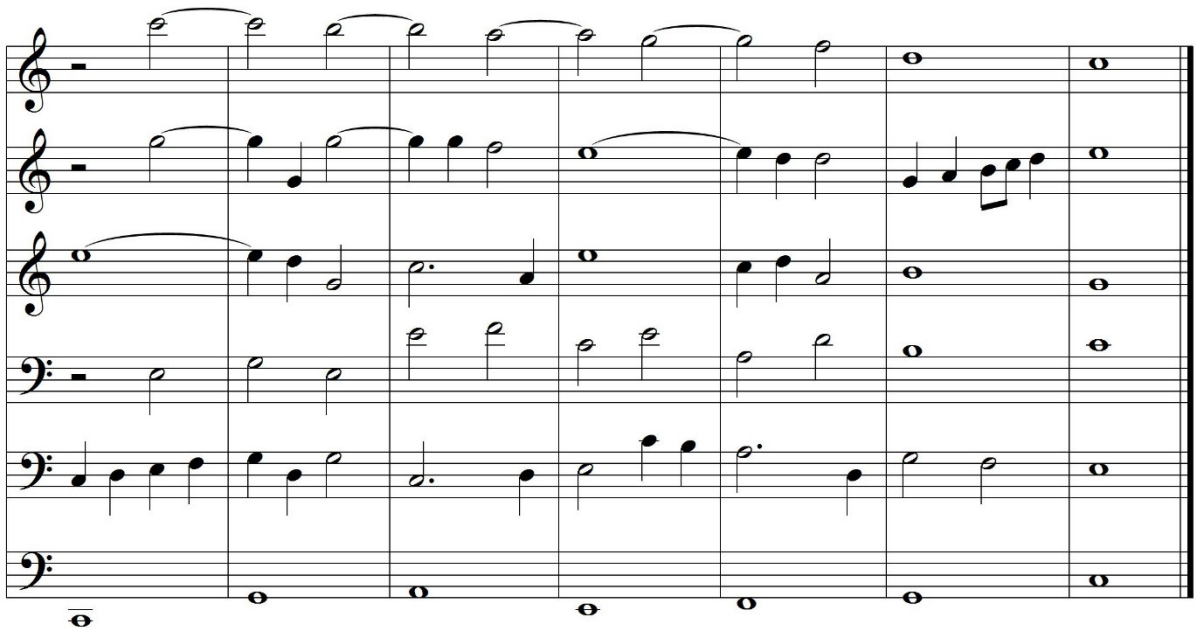


[1]Some teachers allow lateral dissonances in the four florid melodies mode and in contrast multivoice polyphony. I don't show examples of such solutions in this chapter. I am not forbidding lateral dissonances in principle, but I would use them only if there is no other solution, and so I mention this theoretical possibility in the chapter "Additional options"for resolving dissonances.

[2]Relations between soprano and bass in m. 3 can in this case be treated a little more liberally - A in the soprano is repeated, albeit an octave down. In other words, the sustained dissonance still refers to A. See Option II in the chapter on additional options, example 7.

Here it is not difficult to note that the bass and soprano were written first:

3.

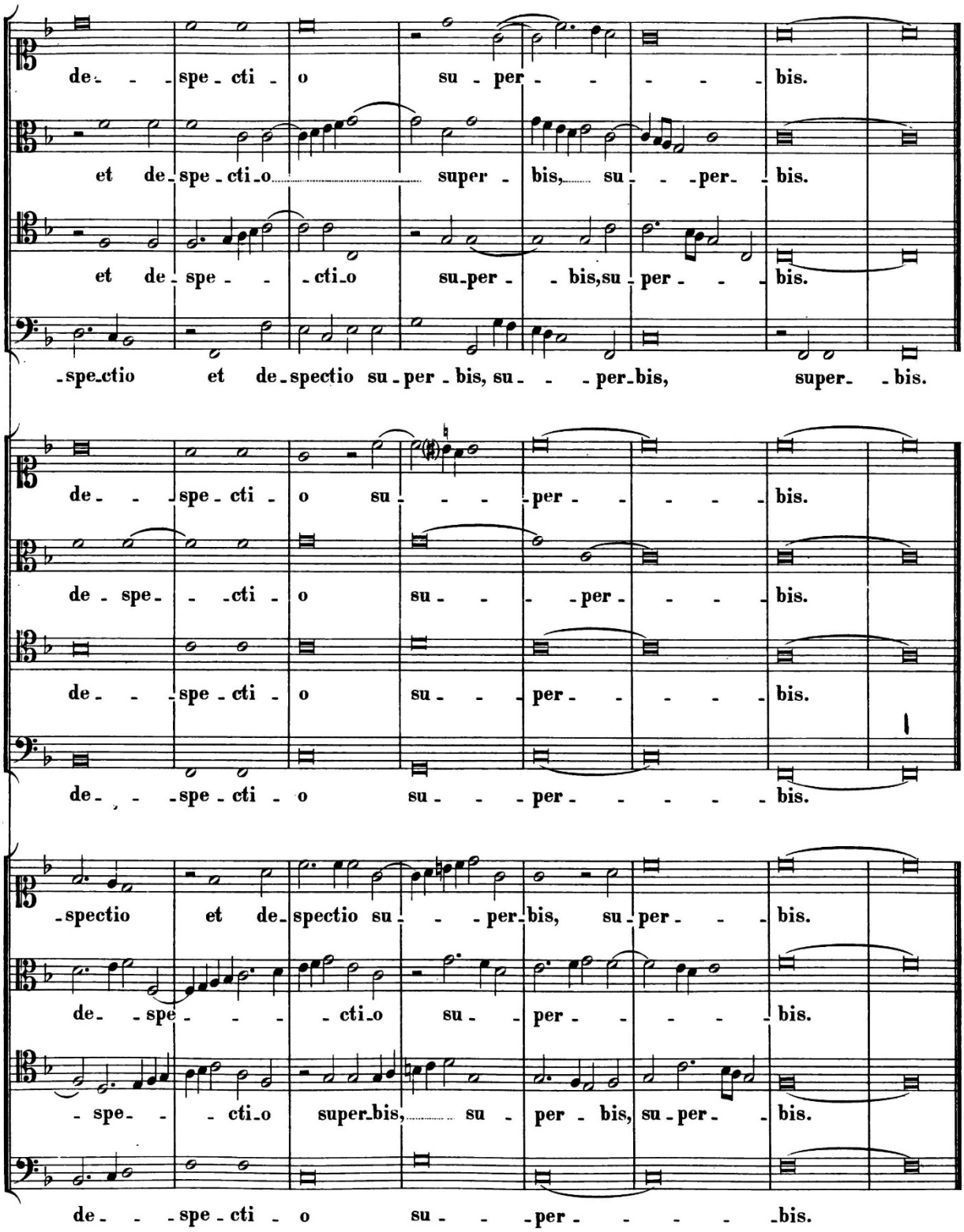


Fux's exercises are rarely written in more than four or five voices. If we have to try to write for singing voices in addition to all the rules, at some point we start to run out of possible solution options. This is especially obvious if there are more than six voices and if large note values are predominant.

In the living Renaissance music, there can be many more than six voices. There are significant exemptions from the rules - these are shown in the “Motet” chapter.

On the next page there isan excerpt from a 12-voice piece - Palestrina's *Ad televavi oculus meus* for three mixed choirs. It is obvious that the voices are “duplicated”. For example, the sopranos from choruses 1, 2 and 3 use almost the same notes but without parallel unisons, fifths and octaves. The same applies to altos, tenors and basses.

The fact that different voices can use the same tone (i.e., perfect unison) on any beat, including the downbeat, makes this technique possible to apply: so I can use 24 voices if I want. But even if the voices “stand” on the same note, their rhythmic is never identical. Without a doubt, 12 independent voices sound in *Ad televavi*.

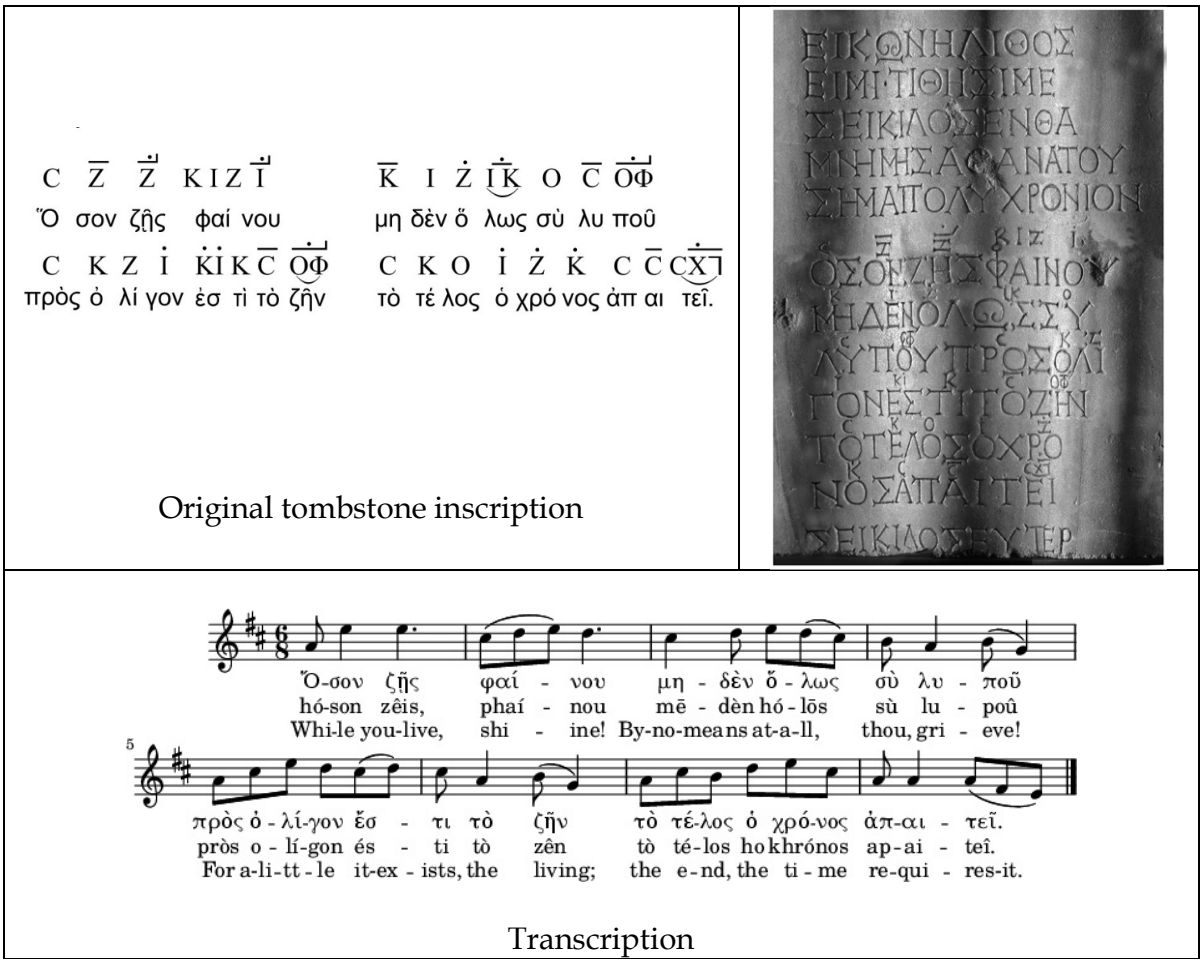

de- - -spe - cti - o su - per - - - - - bis.
et de - spe - cti - o super - bis, su - - per - bis.
et de - spe - - - cti - o su - per - - bis, su - per - - - bis.
- spe - ctio et de - spectio su - per - bis, su - - - per - bis, super - - bis.
de- - -spe - cti - o su - - - - per - - - - - bis.
de - spe - - - cti - o su - - - - per - - - - bis.
de- - -spe - cti - o su - - - per - - - - bis.
de- - -spe - cti - o su - - - per - - - - - bis.
- spectio et de - spectio su - - - per - bis, su - per - - - bis.
de- - spe - - - - cti - o su - - per - - - - bis.
- spe - - - cti - o super - bis, su - per - bis, su - per - - bis.
de- - -spe - cti - o su - - - per - - - - - bis.

Texture Types and Complex Counterpoint

The musical texture is generally classified into three types:

I. Monophony - when there is only one sounding voice. The single-voice texture is a characteristic of the ancient monodic cultures:

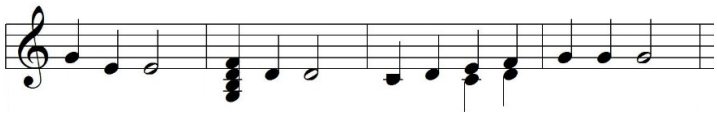
1. The Seikilos epitaph –
the oldest written complete song, from a tombstone (1st - 2nd century AD).



II. Heterophony - when in addition to one musical line there may be some other different element that generally speaking appears and disappears[1]:

[1] One possible interpretation of "different musical elements at the same time": "hetero" – "phony".

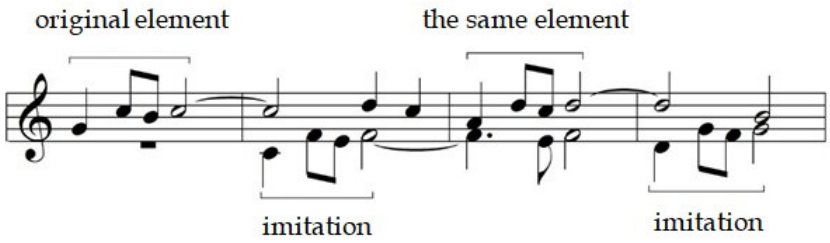
2.



III. Polyphony - when there are at least two simultaneously sounding voices (melodic lines).

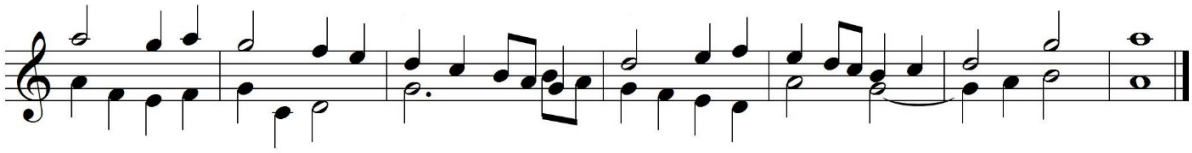
III. A. – Imitative polyphony - elements from one of the lines are used by the other:

3.



III. B. – Contrasting/non-imitative polyphony - there are no shared motifs or ideas in the different lines.

4.



Complex counterpoint[2]

I. It can be classified by the number of voices - double, triple, etc.

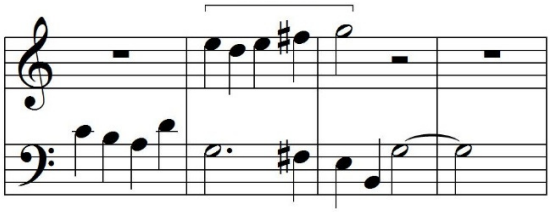
Also by the geometric relations between the voices:

[2] Some types of counterpoint exist mainly in theory and are not part of the curriculum.This is also true for some types of three-voice and four-voice counterpoint, as well as those in multiple voices. Horizontal, uninverted, augmented/diminished and retrograde counterpoint are written using polyphonic projection. This is an important polyphonic technique that is also used a lot for the canon. The principle of projection is briefly explained in the section on canon; it is used to solve more complex polyphonic problems and it represents a technical and compositional tool that seems to have originated in the Renaissance and found its peak in the 17th-18th centuries.

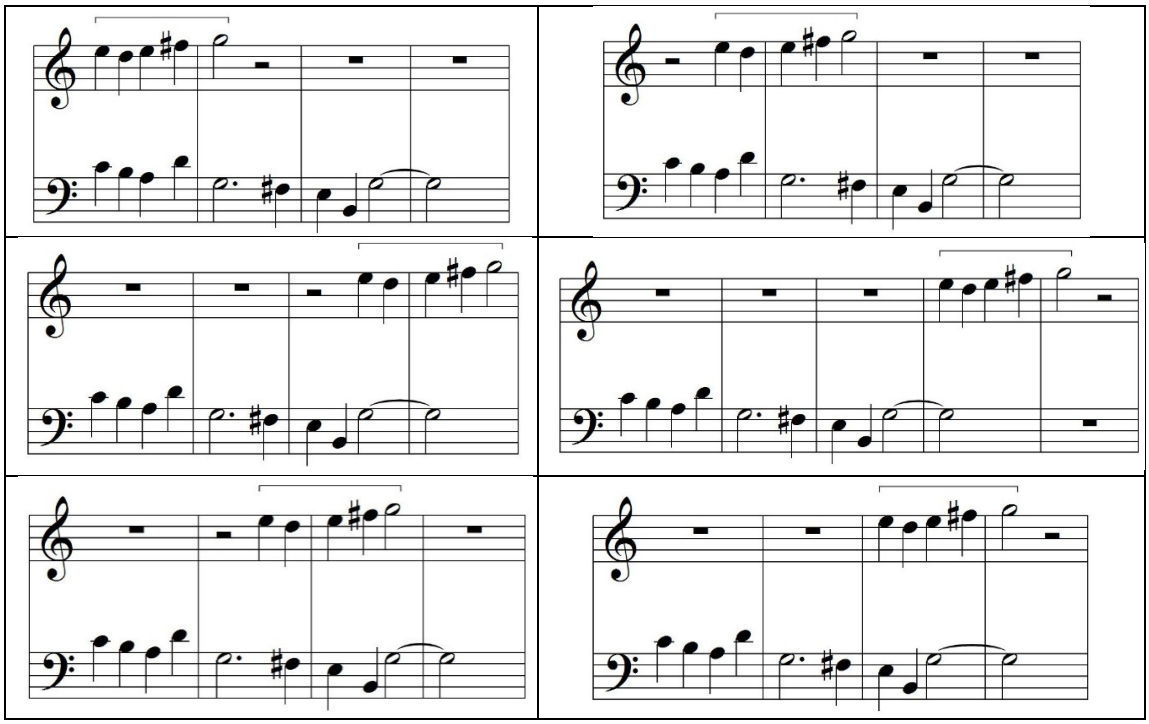
II. Horizontal - the element in one of the voices shifts horizontally forwards or backwards:

5. Model:

original motive



2. Variants of horizontal shifting:

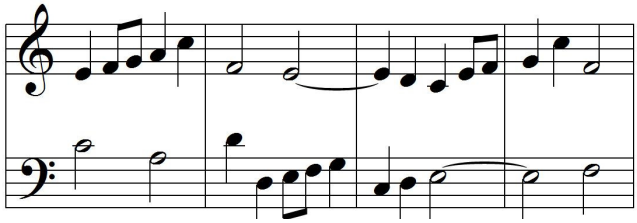


III. Vertical

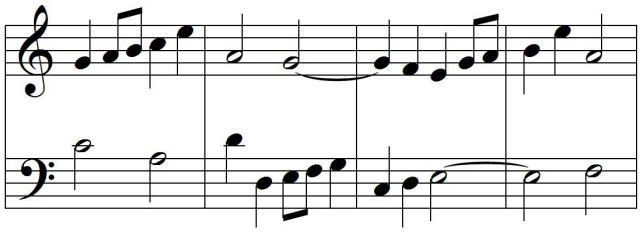
III. A – Uninverted – there is a vertical shift between the voices, but they don't cross, i.e. they don't swap their places:

3.

initial configuration

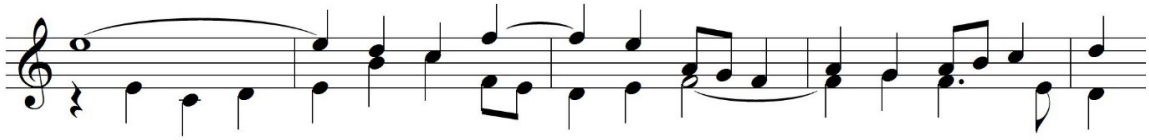


vertical shifting of the upper voice



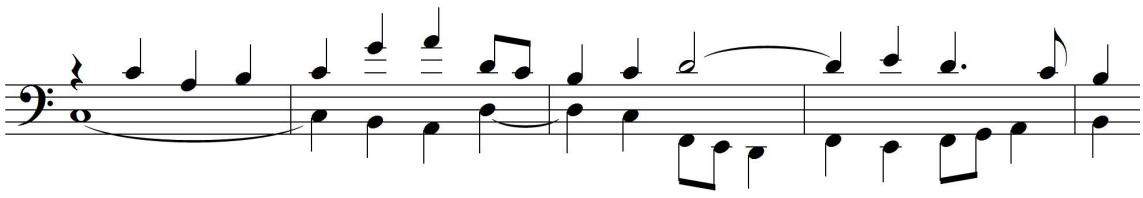
III. B. Invertible counterpoint - the voices swap their places[3]

4. Model



Three different realizations[4]:

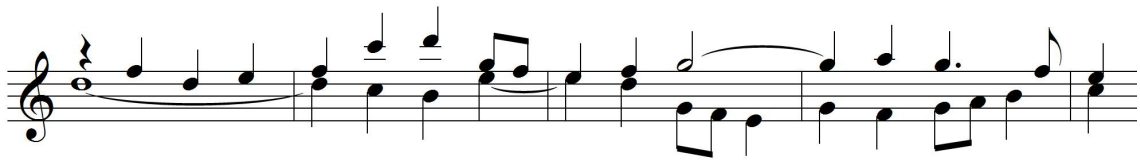
5. Invertible counterpoint with inversion index octave:



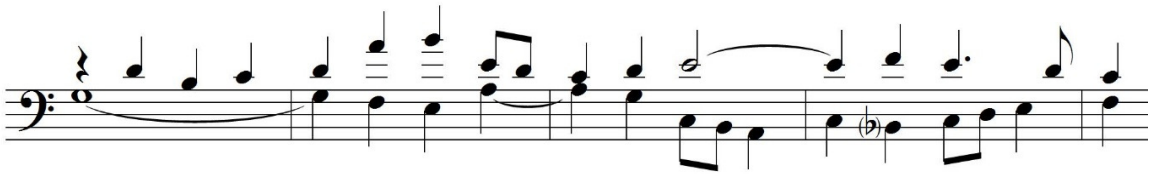
[3] The invertible counterpoint in a two-voices/double polyhony is taught in a separate section. Three-voice/triple and four-voice/quadruple counterpoint are also shown there.

[4] There are seven theoretical possibilities because the tone row consists of seven elements. For more on this subject, see the section on invertible counterpoint.

6. Invertible counterpoint at the tenth



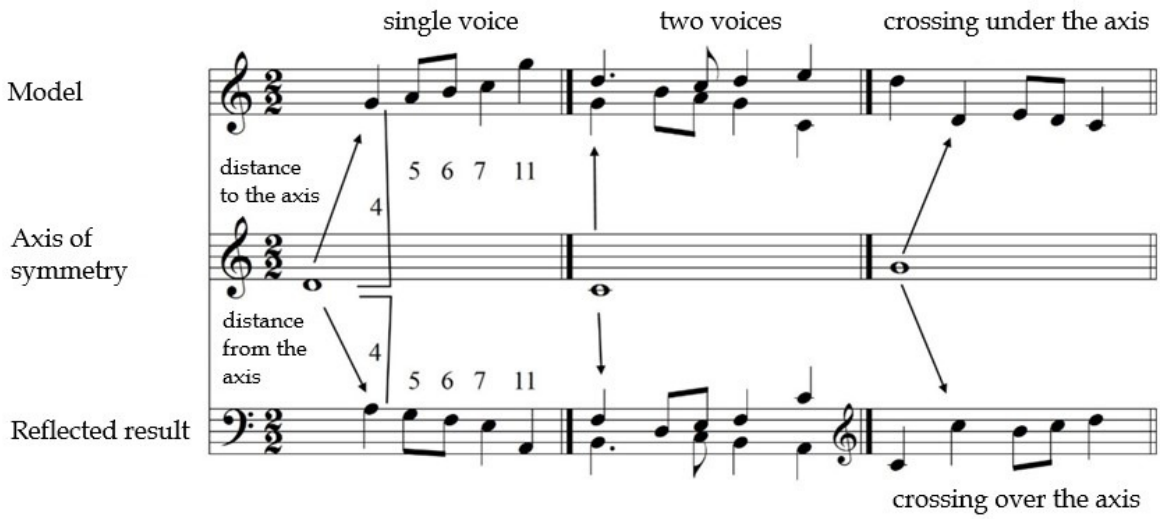
7. Invertible counterpoint at the twelfth



III. Mirror counterpoint, also called inverted[5]

Using one tone for a vertical axis of symmetry, I can "reflect" the music:

8.



On this principle, the upward motion in the model becomes downward in the result and vice versa. The interval from the model tone to the axis is reflected in the opposite direction. The result in many ways depends on the axis of symmetry.

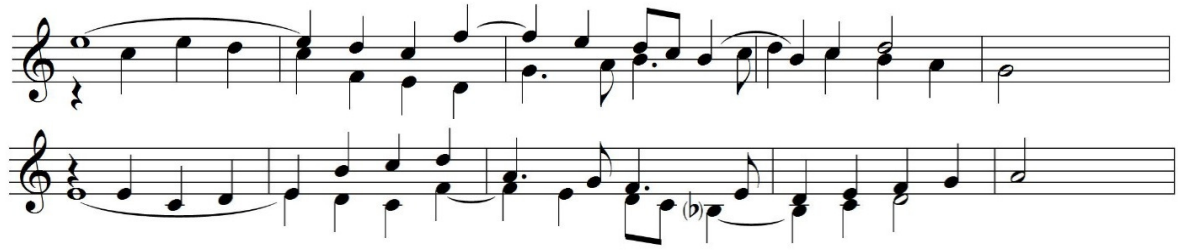
[5] Mirror and retrograde counterpoint are musical applications of the mathematical laws of reflection and symmetry. The mirror counterpoint applies the principle of vertical symmetry (when we read the musical text from top to bottom and vice versa), while the retrograde counterpoint concerns horizontal symmetry - when we read the text from left to right and vice versa.

For example, the fifth in a two-voice in the second measure (the fifth between the two voices) was reflected into a tritone in the result. The choice of axis is important - it doesn't just change the interval characteristics. There are historical examples of fully mirrored forms and genres in the freestyle - depending on the axis chosen, both the tonality/mode and the harmonic functions are changed[6].

In the strict style, this technique is sometimes used - this is especially true for the canon. Here this is explained in more detail in the sections on mirror counterpoint and canon.

III. A. Single mirror

99.
Model

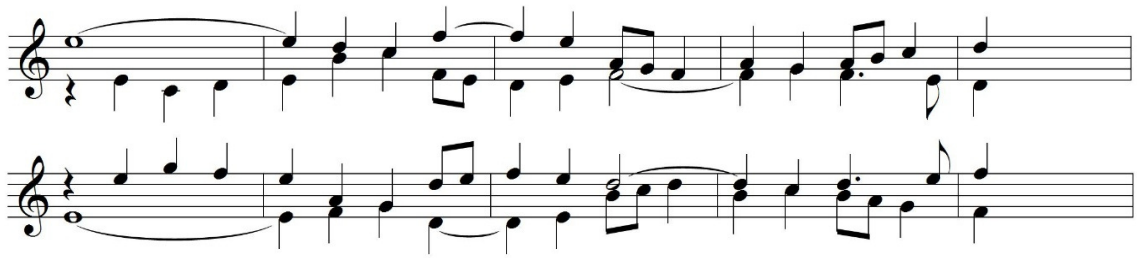


The lower voice, which was upper in the model, has remained unchanged.

The upper voice, which was lower in the model, is imposed mirrored on the lower voice in the result (mirrored to itself).

III. Double mirrored

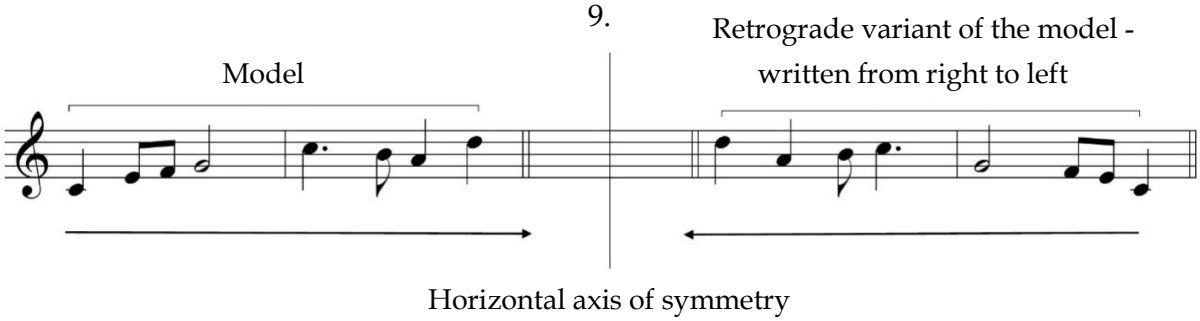
99.
Model



[6] Explained and shown how to do it in practice in the section on mirror fugue in my book "Technitcheski i tvortcheski predizvikatelstva pri pisane na slozhna fuga/Technical and creative challenges in writing a complex fugue", Sofia: In Sacris, 2020. Online at www.musicology-bg.com

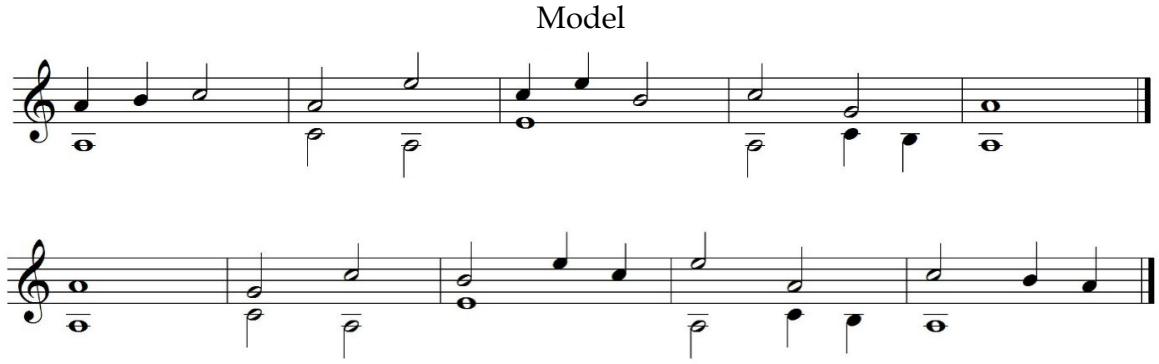
IV. Retrograde counterpoint[7], also called a crab-walk, crab or reverse. It is built on the principle of retrograde (horizontal) reflection of the musical material:

9.



IV. A. Single retrograde:

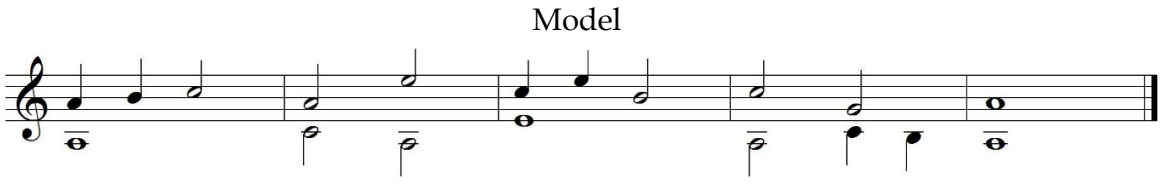
10.



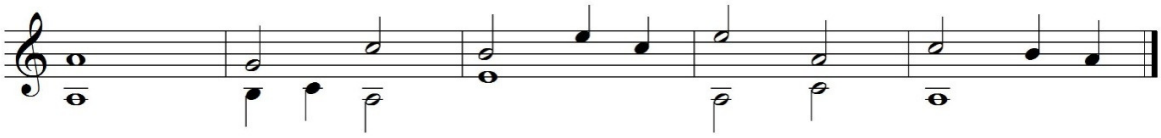
The lower voice has remained unchanged. The upper voice of the model is written from right to left on the lower - retrograde to itself.

IV. B. Double retrograde:

11.



[7] For more information see Artin Poturlian's book "Vazvraten kontrapunkt/Reverse counterpoint", Sofia, 2004.

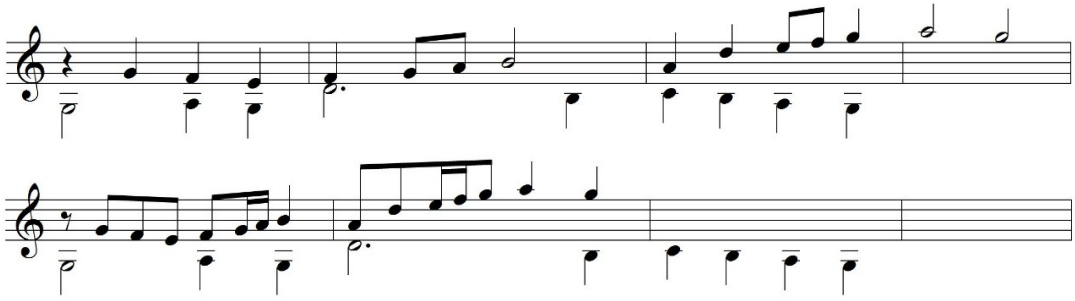


Both voices of the model are written from right to left.

V. Augmented and diminished counterpoint

12.

Model



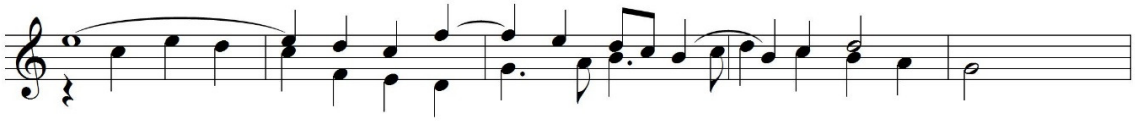
The upper voice of the model is written in twice smaller note values in the result. The lower voice remains unchanged. It can also be interpreted as a partially horizontal counterpoint.

VI. Combined counterpoint

All the varieties listed so far can be combined. They result in many interesting variants.

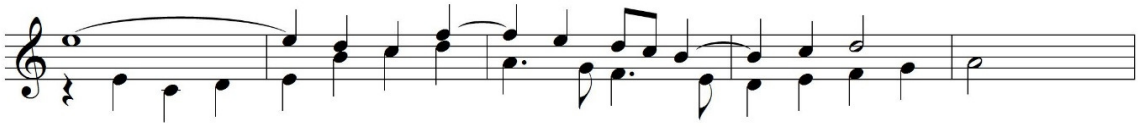
13.

Model



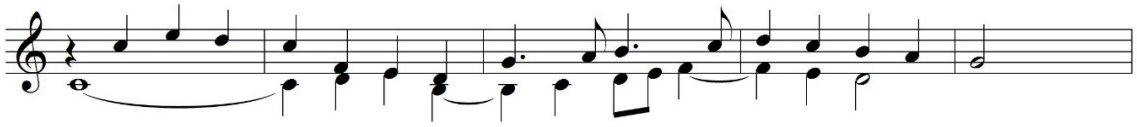
Single mirrored uninverted counterpoint:

14.

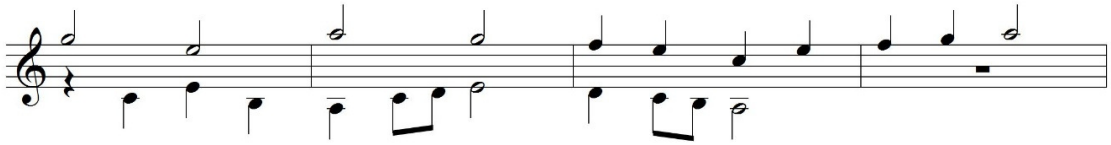


Double mirrored invertible counterpoint (according to the model)

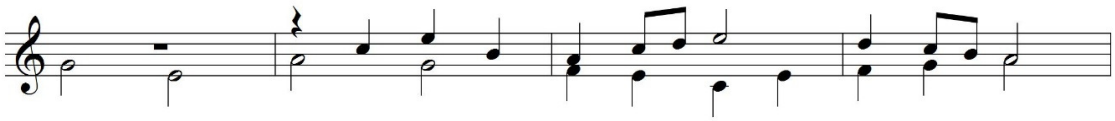
15.



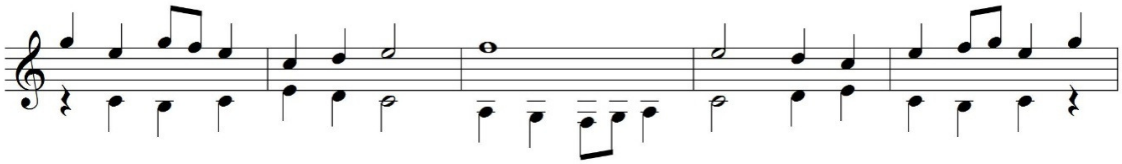
16.
Model



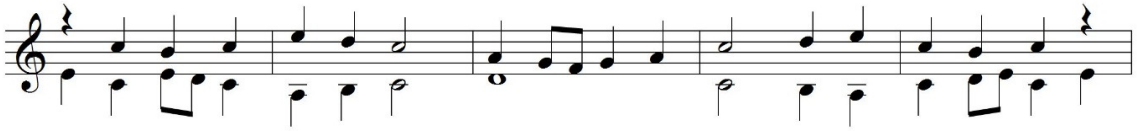
17. Horizontal double invertible counterpoint



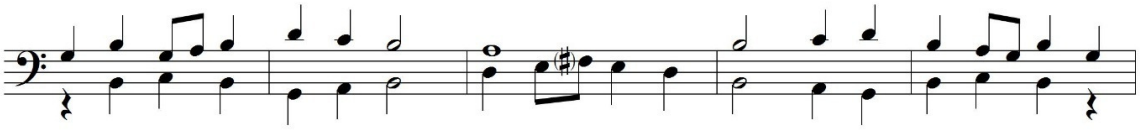
18.
Model



19. Retrograde double invertible counterpoint at the tenth



20. ... and its mirror variant:



Thus, calculated from the model (example 18.), this is double mirrored, double retrograde, double invertible counterpoint at the tenth.

For the examples 18.-20. I used a little trick. In both voices of the model I used for the most part musical material that is horizontally symmetrical to itself, i.e. self-

reversible. Such an element - a palindrome - is used in both poetry and literature - a text which, when being read backwards, gives the same result[8].

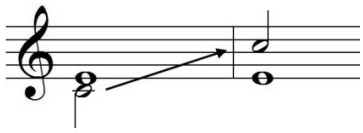
[8] Examples of palindromes: “bird rib”, “level”, “Sir, I demand, I am a maid named Iris.”, “noon”, “Was it a car or a cat I saw?”, “Don’t nod!”. There are poems and even whole novels that are palindromes.

Invertible Counterpoint - Introduction

When two or more voices exchange places (on the vertical), we speak of an invertible counterpoint. This phenomenon is directly related to another - the interval inversion.

If I have two tones and decide to move the lower an octave upwards, the following happens:

1.



That means the third on the vertical in the first measure has been inverted into a sixth in the second measure. The voices have crossed - the upper voice has become lower and vice versa.

If the transposed interval is an octave, as shown above, then the inversion of the intervals would be as follows:

2.

Perfect unison – 1	Will be inverted into Perfect octave - 8
Second - 2	into Seventh - 7
Third - 3	Sixth - 6
Fourth - 4	Fifth - 5
Fifth - 5	Fourth - 4
Sixth - 6	Third - 3
Seventh - 7	Second - 2
Octave - 8	Perfect unison - 1

Or written another way:

1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1

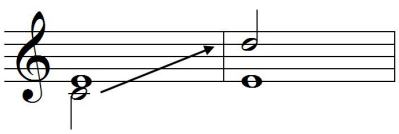
The interval in the upper row inverts to the interval in the lower row and vice versa.

All dissonances except one invert into dissonances. However, the fifth, which in a two-voice polyphony is a dissonance, is inverted into a fourth.

Conversely, all consonances except one invert into consonances. However, the fifth, which is a consonance, is turned into a fourth, which is a dissonance.

What if I transpose to a ninth instead of an octave?

3.



The third has been inverted into a seventh. Thus it appears that the perfect unison will be inverted into a ninth, the second - into an octave...:

4.

Perfect unison – 1	Will be inverted into a Ninth – 9
Second - 2	into Perfect octave - 8
Third - 3	Seventh – 7
Fourth - 4	Sixth – 6
Fifth - 5	Fifth – 5
Sixth - 6	Fourth – 4
Seventh - 7	Third – 3
Octave - 8	Second – 2
Ninth - 9	Perfect unison – 1

Or written another way:

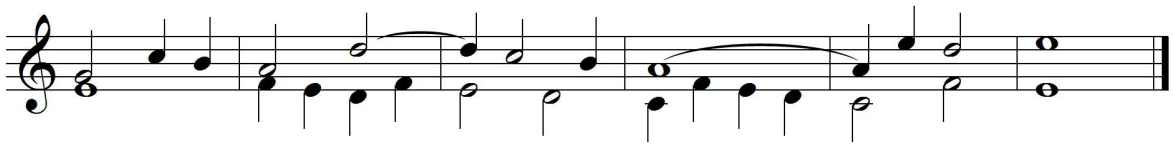
1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1

All consonances except the fifth are inverted into dissonances.

If for some reason (e.g. a deliberately chosen compositional means of expression) I present the voices first in one vertical relation, and then I swap their places, it means that I am using invertible counterpoint.

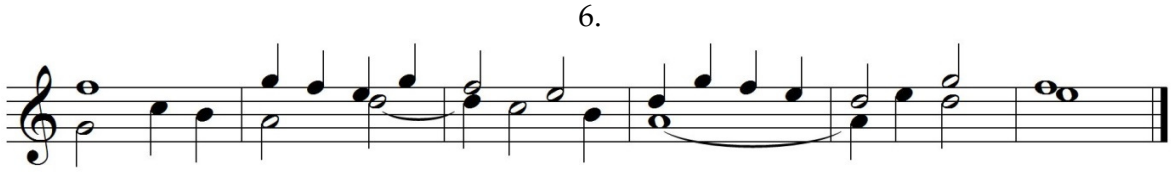
As you can see, if I know, or have chosen in advance, what interval to transpose to, I will also know in advance what vertical ratios my result will have. If I have chosen a polyphonically correct example:

5. Model



and I have transposed the lower voice a ninth upwards, the following will result:

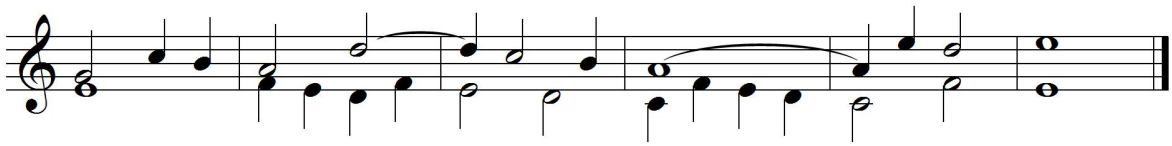
6.



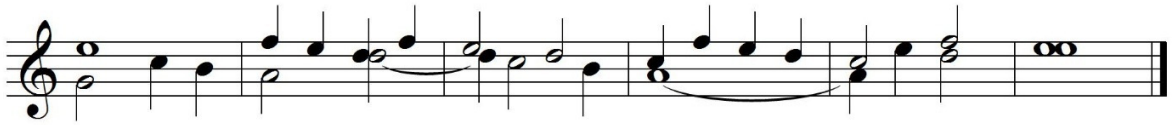
This double counterpoint is incorrect (and sounds terrible). Why? Because I have chosen the wrong interval of transposition. The table in Example 4. has warned me that any consonance on the vertical in the model will turn into dissonance.

What will happen if I transpose the lower voice of the model (example 5.) an octave upwards:

7. Model



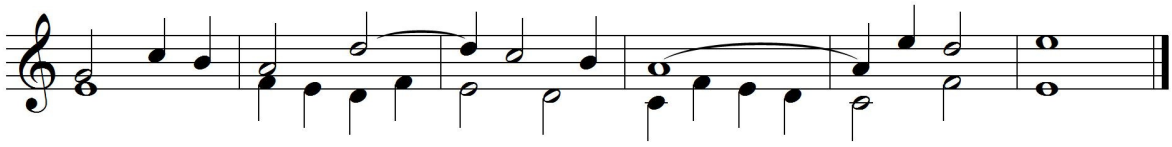
8. Result



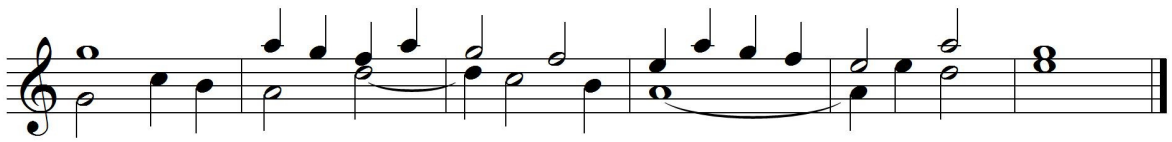
There is not a single error.

And if I transfer to a tenth?

9. Model



10. Result



There are some errors. Accented octaves, parallel and accented fifths.

That is to say, each different interval of transposition/transfer (also called index) has its own laws. These laws are mathematical, algebraic. And if I know them, I will be able to write a model in advance that will sound correct in my predefined index. In theory, there are seven indexes.

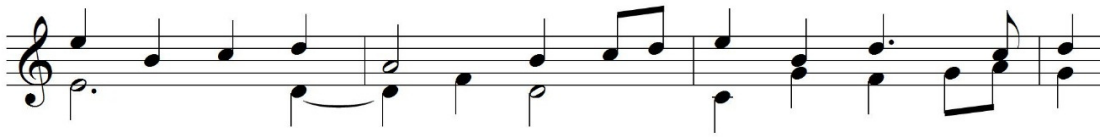
Some indexes are impractical and *de facto* not used in the strict style. The ninth index is not used at all, to my knowledge. Some others are of limited applicability in the freestyle (Baroque) polyphony.

The next four chapters in this section show what the rules are for writing double (two-voice) invertible counterpoint that has the index of perfect octave, tenth, twelfth, and mirror (inverted) counterpoint. These are used most often.

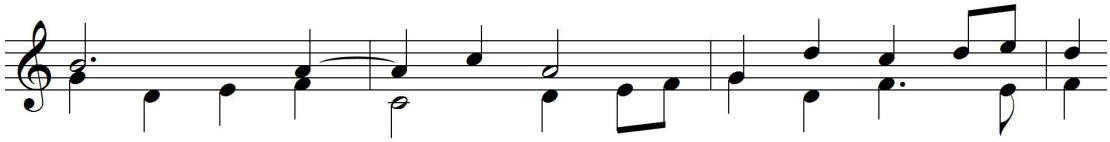
Identification of the index

If I have already a written piece that has both a model and a result, I can identify the index (the transferred interval) in a simple way.

11. Model



12. Result



Both voices in the result are transferred (transposed). But this is not important. It is enough to know that each voice interval-wise is identical to the one in the model[1].

I take the first vertical interval of the model - an octave, E-E - 8. To it I add the first interval of the result - a third, G-B - 3. Then I subtract the digit 1:

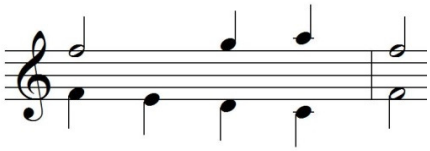
13.

$$8 + 3 - 1 = 10$$

This is the answer, the index is a tenth (10).

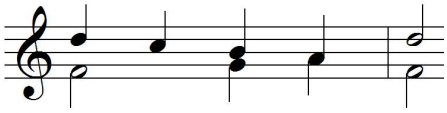
One more example:

14. Model



[1]Interval size is irrelevant, i.e. it is not important whether we are talking about a large or a small third, etc. At octave index, large intervals are reversed to small intervals (and vice versa), perfect to perfect. In the transposition, a perfect interval can turn into a tritone, and vice versa. These are regular phenomena in diatonics. If we want to get rid of the resultant tritone, we can sometimes use a ficta, as I have already shown. This practice has existed since the Middle Ages and has been widely used since then and to the present day.

15. Result



Octave in the model plus a sixth in the result: 8 + 6 - 1 = 13. The answer is 13 - thirteenth (sixth over an octave).

Three-voiced counterpoint

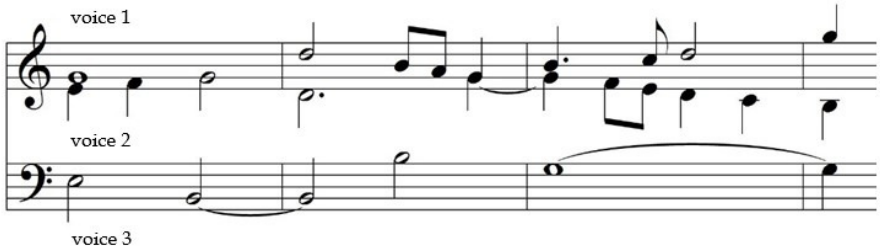
If there are three voices, there are six possibilities for an inversion - here they are marked with digits:

16.

Voice	Variant 1	Variant 2	V. 3	V. 4	V. 5	V. 6
	1	1	2	2	3	3
	2	3	1	3	1	2
	3	2	3	1	2	1

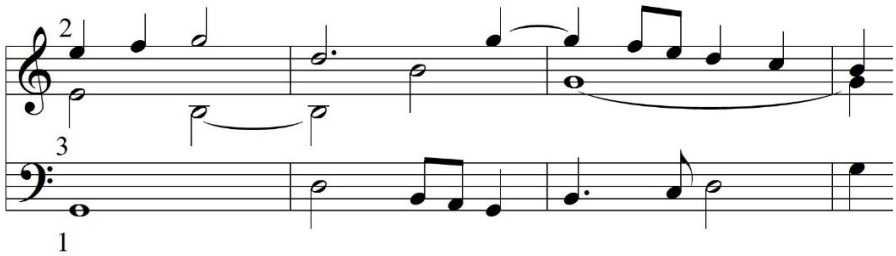
Most often index of inversion is octave in all three voices. Occasionally, both a tenth and an octave may be used, as well as a twelfth.

17. Model



The original order of voices (read from top to bottom) 1-2-3 has now been changed to a 2-3-1:

18. Result



What index of inversion have I used?

In the model, the interval between the first note of voice 1 and voice 2 is a third - 3. The result is a sixth - 6 (over two octaves, but these are irrelevant; one or more added octaves don't change the principle):

$$3 + 6 - 1 = 8$$

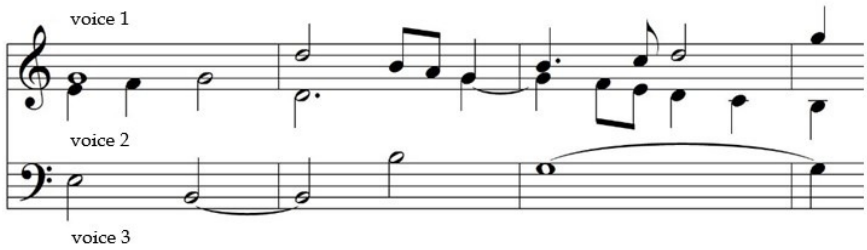
In the model, the interval between voice 2 and voice 3 is an octave - 8. In the result - also. In this case, there is no need for calculations - the result is an octave[2].

In the model, the interval between voice one and voice three is a tenth, i.e. a third over an octave - 3. The result is a sixth over an octave - 6:

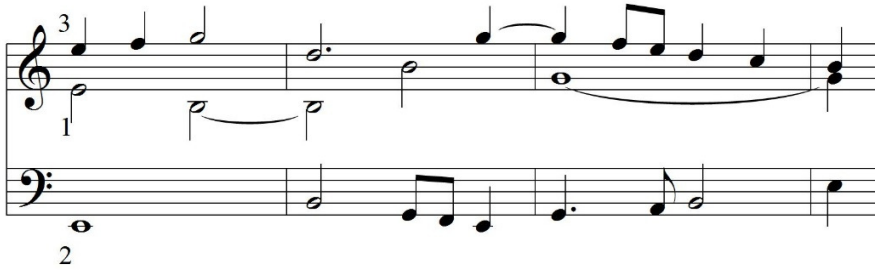
$$3 + 6 - 1 = 8$$

That means the index used between all voices is octave.

In the following example, the configuration is changed. First I show the same model:



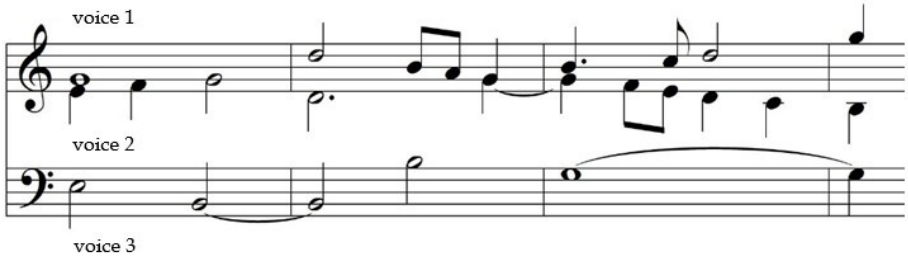
19. Result



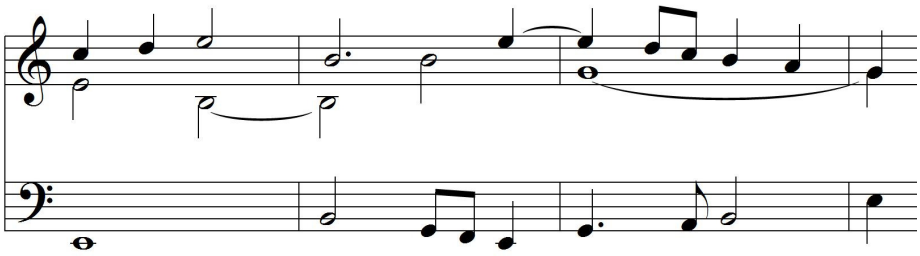
Initial distance	Distance in the result	Index
Between voice one and two = 3	8	3 + 8 – 1 = 10 tenth
Between voice two and three = 8	8	not crossed
Between voice one and three = 3	8	3 + 8 – 1 = 10 tenth

[2]But these two voices do not cross, there is no invertible counterpoint here.

The same model:

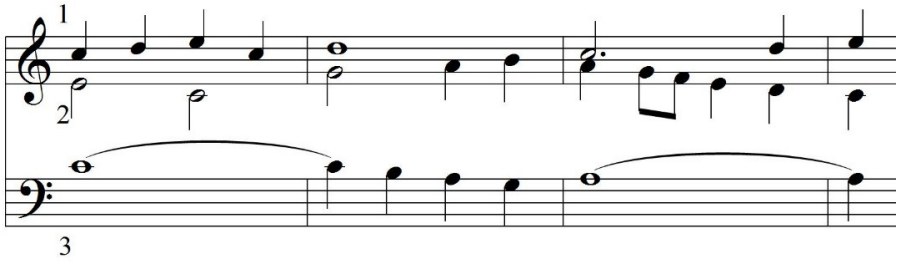


20. Another result:

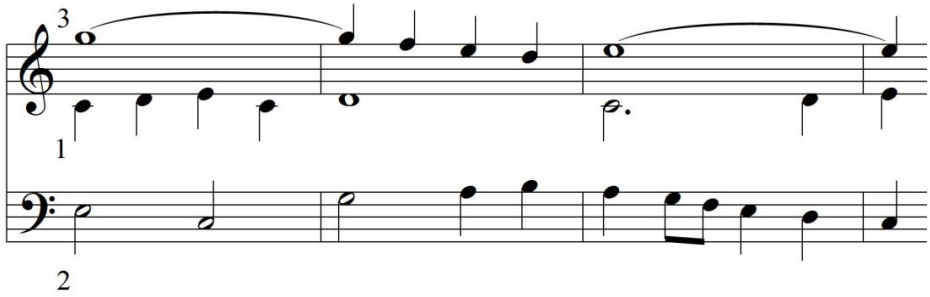


Initial distance	Distance in the result	Index
Between voice one and two = 3	6	3 + 6 – 1 = 8 octave
Between voice two and three = 8	6^3	are not crossed
Between voice one and three = 3	8	3 + 8 – 1 = 10 tenth

21. New model



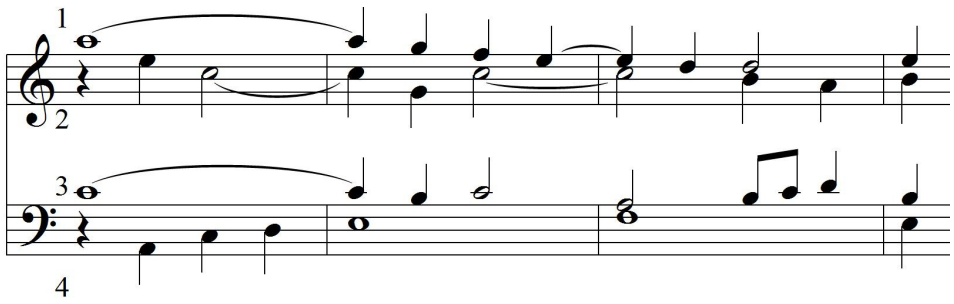
22. Result



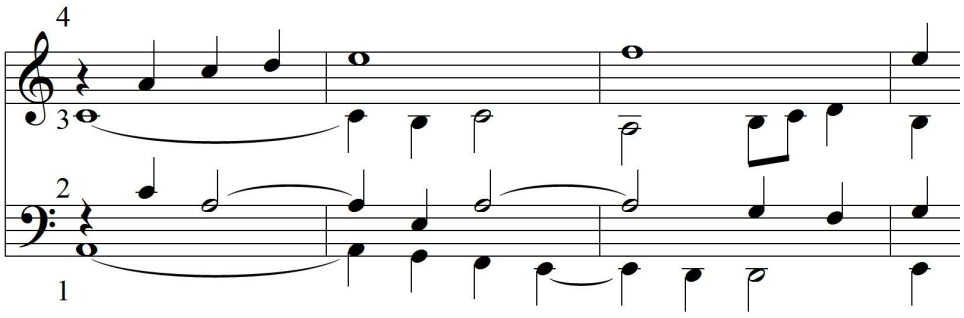
Initial distance	Distance in the result	Index
Between voice one and two = 6	6	are not crossed
Between voice two and three = 3	3	3 + 3 – 1 = 5 fifth[3]
Between voice one and three = 8	5	8 + 5 – 1 = 12 twelfth

On the same principle is the invertible counterpoint in four or more voices. The possible variants are 24.

23. Model



24. Result



In this example, a slightly complicated formula is used to fragment a dissonant syncope. It has no uncrossed voices and uses invertible counterpoint at the octave (twice), tenth and thirteenth. This counterpoint is also linearly and asymmetrically mirrored.

In the following chapters, two-voice invertible counterpoint with index perfect octave, tenth and eleventh and double mirror counterpoint are used.

[3]That means twelfth. Bach uses the terms"invertible counterpoint at the third", and this means both the tenth and the"invertible counterpoint at the fifth", meaning a twelfth.

Invertible Counterpoint on/at the Octave[1]

All exercises are based on the same principle - we prepare a two-voice model, taking the model of two florid melodies as a basis (the last chapter on Fux's two-voice counterpoint). One melody is assigned in advance; to it we add one more. All Fux's rules for two florid melodies remain valid[2].

A realization of our model is not necessary because we know in advance how each of its intervals will turn out in the eventual result. I do show examples of realizations, but these are not required from the reader.

Using the table shown in the introduction to the invertible counterpoint, I can draw the following conclusions:

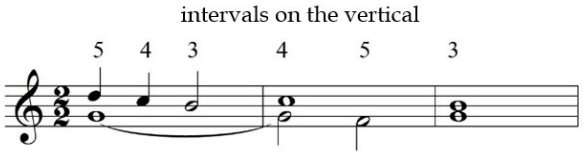
1.

1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1

The perfect intervals - the unison and the octave - are inverted into perfect intervals. Major intervals are reversed into minor intervals and vice versa (for example, a major seventh will be inverted into a minor second and vice versa). The most important consequence - the fifth in the model will be inverted into a fourth and vice versa.

Therefore, if there is a transient fifth in the model, it will be inverted to a transient fourth and vice versa. There can't be a fifth on the downbeat because the fourth on the downbeat is forbidden in the result. But if I write a transient fifth, that would be fine. I can also use a suspended dissonance.

2. Model



The fourth in measure 1 is transient and in measure 2 – suspended (held).

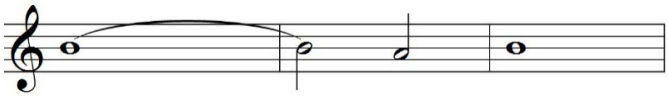
[1]I recommend that the reader first have a look at "Introduction to invertible counterpoint."

[2]Depending on the interval of invertion, a tritone - a diminished fifth or an augmented fourth – may appear in the result instead of a fifth or fourth. This is legitimate; but for us it is not essential, because we don't make a realization of the model. We are only talking about technical exercises, not real composition work.

Realization of the double counterpoint

It goes like this. First I have to cross the two voices. I choose a random note to begin with the lower voice - the same one that was the upper one in the model. For example, now my new lower voice will start from B.

2.



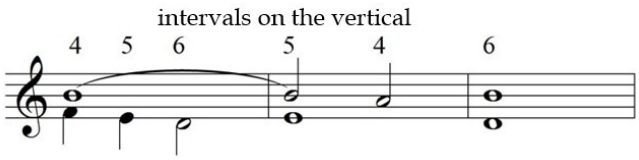
Then I take my octave index (8), and subtract the interval of my first vertical of the model (G-D, fifth, i.e. 5) and add the digit 1:

3.

8 – 5 + 1 = 4

This is how I figure out which note my new lower voice should start on. The fourth downwards from B (the first note of the result) is F. I write the new lower voice from F:

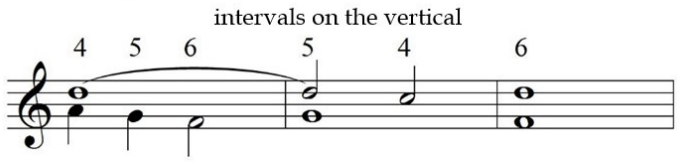
4. Result



Observations

The fifth in the model has been inverted into a fourth (as a result of my calculation). The fourth has been inverted into a fifth, the third into a sixth.
Deciding to start the new upper voice (in the result) from B, I have inverted the fifth into a tritone - augmented fourth.
The exercise has begun with a fourth on the vertical - this is impossible in two-voice counterpoint.There can be no fourth on the downbeat anywhere (unless it is suspended).
In measure 2, the fifth on the second beat in the model (lateral fifth, F-C) has been inverted into a lateral fourth. This is a serious mistake.

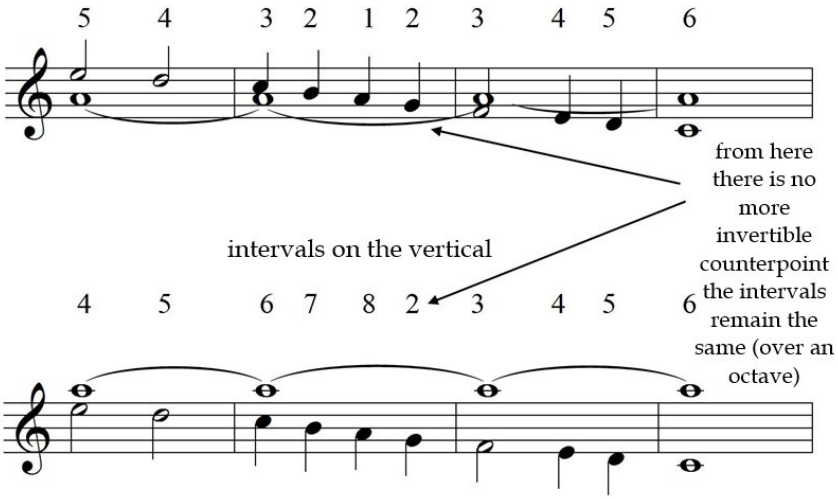
5. Here's a transfer from another note:



There are no tritones here. But no matter from which note I realize the model, the numerical result would remain the same.

If the two voices cross in the model, then the interval inversion disappears in the result - model intervals become identical to those of the result:

6.

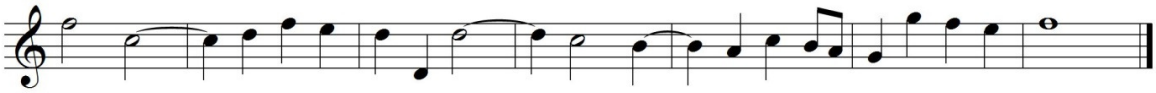


Because this way there is no invertible counterpoint, crossing of voices in the model is not allowed in all species.

Writing

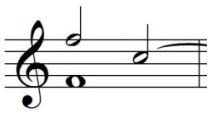
Here's my assigned upper voice.

7.



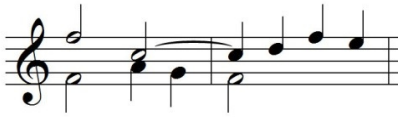
To begin with a fifth is impossible - OK. But if I write a whole note F1, then on the second beat against C there will be a fifth:

8.



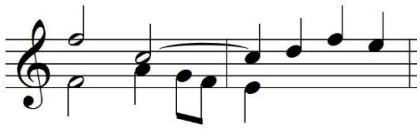
A fifth leap will obviously invert into a fourth leap, this isn't good. But I can try this:

9.



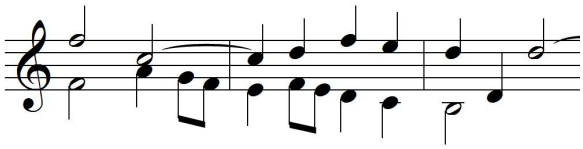
I avoided the fifth in the first measure. In the second measure, the fifth is suspended and then resolved with an ascending motion into a sixth (F-D). Unfortunately, this is also not a viable option: the suspended fifth that resolves upwards will be inverted into a suspended fourth that resolves upwards, and until the early 17th century no dissonance could be resolved upwards. I correct it again.

10.



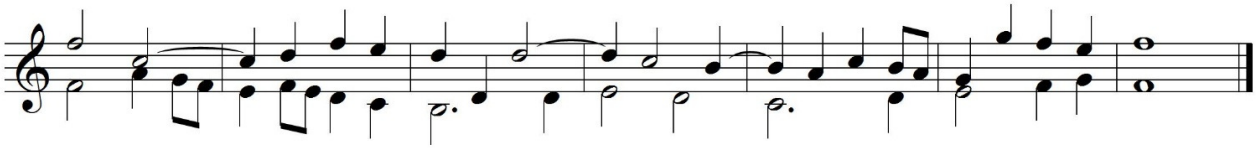
It's fine now. The fifth at the end of the first measure is transient.

11.



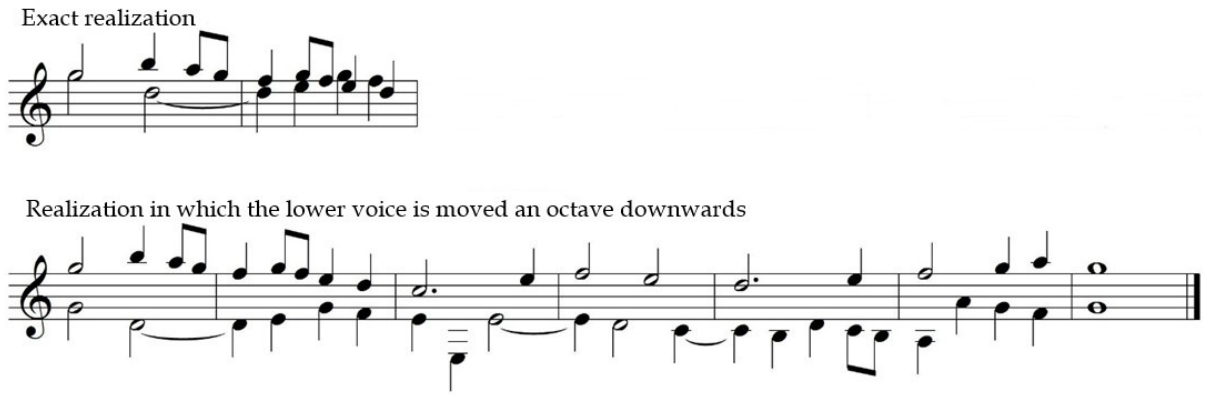
In the second measure I moved downwards a bit, to D. My instinct tells me to always move in a contrary motion; but here, besides instinct, I have also a rational argument - I have noticed that in measure 3 the upper voice leaps an octave down. I can't allow the two voices to cross; so I moved downwards already in m. 2 - preparing the leap that will follow in m. 3.

12.



Until the end, the solution is with a slightly more static lower voice. Although not necessary, I show one possible result of this model:

12.

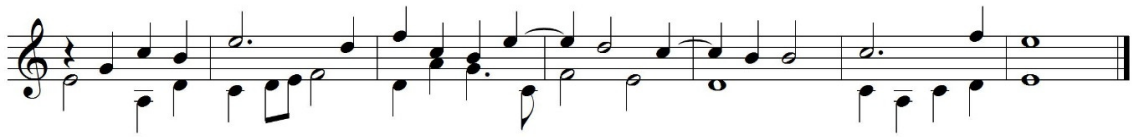


In a correct realization, the two voices are crossed from time to time. This doesn't make the exercise polyphonically incorrect. But if I want there to be no crossing in the result - as there was none in the model - I additionally transfer the lower voice an octave downwards (or the upper - an octave upwards).

Sometimes the index is marked with a digit - 8'. Some teachers also use the 8' + 8' formulation, with the first eight representing the index and the second representing the additional transfer I showed. For me, the second digit is not really necessary.

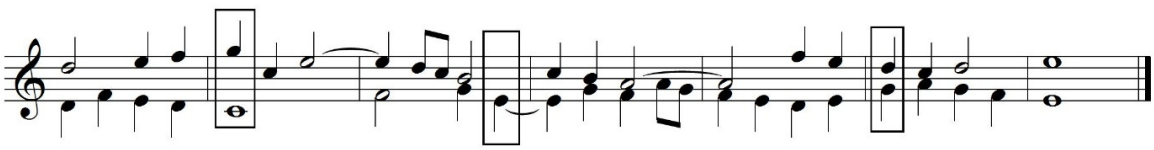
Here's another complete solution - in this case, the assigned voice is the lower one. There is no realization.

13.



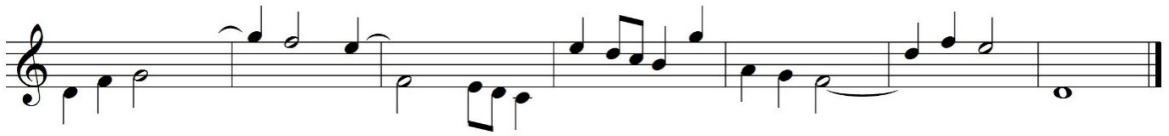
Incorrect example - the errors only concern the specific index. No realization.

14.

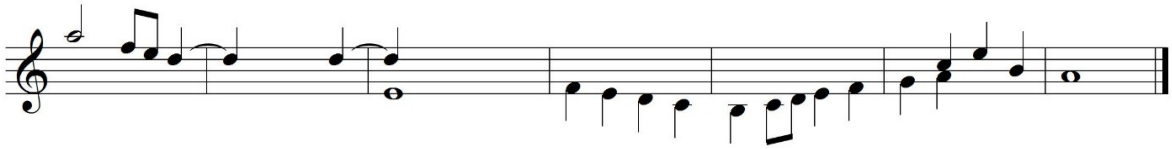


Exercises

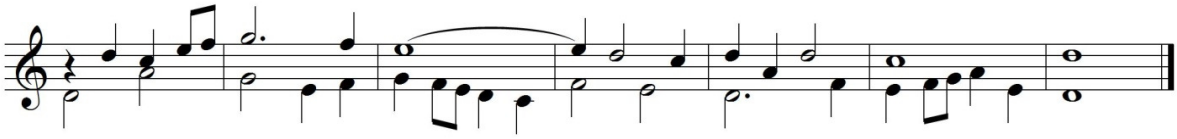
16.



17.

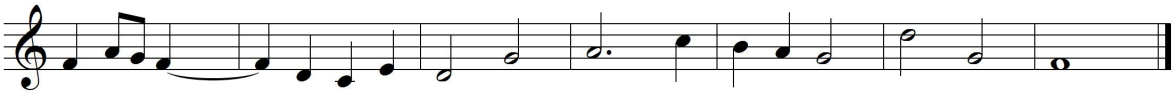


18. This double counterpoint is correct but not in index 8'.
Can it be corrected in 8'?

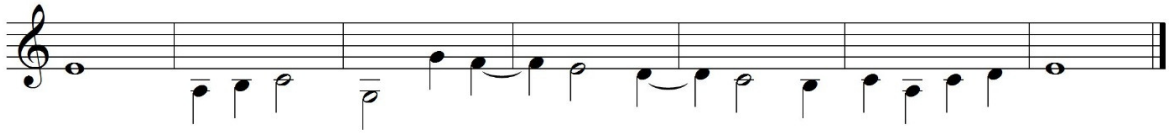


Assigned florid melodies for solving

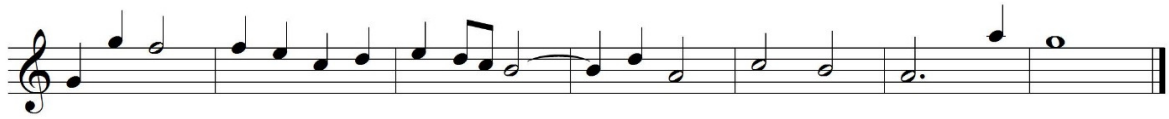
19.



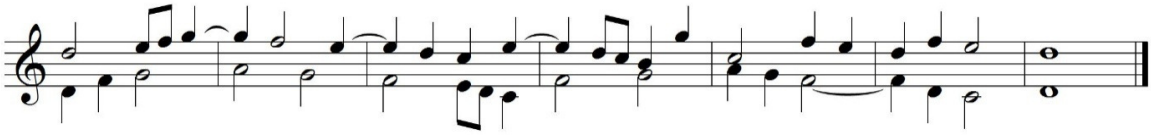
20.



21.



22. Solution of example 16



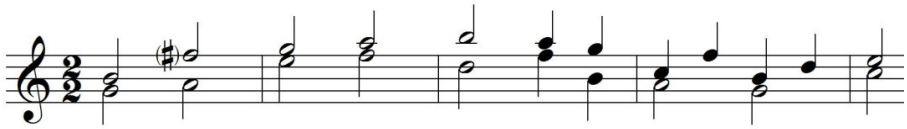
Invertible Counterpoint On the Tenth

1.

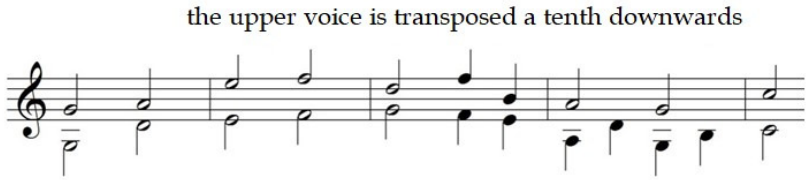
1	2	3	4	5	6	7	8	9	10
10	9	8	7	6	5	4	3	2	1

At first glance, it seems that this index is quite convenient - every consonance turns into a consonance. The problem is that if there are parallel thirds, they will turn into parallel octaves; parallel sixths will turn into parallel fifths. In fact, any similar motion will lead to one problem or another:

2. Model



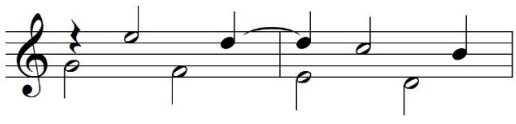
3. Result



In the result there are not only parallel intervals, there are also hidden and accented ones.

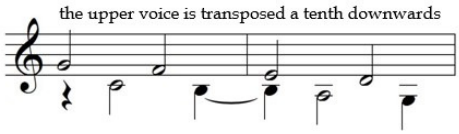
Therefore, an important rule in this mode is to not allow any similar motion in the model. Lateral and contrary motion are possible - it may not be immediately apparent, but in the example below there is no similar motion:

4.



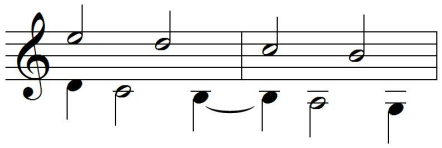
However, this example can't be used. My table indicates that the sixths will turn into fifths:

5.



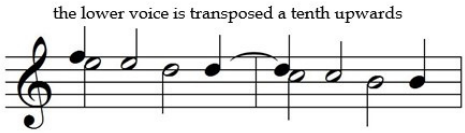
This chain syncope also doesn't seem possible:

6.



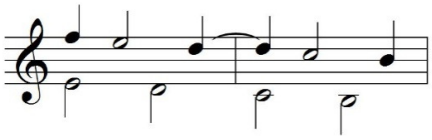
A chain of faux seconds would occur:

7.



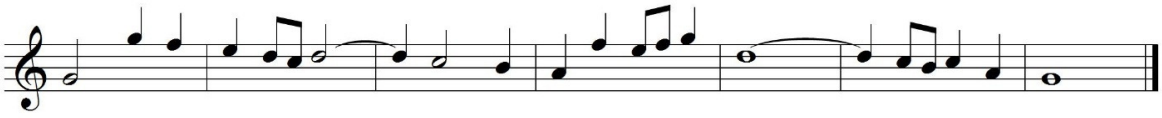
And if another octave is added in the realization (10' + 8'), we obtain parallel octaves:

8.



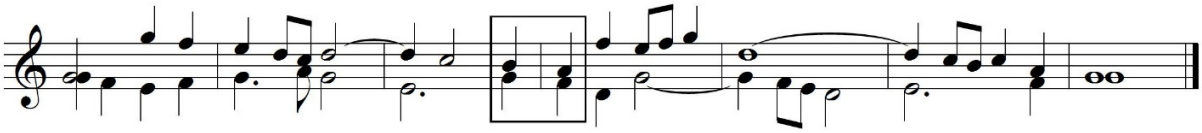
Writing

9.



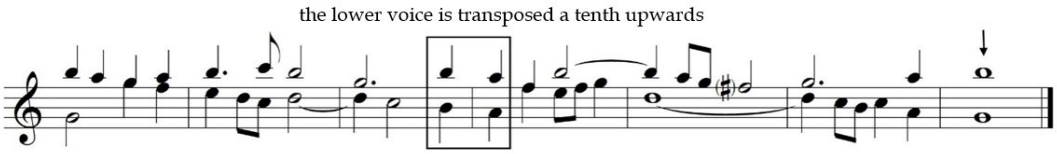
This melody contains a formula for fragmenting a dissonant syncope.

10. Model



I have deliberately made a mistake here - it is in the rectangle. There is a case of similar motion. I do this in order to see what will happen - will problems arise and of what nature. Here is a realization.

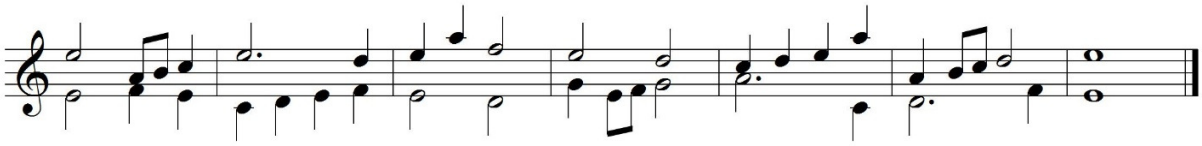
11.



Unallowable parallel octaves as seen. In the last measure the perfect unison has turned into a tenth - this is legitimate and normal.

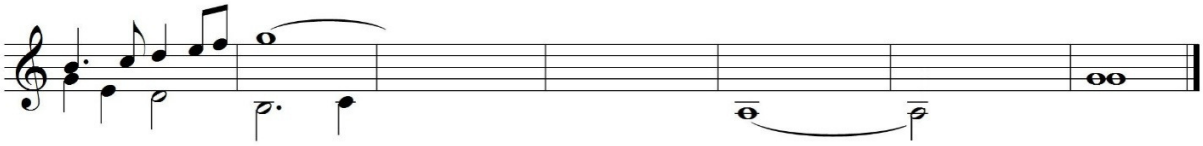
Incorrect model with no marked errors:

12.

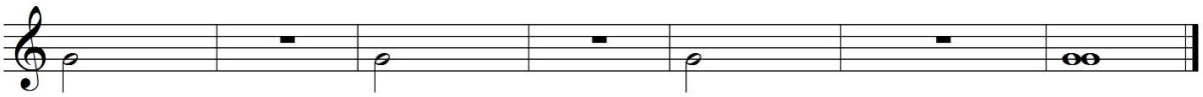


Exercises

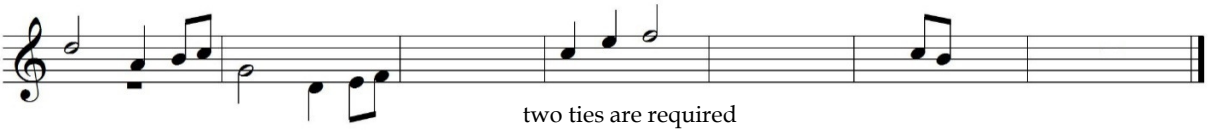
13.



14.

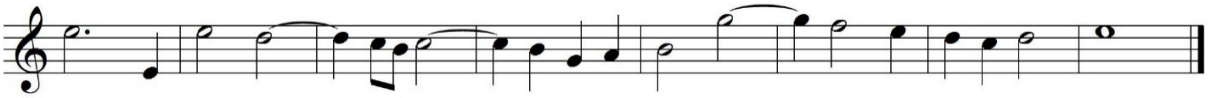


15.

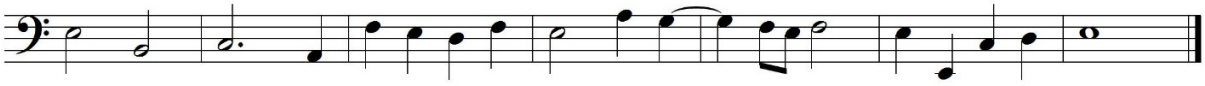


Florid melodies for solving

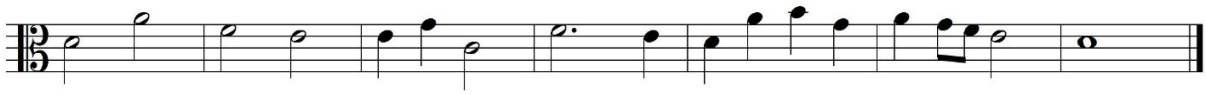
16.



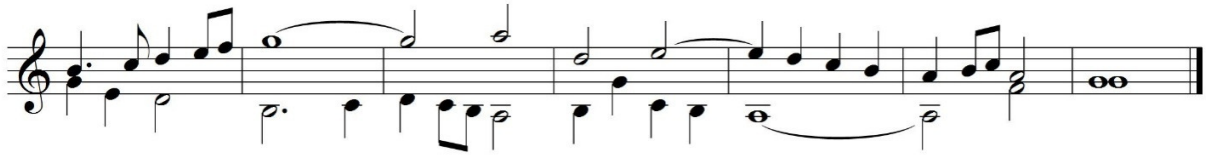
17.



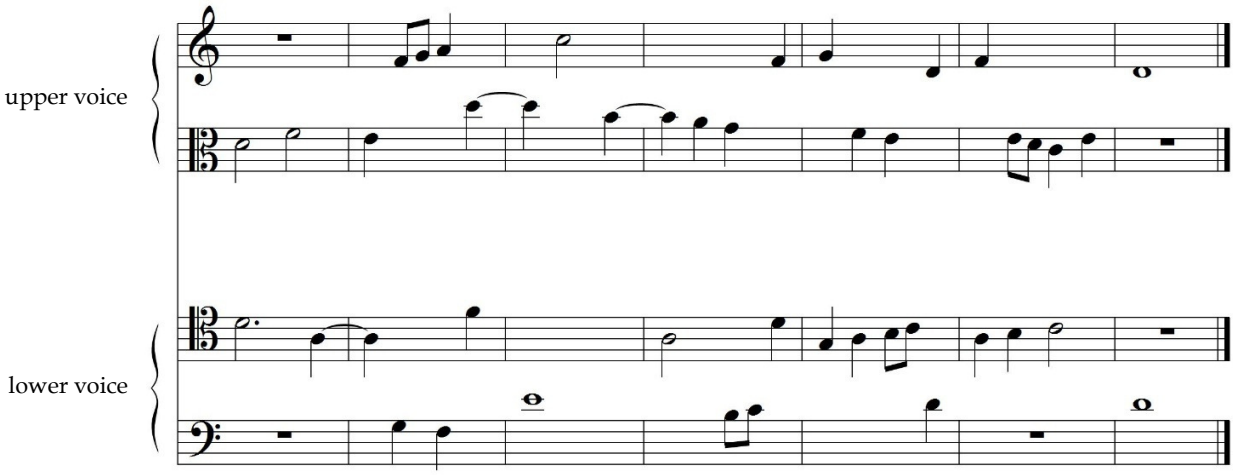
18.



19. Solution of example 13



20. Is the solution correct?



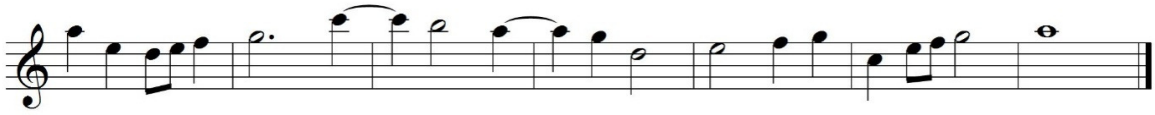
Invertible Counterpoint On the Twelfth

1.

1	2	3	4	5	6	7	8	9	10	11	12
12	11	10	9	8	7	6	5	4	3	2	1

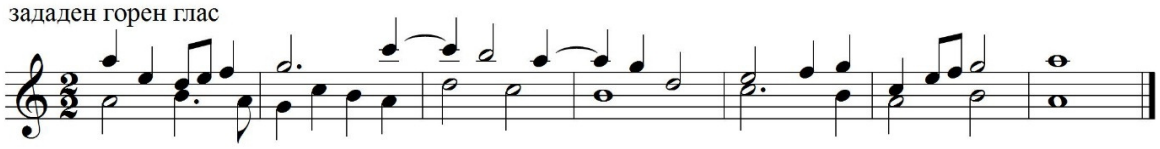
The only problem here is the sixth - it turns into a seventh.

2.



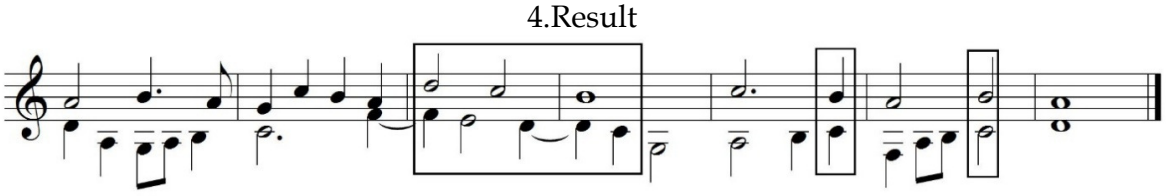
Here's a standard solution - a mode of two florid melodies:

3.Model



I move the upper voice a twelfth downwards[1]:

4.Result



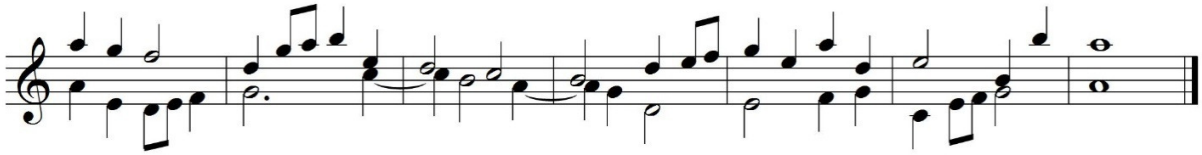
Terrible. But fair enough - I haven't been forgiven a single non-transient sixth, nor a leap from a sixth, nor a syncopated chain containing sixths. The “ordinary” sixth in the penultimate measure has also turned into a seventh. There are also parallel sevenths.

Here's the “iron” rule: if there are sixths on the vertical, they must be transient or suspended.

Repeated attempt. I use the same cantus firmus as a lower voice.

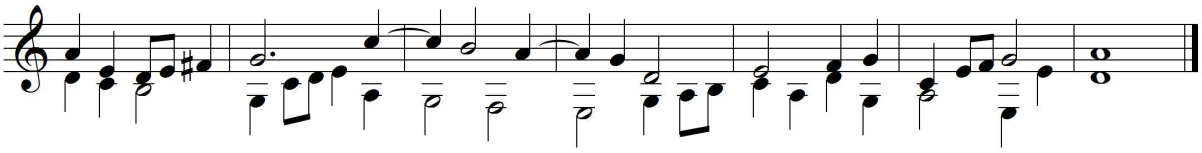
[1]If I transpose the cantus firmus below the counterpoint, the mode will change. This is also often true when moving the counterpoint.

5.Model



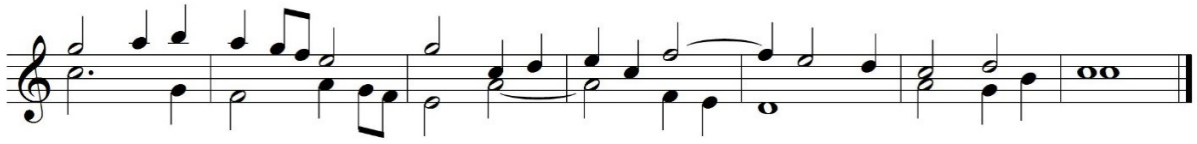
In order to reach the main tone as the last one, I wrote the fifth degree of the mode in the counterpoint (A). Below, the upper voice is transferred a twelfth downwards.

6. Result

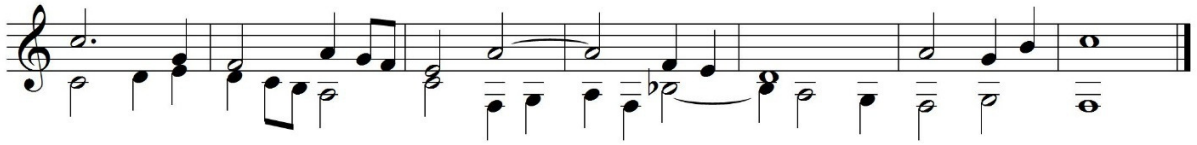


The mode at the beginning and at the end can often be different.

7. Model

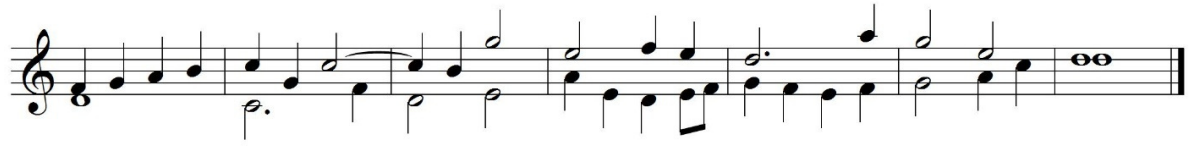


8. Result



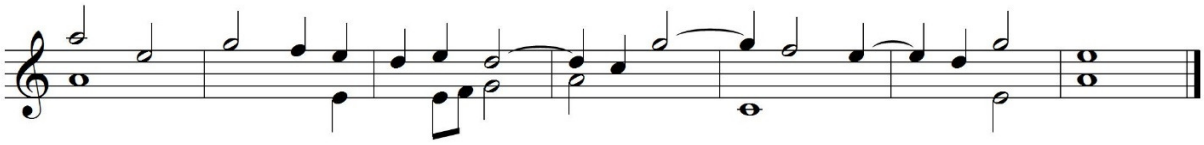
In the following example, which has no realization, it is possible that there are errors.

9.

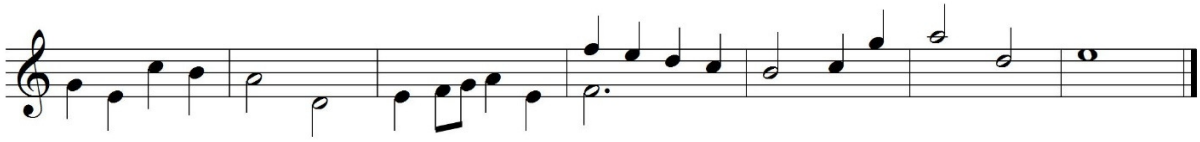


Exercises

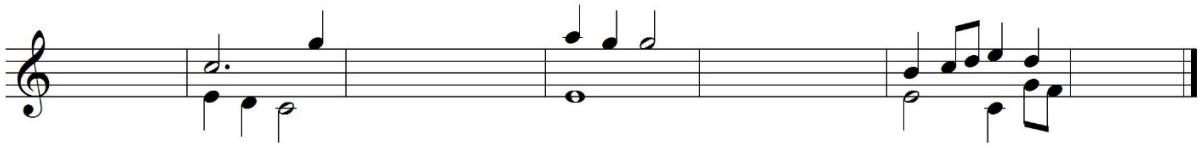
10.In this exercise the blank spaces originally had sixths on the vertical.



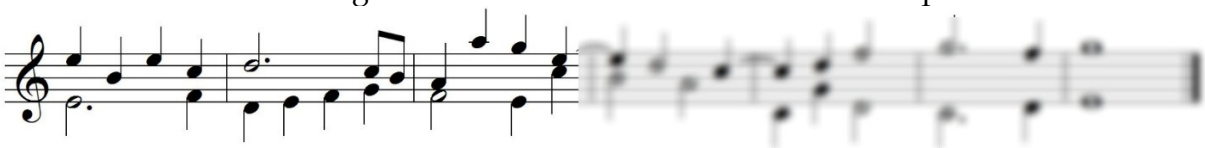
11.



12. The first and last tone of the lower voice is E.



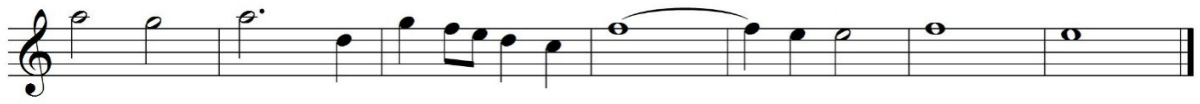
13. This image blurred after the textbook came out of print...



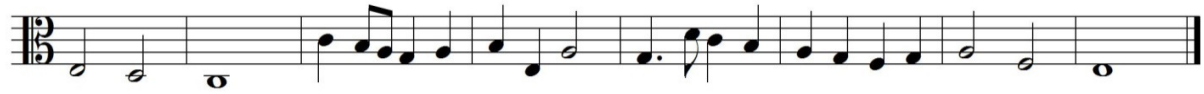
Florid melodies for solving

14.

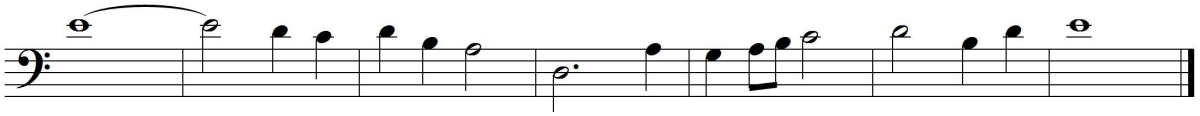
Aeolian mode



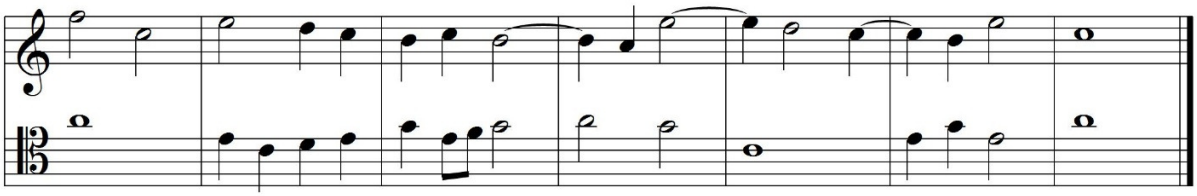
15.



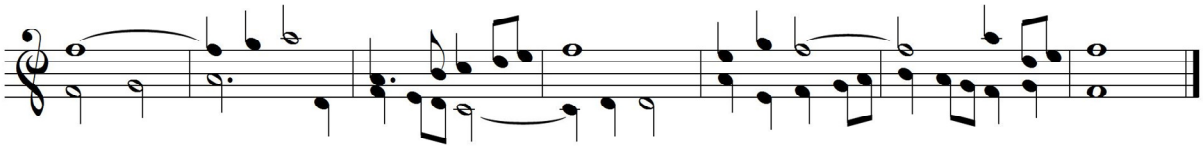
16.



17. Solution of example 10



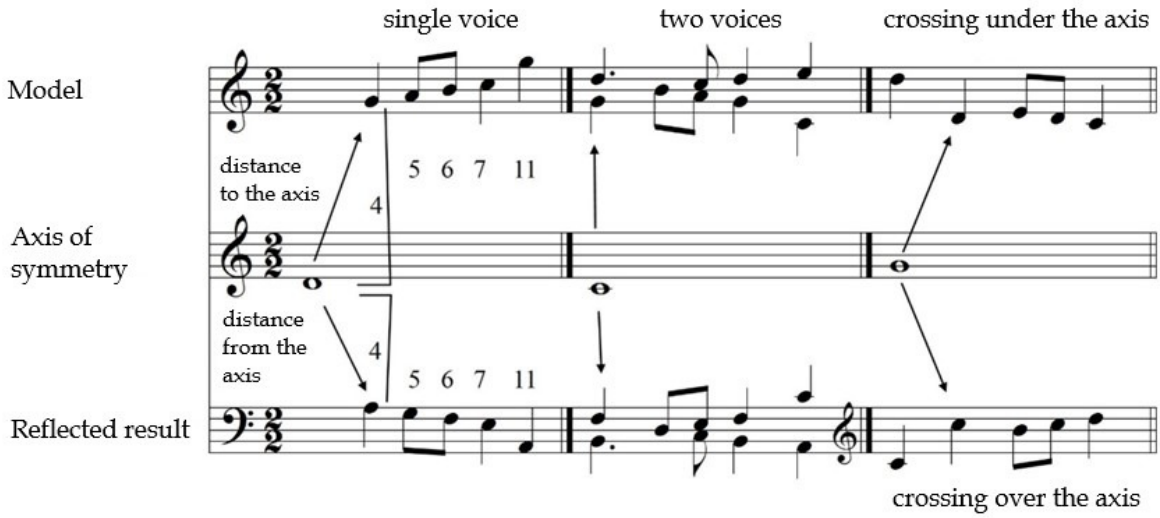
18.Solution of example 12



Mirror Counterpoint

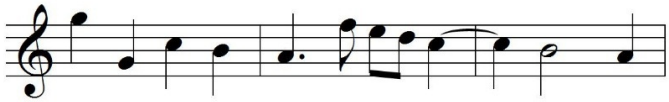
In this chapter a double invertible counterpoint is exercised in a two-voice polyphony. Again, I show the example I used in the chapter on the types of texture and complex counterpoint:

1.



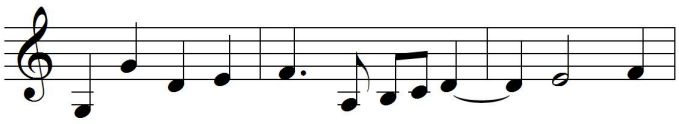
That is, this theme:

2. Model



would appear inverted like this:

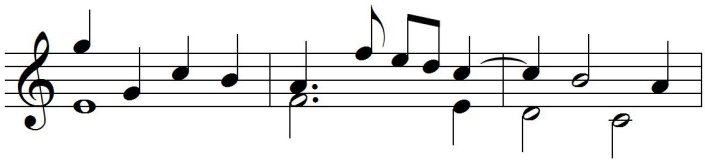
3. Result



with the axis of symmetry in this case being G.

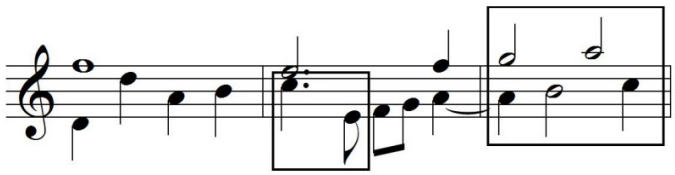
If I add a lower voice to the model:

4.



this example would appear mirrored like this (axis: B):

5.

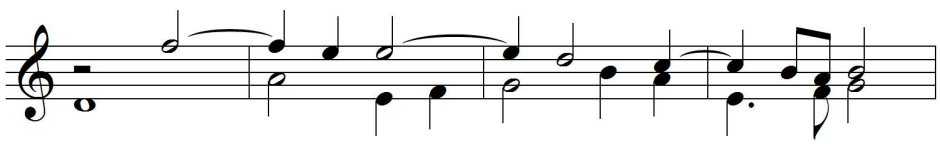


The upward sixth leap in the model has been inverted into a downward sixth leap; the suspended chain dissonances in the model are resolved with downward motion - in the result they are resolved with upward motion.

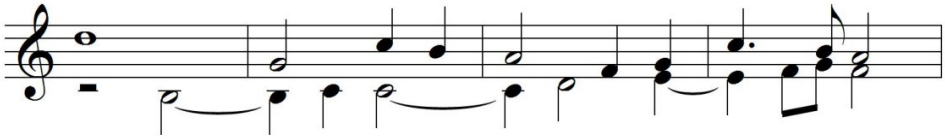
This mode is a good illustration of the importance of planning in advance: any motion will be reflected in the opposite direction, so it's good to think about the consequences of my actions. This also sets the rules for this species - to avoid sixth leaps upwards and suspended and chain dissonances (including the formulas for fragmenting a dissonant syncope); but the cambiata is allowed - the leap from dissonance is possible both upwards and downwards.

In fact, both the syncopated chain and the dissonant formula can be counterpointed correctly in mirror counterpoint - given that there is no suspended dissonance anywhere:

6. Model

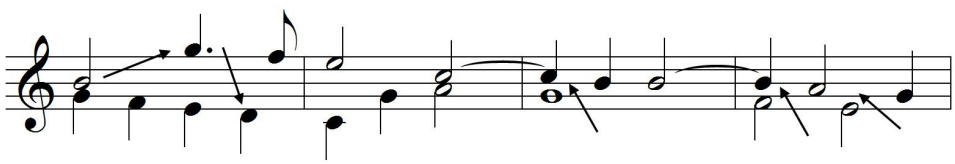


7. Result - axis of symmetry - D[1]



Here is an incorrect model (without realization):

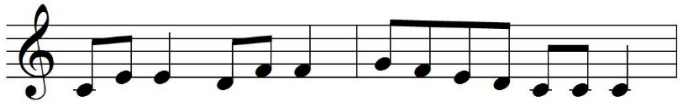
8.



[1]With a different axis of symmetry, the fifth or fourth would be inverted into a tritone. But this consequence is valid for all modes of inversion and invertible counterpoint.

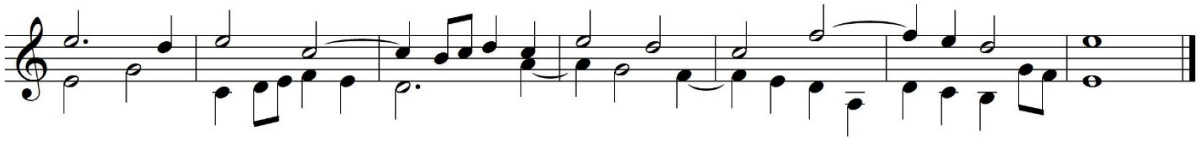
Unlike retrograde inversions, mirror inversions "sound" - the brain registers the mirror variant of the straight one:

9. "Stuck hippopotamus"[2]

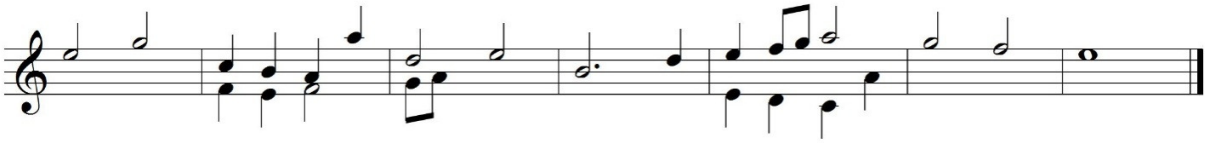


Exercises

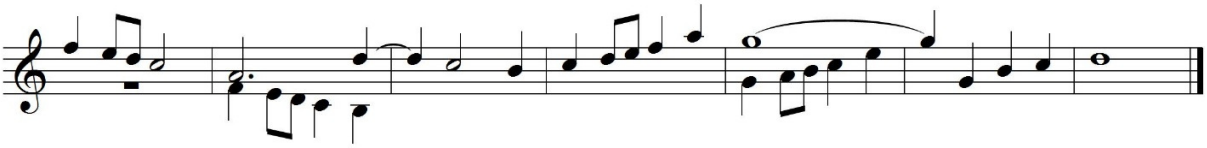
10. Are there any errors?



11. There are missing notes in the lower voice here, but then again those are present, are incorrect

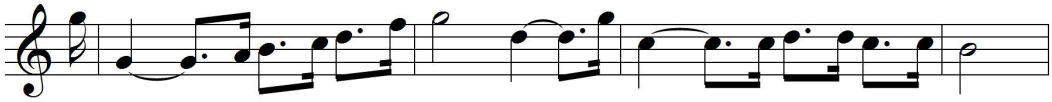


12.

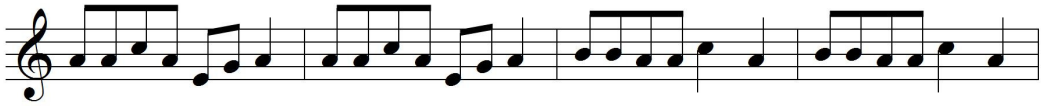


Do these melodies look familiar?

13.



14[3].



[2]Known in its straight variant as "Hänschen klein" – in Bulgarian: "The Blackbird Went A'Walking" («Тръгнал кос»). A joke in Bulgarian: a stuck hippopotamus is supposed to be the inverted variant of a walking blackbird.

[3] inverted variants of a Bulgarian children's song and the Bulgarian National Anthem.

Combined Invertible Counterpoint

The four types of counterpoint taught can be combined by following their rules simultaneously. There are eleven practical variants[1].

With two variants out of three possible, the options are three:

12 23 13

With two variants out of four possible[2]–six:

12 13 14 24 23 34

If we don't count the recurring options, their total number so far is six.

With three variants out of four possible – four:

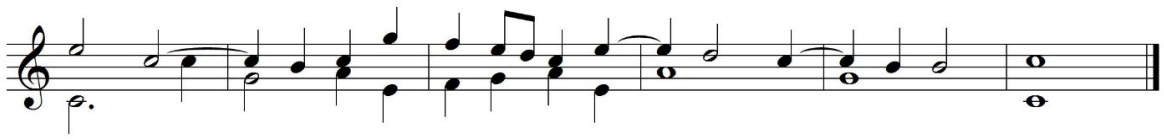
123 124 234 134

plus one more option in which all variants are used:

1234

For example, following the rule of no fifths and sixths except transient ones, I can write a combined counterpoint with the index octave and twelfth:

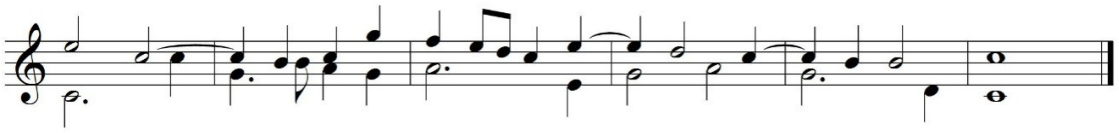
1.



Here is a combination of an octave counterpoint with a twelfth one where there can't be a similar motion:

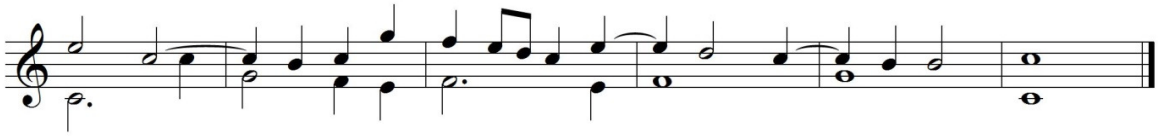
[1]There are no realizations in this chapter – they are not necessary. The technical ability to combine and follow the various rules is sufficient in itself. Making an actual realization (depending on what its purpose might be) is an operation that belongs mainly to the composer.
[2]The four variants referred to are the four types/species of invertible counterpoint covered in this text book – at the octave, at the tenth, at the twelfth and the mirror (inverted) counterpoint.

2.



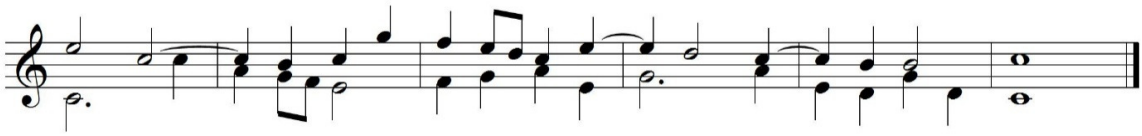
Invertible counterpoint at the tenth and fifth (at the twelfth).

3.



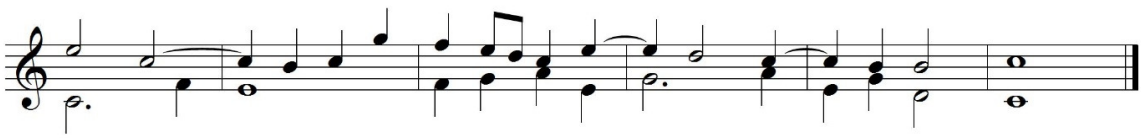
At the octave and mirror counterpoint (where there is no upward sixth leap and suspended dissonances).

4.



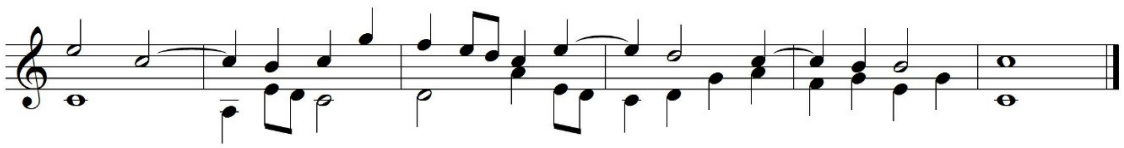
At the tenth and mirror counterpoint.

5.



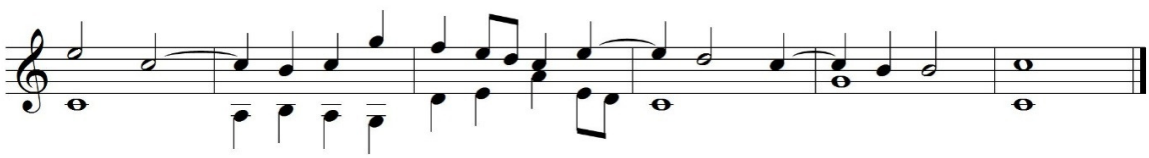
At the tenth, fifth and mirror counterpoint.

6.



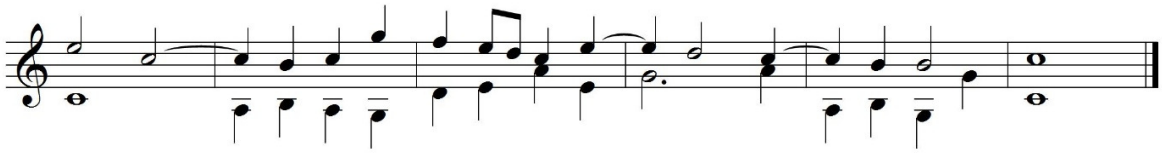
At the octave, fifth and tenth.

7.



And the last combination - the four types together:

8.



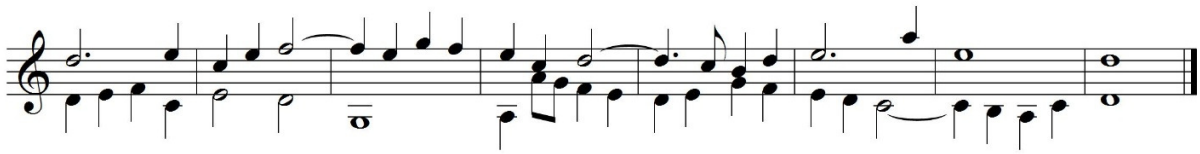
Thus I have demonstrated eight variants out of eleven possible.

The combined counterpoint is not difficult - it mainly requires concentration. The two-voice exercises are somewhat abstract, synthetic, but the importance of these techniques grew with the course of history and reached its peak in the Baroque period.

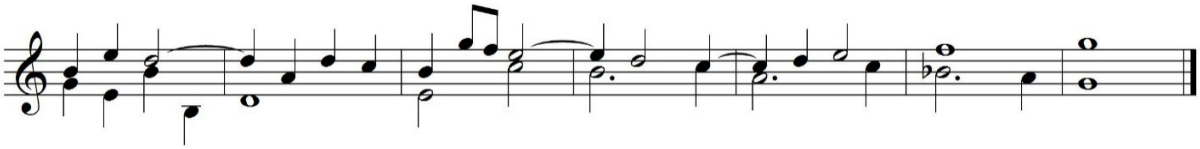
Exercises

Identify in what combination of indexes the following exercises were written:

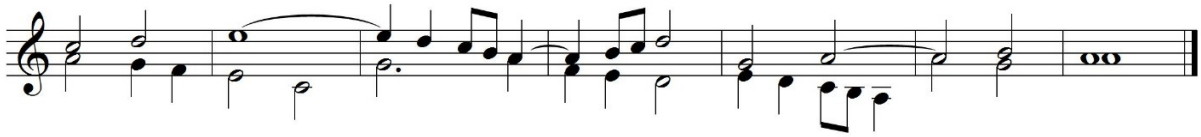
9.



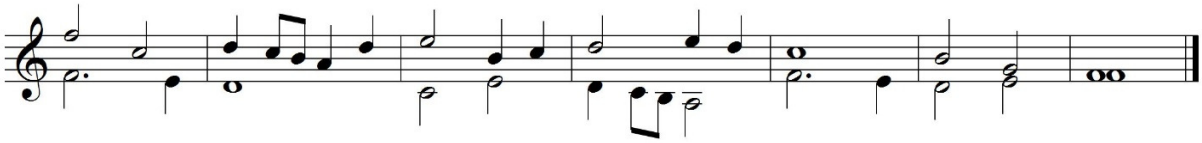
10.



11.



12.There are two indexes here, but something appears to be wrong.



The solution is possible with a correction of only one note.

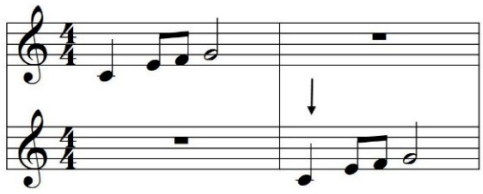
Imitation

"The direct transposition of a melodic fragment from one voice to another is called imitation"[1].

Types:

I. Strict/real - often marked with the Latin letter T (Theme) or P(Prime)

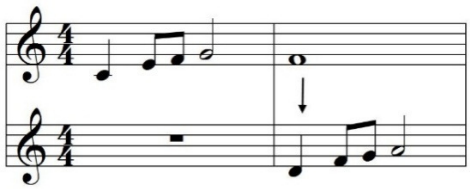
1.



Here the imitation is in the lower voice - it is marked with an arrow. In the upper voice the counterpoint has not yet been written. The interval between the theme in the first measure and its imitation in the lower voice is an unison. That means it is an imitation on a unison. There are two voices here, but of course there might be more.

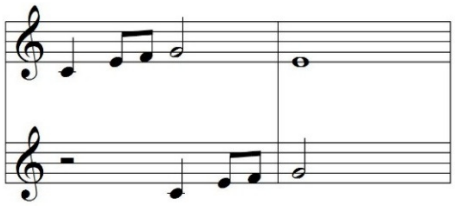
The imitation interval may be different. Here is an imitation on a second:

2.



In the second measure there is already a counterpoint (counterposition) in the upper voice - the whole note F.And here the imitation is shifted horizontally (it is called contratant (stretto) imitation):

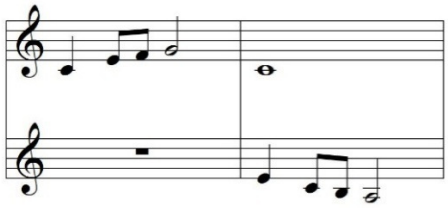
3.



[1]**Манолов**, Здравко,**Христов,** Димитър. Полифония. София: Музика, 1977, с. 133. [Manolov, Zdravko, Hristov, Dimitar. Polifonia/Polyphony. Sofia: Muzika, 1977, p. 133]. For more information on the imitation see the cited work of Manolov and Hristov,p. 133-148 ff. and **Карастоянов**, Асен. Контрапункт. Полифония. Второ издание. Ред. Горица Найденова. София: In Sacris, 2017, с. 168 – 168. [Karastoyanov, Asen. Kontrapunkt. Polifonia./Counterpoint. Polyphony. Second edition. Editor Goritsa Naydenova. Sofia: In Sacris, 2017, p. 168 – 168]

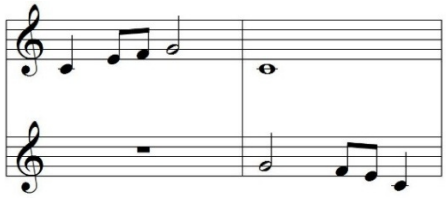
II. Inverted (mirrored) – marked with the letter I(Inverted):

4.



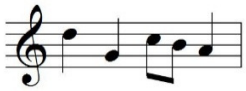
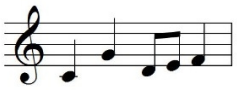
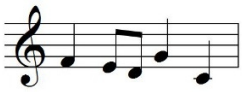
III. Retrograde (crab, backward, cancrizans) – marked with the letter R(Retrograde):

5.



IV. Retrograde of the inverted or invertion of the retrograde –RI(Retrograde [of the] Inverted):

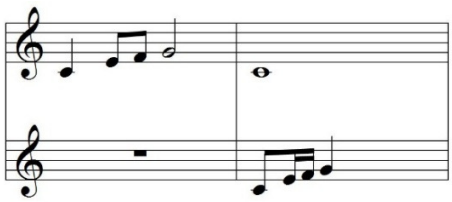
6.

Model	Mirror variant	Retrograde variant of the mirrored
		

The imitation can also be classified by the rhythmic correlation between the original model and the result:

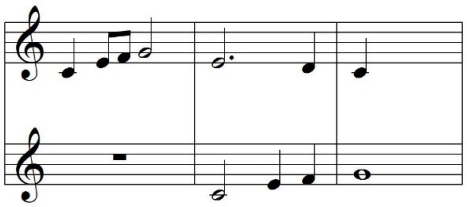
1. Imitation in smaller note values(diminution):

7.



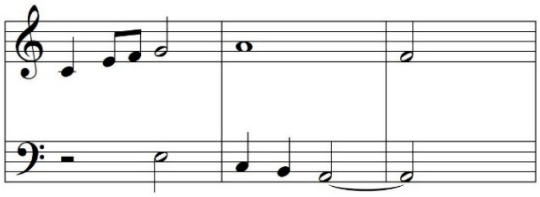
2. Or vice versa - in larger note values (augmentation):

8.



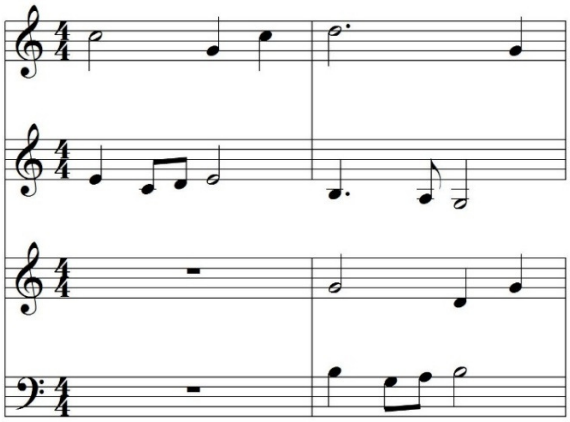
A combined imitation at the sixth is shown here. It is simultaneously mirrored, contratant and in augmentation:

9.



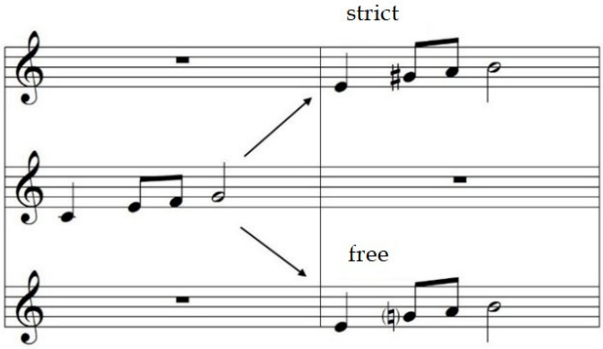
Imitation can also be present simultaneously in more than one voice. A double imitation is shown here, where the two upper voices later appear in the two lower ones at a fourth:

10.



In addition, imitation can be with observed interval ratios between the tones (strict) or without observed ratios (free):

50.



The ratios can also refer to rhythmic characteristics.

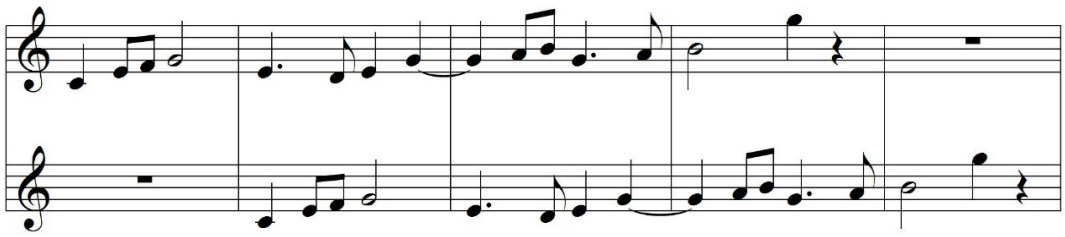
If we take the four possible variants – straight, inverted, retrograde, and mirrored-retrograde - and apply them in normal rhythm, in augmentation, and in diminution, we get 4x3 variants - 12. These are the twelve basic principle variants of imitation. There are many others, even if we don't count the combinations[2].

[2]Inaccurate, rhythmic without a melodic component, melodic without a rhythmic component, solmizational, semantic, micro- and macro- imitations, fragmented, encrypted – and generally following any other, different interpretation of the term "imitation".

Canon - Introduction

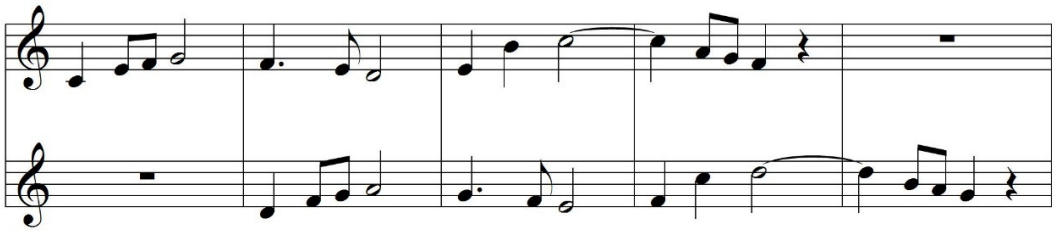
The canon is a direct consequence of the imitation. All types of imitation that have been described can be largely applied to the canon. Here is an example of a unison canon[1]:

1.



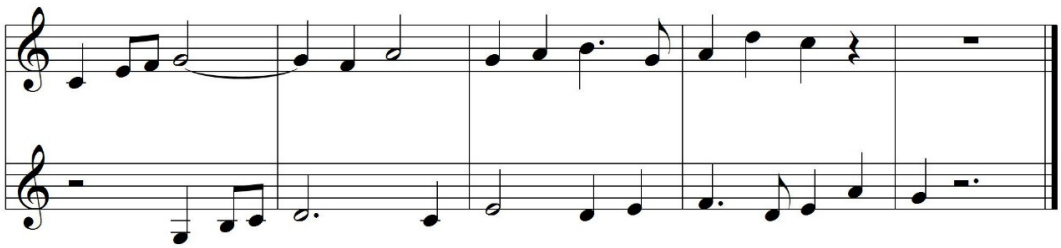
Or canon at the second (any diatonic interval of imitation is possible for constructing a canon in the strict style):

2.



Contratant[2] canon at the fourth:

3.

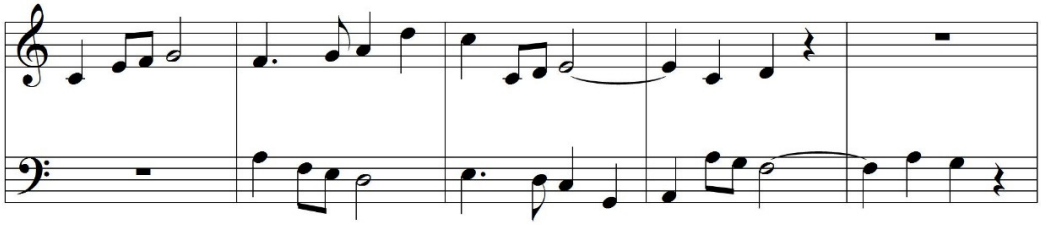


[1] A perfect unison is the interval from the first note of the theme in the first voice to the first note of the theme in the second voice.

[2] The second (lower) voice doesn't start on the downbeat like the first. That is, it is a contratant performance/realization of the lower voice against the upper.

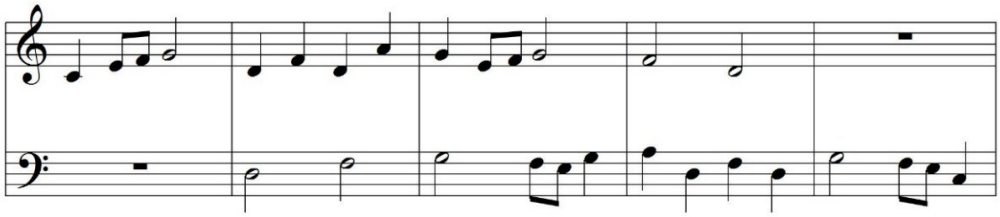
Mirror canon at the third:

4.



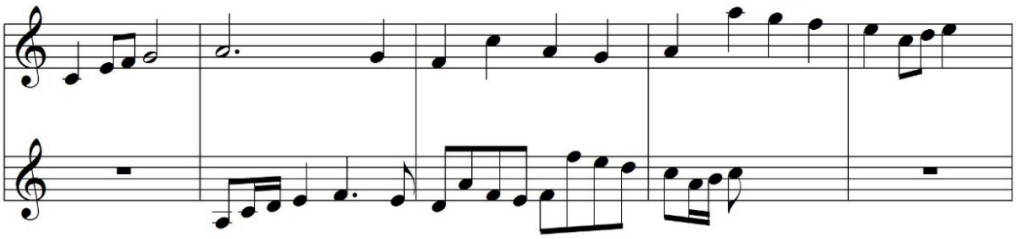
Retrograde canon at the octave:

5.



Canon in diminution at the third:

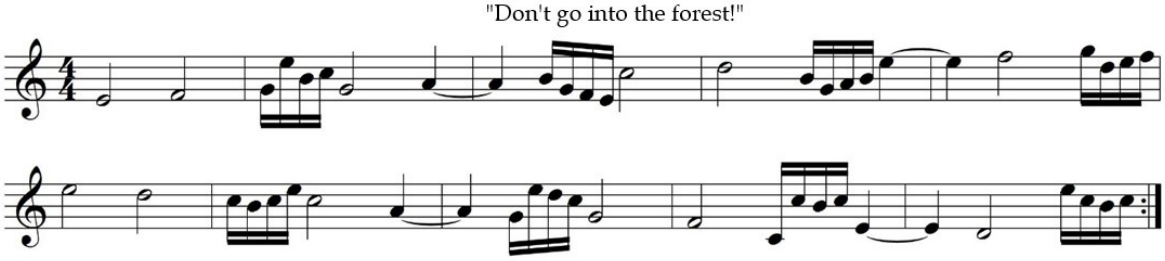
6.



Enigma canon

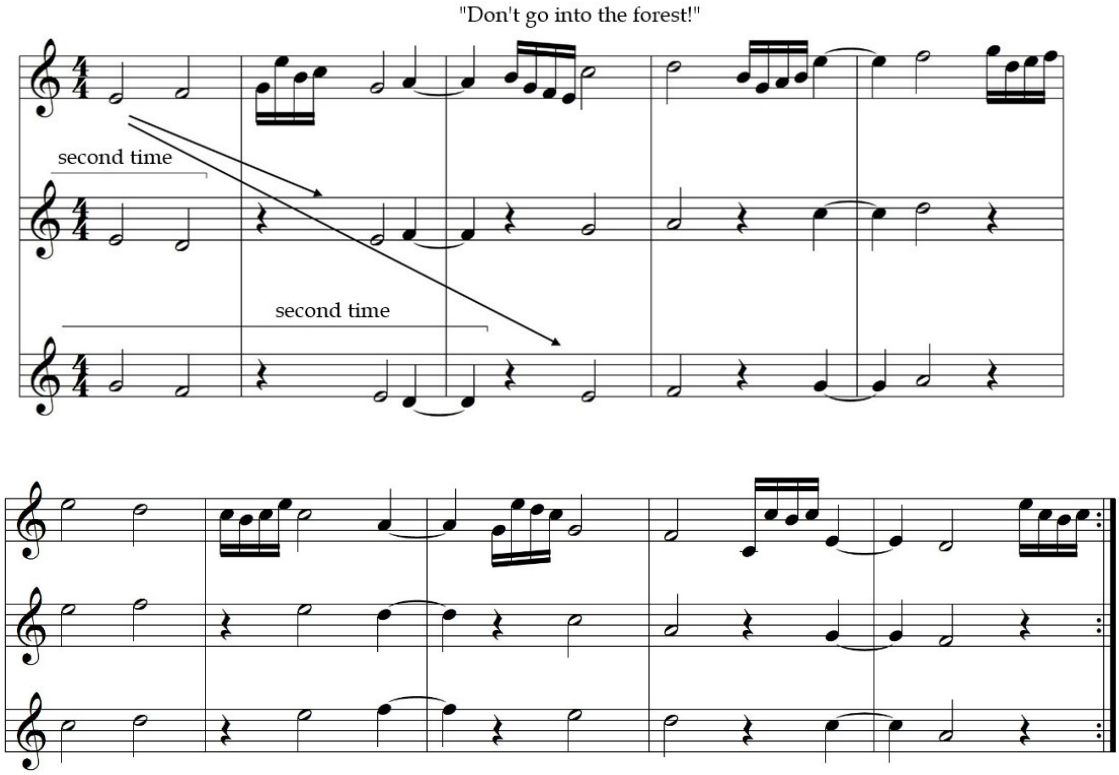
This is a canon that hasn't been realized and there are no instructions on how to do this. Sometimes there is a hint or some obscure remark:

7.



What is being meant? By my logic (the canon is mine) there are trees, branches, and leaves in the forest. These are supposed to be the sixteenth notes. The solution is possible in two voices as well as in three. Below I show the three-voice solution - in it the imitating voices simply don't perform the sixteenth notes - they skip them.

8.



This isn't a Fux's canon - there is a fourth with the bass here, hidden fifths between the outer voices. But otherwise it suits a strict-style canon. Such intellectual-musical entertainments were very common in the past (mainly before TV and the Internet). There have been paintings, prints, even furniture with enigma canons carved on them. In his famous portrait, Bach is holding an enigma canon.

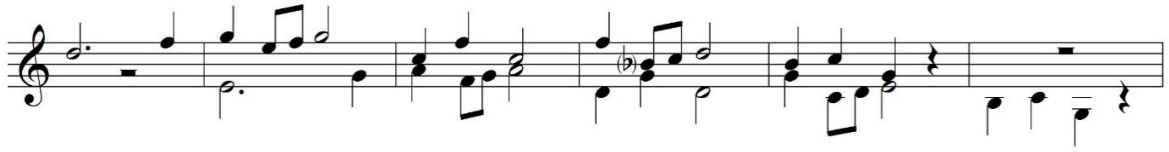
The additional conditions or instructions are what define the term canon. If we have a melody to which some other instruction is added (including hidden or deliberately vaguely explained), that is a canon. The actual word “canon” in Latin means a rule or condition.

Such additional conditions might be many and on different principles. Here we may be talking about geometric or algebraic instructions, solmizational, semantic. Therefore all the possible variants of a canon are infinitely numerous.

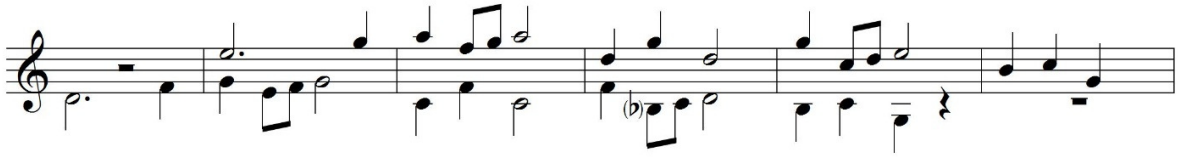
Canon and invertible counterpoint

If, for some reason, the author decides to swap the places of the voices, then the rules for the invertible counterpoint come into play.

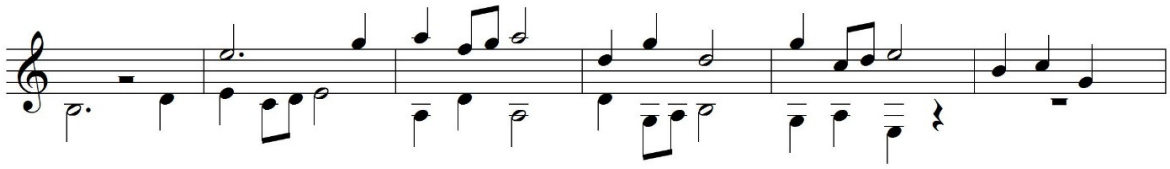
12. Model



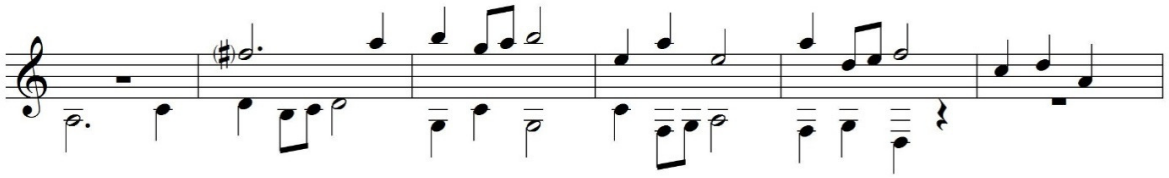
13. Swapped voices and invertible counterpoint at the octave



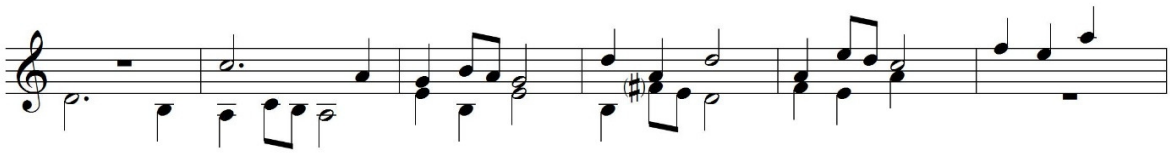
14. Invertible counterpoint at the tenth



15. Invertible counterpoint at the twelfth



16. Double mirror counterpoint

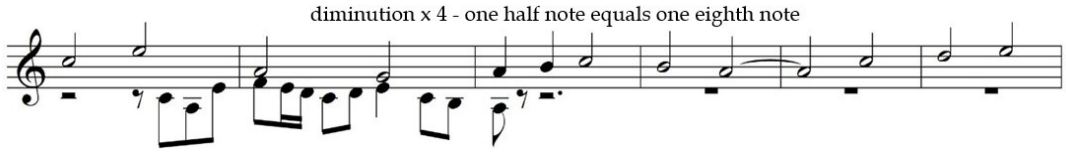


If I know the rules for the invertible counterpoint (and I claim to know them), I can write a canon that will lend itself to any index or indexes I desire.

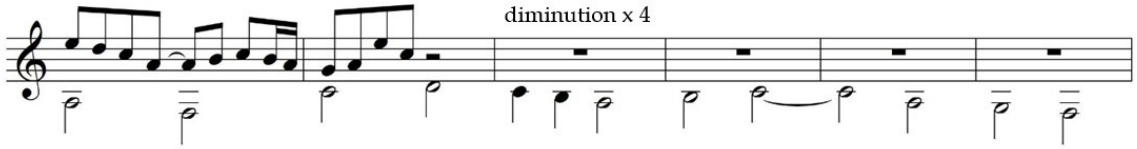
Combined canon

Here I show four examples of canons where there is more than one condition.

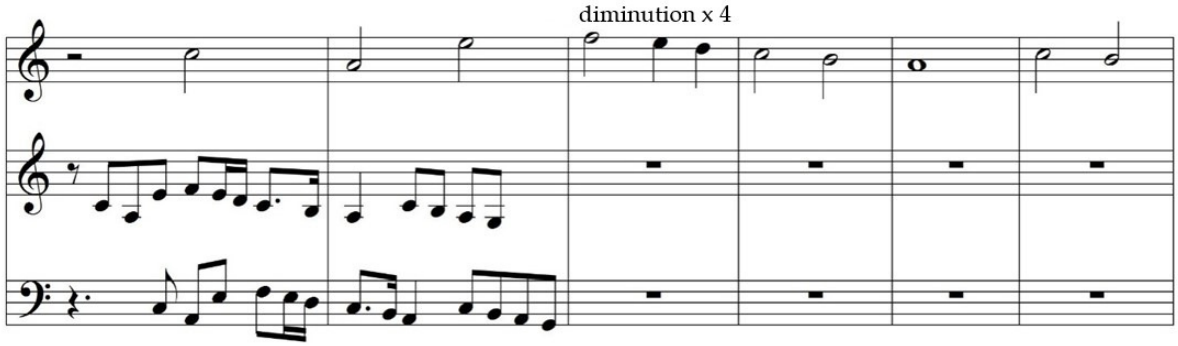
17. Mirror canon at the octave in diminution



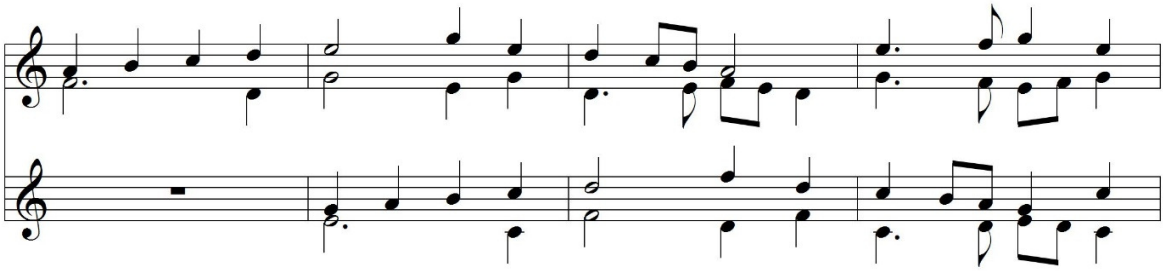
18. Mirror-retrograde canon in diminution (at the fifth)



19. Triple/three-voiced contratant canon at the octave in diminution



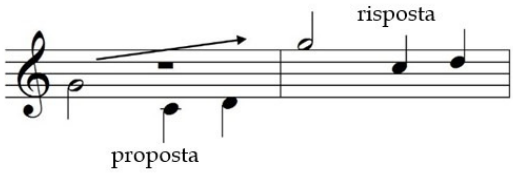
20. Double quadruple/four-voice canon at the second



Writing

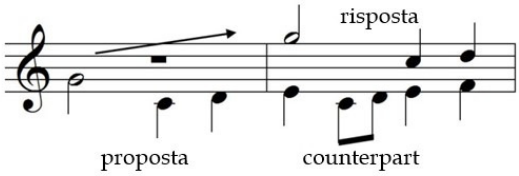
A simple canon can be written by taking its theme (sometimes called proposta) and transposing it into another voice where there is some kind of horizontal shift. The imitated theme in another voice is called risposta.

21.



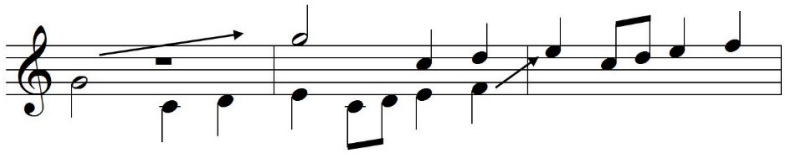
Then write a counterpoint in the lower voice in measure 2.

22.



Then I transpose the counterpoint to the upper voice in measure 3.

23.



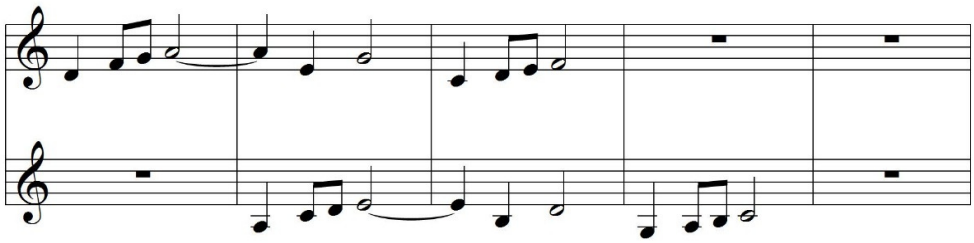
The next step is obvious - I write a counterpoint to this in the lower voice of measure 3 - and so on.

If we followed all of the Fux's rules, the canon in more than two voices would be very difficult to write. With more lenient rules (see the chapter on motet) we get the opportunity to write complex and beautiful pieces with great creative potential and in many voices.

Canonical projection

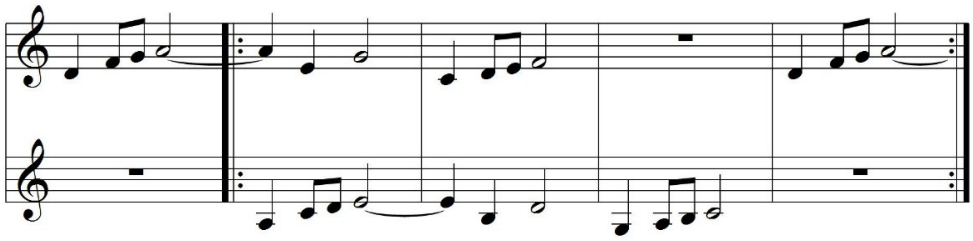
One canonical technique that is important and used in both strict and free style is the technique of projection[3]. It is used when the canonical segment needs to be closed, thus forming an infinite canon. Let's look at the following example of a canon at the fourth:

24.



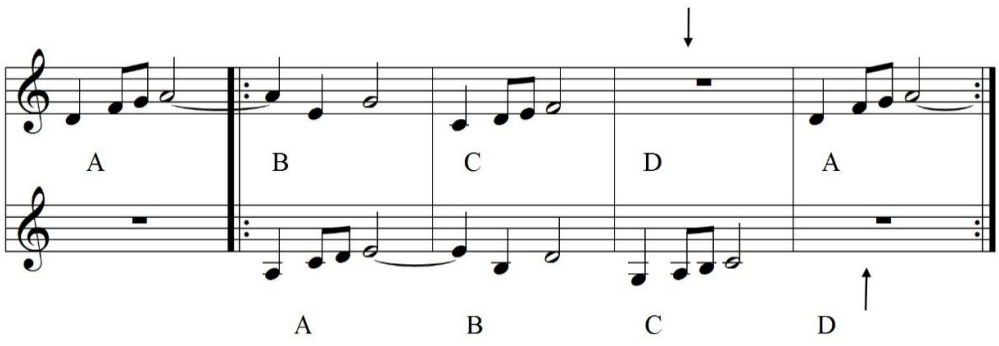
Let's assume that the canon must start over from there, i.e. become an infinite canon (also called a school canon). For this purpose, I am transcribing the material from the first measure into the last, in the original voice, and closing the construction:

25.



I am marking in letters each segment of the canon - there are four, one in each measure. Segment D in the upper voice in m. 4, currently empty, should appear transposed a fourth downwards in the lower voice in m. 5, and fit perfectly to segment A in the upper voice (same measure):

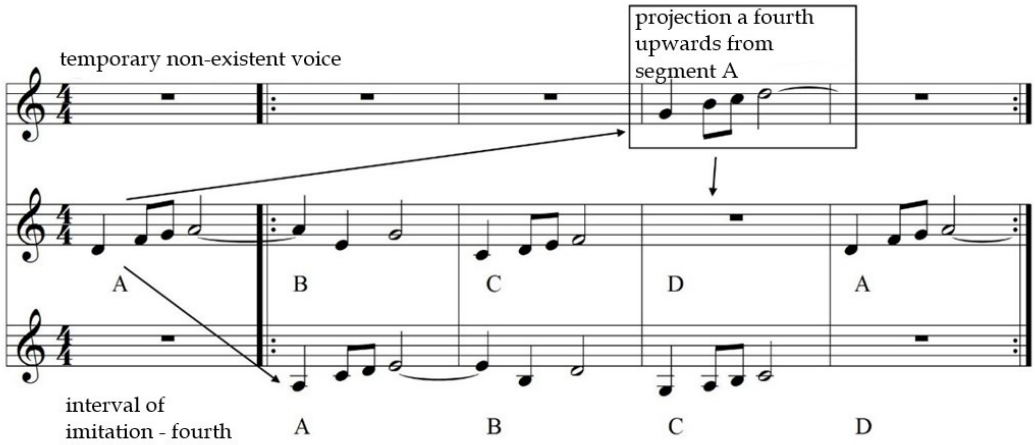
26.



[3] This technique, which comes from the late Middle Ages or early Renaissance, is shown by Sergey Taneyev in his groundbreaking study: **Танеев,** Сергей.Учение о каноне. Учебное пособие. 2-е изд. Москва: Лань, 2017 [Taneyev, Sergey. Uchenie o kanone. Uchebnoe posobie/Doctrine of the Canon. Textbook. 2nd ed. Moskow: Lan/Лань, 2017], p. 39, where he speaks of a "faux unity" (my translation).

For this purpose, I am writing my projection in a staff above segment D. The projection represents segment A, measure 1, transposed at the interval of imitation, but in the opposite direction - a fourth upwards in a non-existent voice (I will only use it temporarily):

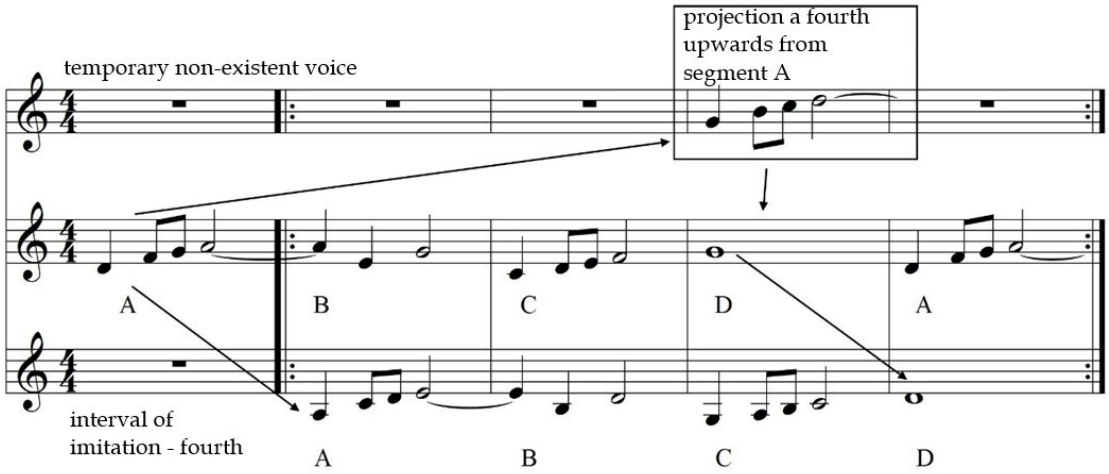
27.



My next step is to write segment D in the upper voice of the canon. How? Segment D needs to be consonant with the lower voice of the canon and *at the same time* - with the projection. All the laws and rules for voice-leading must be followed, both regarding m. 4 and the voice-leading between m. 3, 4 and 5.

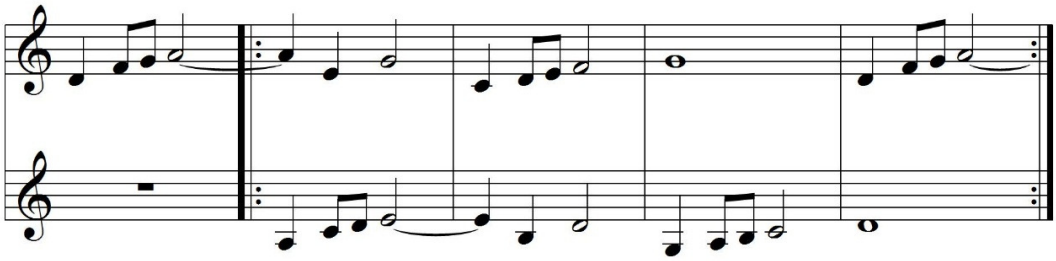
I am discovering that one possible solution is the note G. This whole note is moved a fourth downwards to segment D of the lower voice and so the goal is achieved:

28.



I don't need the projection anymore[4]. Once I have used it, I delete it. This is what the final variant of this infinite four-segment (also called four-part) canon looks like, with the projection deleted and without letters on the segments:

29.



In this textbook four types of Taneyev canon are also taught - they are on a slightly different, mathematized principle. A separate chapter is devoted to each type.

[4] The projection never sounds right with the other voice of the canon, in this case - segment C. That's why it's also called a “false risposta”.

Two-Segmented Canon After Taneyev

The Russian composer, pianist and music theorist Sergey Taneyev (1856-1915) was one of the most important polyphonic scholars (along with Fux and Jeppesen). He has two massive studies devoted to the strict style: *Подвижной контрапункт строгого письма* [Convertible Counterpoint in the Strict Style] and *Учение о каноне* [Doctrine of the Canon]. In these complex and profound scientific works, Taneyev has made many discoveries - mainly mathematical rules that underlie the strict style. The next two chapters of the textbook are taken from these books, beginning with this one. Very little of Taneyev's research is used here, but I have found that components of his theory are taught in several places around the world, not just in Russia - for example, in Israel[1].

In the Taneyev's system the intervals are marked with different digits - not the ones we have been using until now. His numbering is based on the distance between two tones (diatonic). For example, Taneyev marks the perfect unison with 0. The second is 1 because there is a distance of one diatonic unit between the two tones of the second, for example between C and D:

1.

Interval	Perfect unison	Second	Third	Fourth	Fifth	Sixth	Seventh
Digit	0	1	2	3	4	5	6

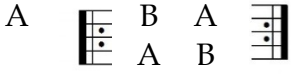
Octave	Ninth	Tenth	Eleventh	Twelfth	Thirteenth	Fourteenth
7	8	9	10	11	12	13

and so on.

When we speak of the canon, viewed from a criterion of length, it can consist of either two or three principle rhythmic/metric components. Four components would consist of two two-segment ones. Five components would consist of a two-segment and a three-segment one, and so on.

Speaking of a two-voice polyphony, we can try to write a two-segment canon that is also infinite. For example, the proposta of one such canon is marked with A and the second segment with B (one segment = one measure):

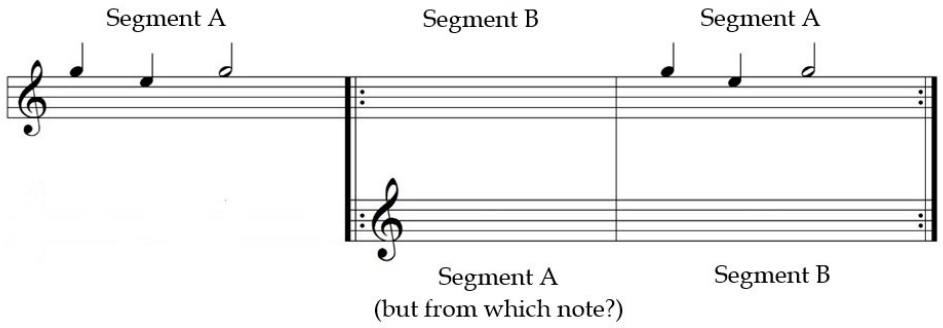
2.



On notes:

[1] This was true for 2014. I believe that polyphony specialists use some of his material in their courses in other countries as well.

2.

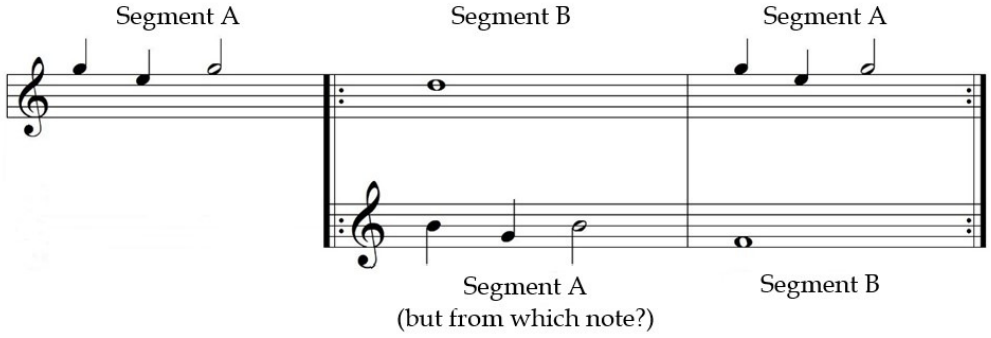


As you can see, the voices change their places - the upper becomes the lower and vice versa. The vertical configuration of A/B has become B/A. This is important, it means that we are talking about invertible counterpoint.

In this situation, Taneyev asks the question I wrote in Example 2 in the brackets - where (from which note) should start segment A in the lower voice?

For example - I can try to write Segment A from B to see what happens:

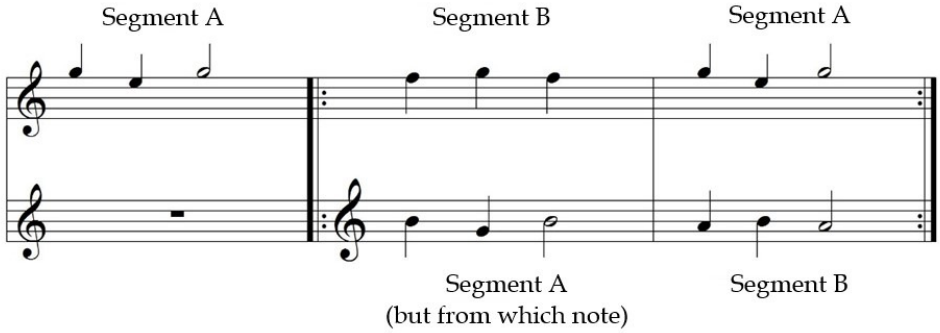
3.



It doesn't work.

I write a different counterpoint in segment B:

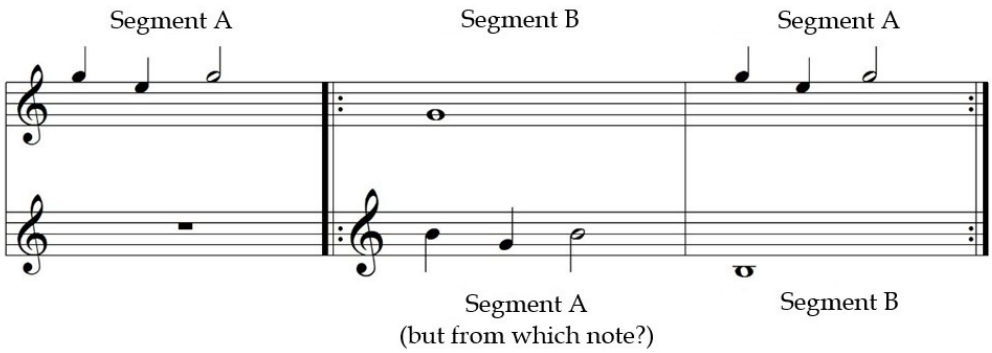
4.



Again it doesn't work.

Third attempt:

5.

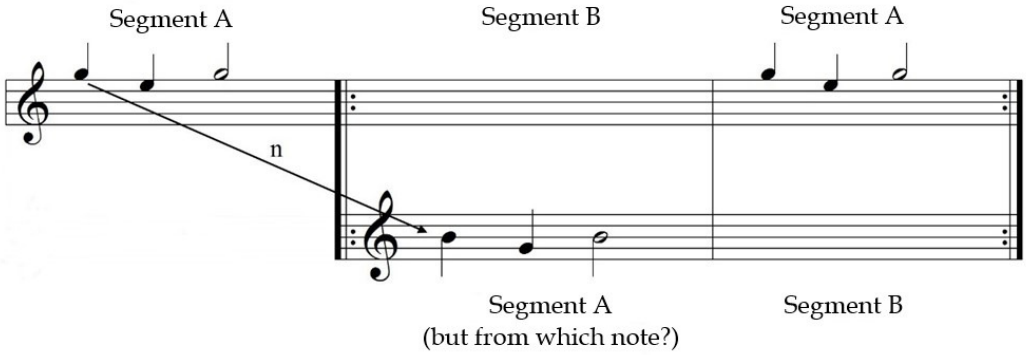


No, it's not working. Any Segment B in the upper voice (m. 2), transferred to measure 3 in the lower voice, doesn't counterpoint with Segment A in m. 3.

Then Taneyev tells us that we should bear in mind the invertible counterpoint. Taneyev calls his index “Index verticalis” and marks it with the symbol Iv.

Taneyev's index of invertible counterpoint at the octave would therefore be Iv = 7. And because the segments switch their places, Taneyev argues that the index should be divided in two - in order to reach the interval from the first tone of A in the first voice to the first tone of A in the second. This interval is marked by the letter n - interval of imitation:

6.



However, there is one problem. Seven can't be divided in two. The diatonic tone row consists of seven components, that is, component 3.5 doesn't exist or, to put it in more modern language, there can be no fractions. To put it in even more literal language, there is no key between E and F; and there are no black keys at all.

Here is the equation that Taneyev derives:

7.

Iv = 2n

The index of the invertible counterpoint (Iv) divided by 2 gives the interval of imitation (n) from the first note of segment A to the first note of segment A in the other voice.

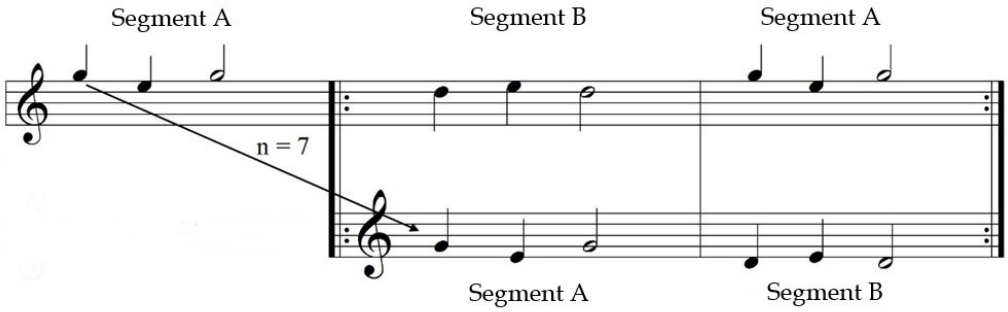
As I have previously shown, sometimes it is necessary to add another octave to the realization of the invertible counterpoint. Taneyev does the same in this equation:

8.

Iv = 2n	
Original formula	Modified formula
7 = 2n	7 + 7 = 2n

So Iv can be divided in 2 and the answer is 7, i.e. octave.
I immediately try to write a canon at the octave:

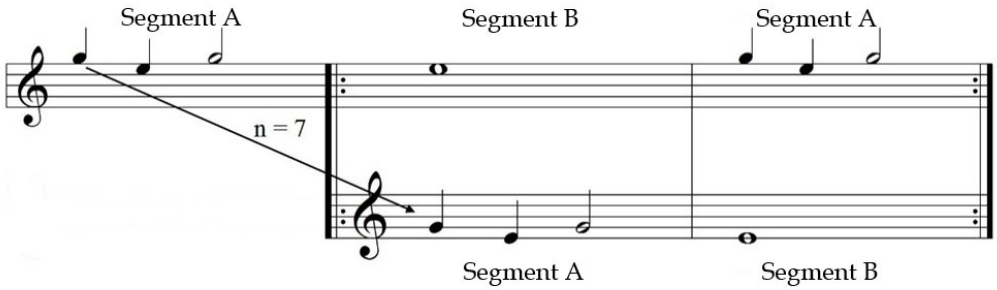
9.



It didn't work again, but the fault here is all mine. I had forgotten that in the invertible counterpoint, there should be no fifth on the vertical unless it is a transient one. It will turn into a fourth. I have allowed a fifth between the first note of segment B in the upper voice, measure 2 (D) and the first note of segment A in the lower voice, measure 2 (G). As a result, there is a fourth interval at the beginning of m. 3.

In my next attempt, I will keep this condition - it's obligatory. At last I find that this time my attempt has succeeded:

10.



In this way, Taneyev uses an algebraic formula in order to explain a polyphonic/musical problem. This is unique in its own way. The conclusions are two:

1. With the invertible counterpoint, the only interval that can be used in the canon (n) is the octave (and in theory, the perfect unison).

2. If we use an invertible counterpoint (at the octave, tenth, or twelfth), we must follow its rules.

The next "sounding [2]" invertible counterpoint is that at the tenth. After Taneyev:

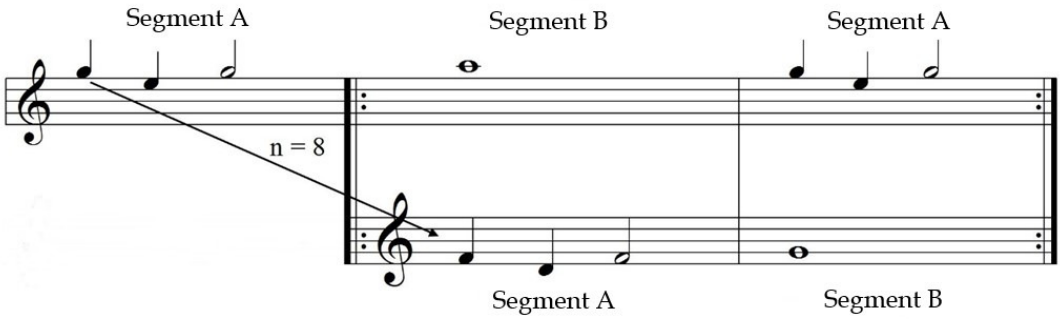
11.

$$9 = 2n$$

Nine is not divisible by two, again we add seven:

$$16 = 2n$$
$$n = 8$$ – i.e. ninth

12. (I remember not to allow similar motion between the voices)



[2] This Taneyev's formula will also give a correct answer for impractical indexes of the invertible counterpoint. For example, for one at the ninth (8 after Taneyev) the answer will be 4, i.e. fifth. But the canon would not work out. All consonances (except the fifth) will turn into dissonances.

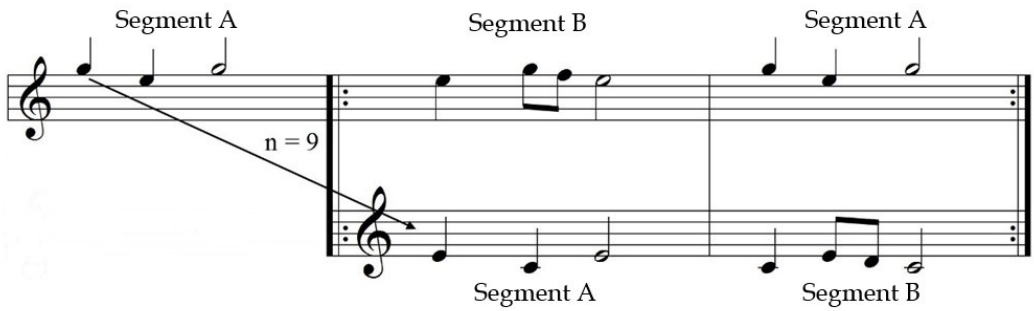
Invertible canon at the twelfth:

13.

Iv = 2n
11 (twelfth) = 2n
11 + 7 = 18
18 = 2n
n = 9 (tenth)

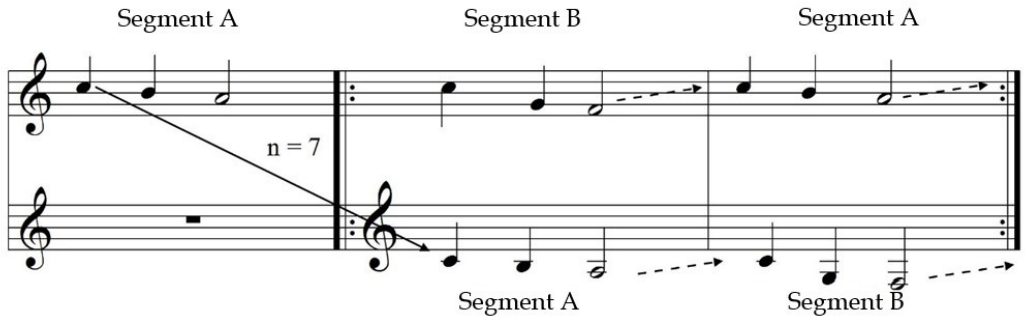
I shouldn't allow a sixth here on the vertical unless it's a transient one.

14.



As can be seen, all the rules of the Fux's counterpoint are followed, as well as those of the invertible counterpoint. But we have to be careful that there are no mistakes between the measures - a frequent problem:

15.



The hidden octaves and fifths and the two leaps in the same direction are the most common inter-measure problems. These can be most easily prevented by avoiding similar motion and leaps whenever this is possible.

Using this formula, I know in advance what the interval of imitation will be:

16.

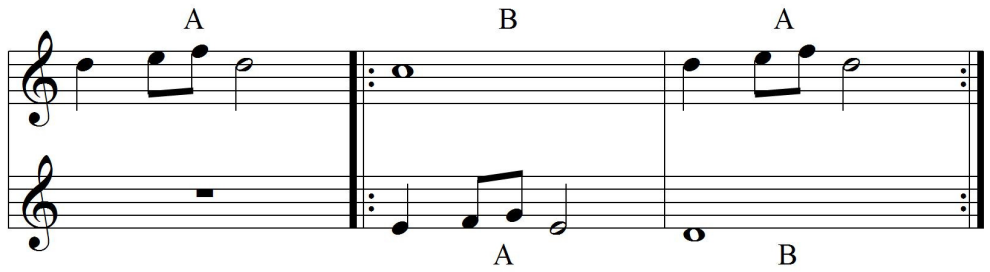
Iv = 7 (octave)	n = 7 – octave (in theory, also unison)
Iv = 9 (tenth)	n = 8 – ninth (in theory, also second)
Iv = 11 (twelfth)	n = 9 – tenth (in theory, also third)

There are no other options.

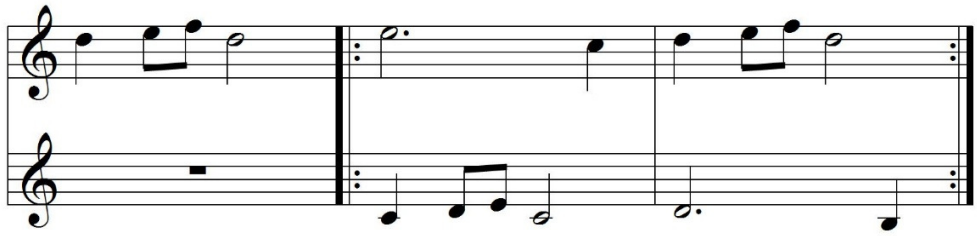
Exercises

In the following five examples I have solved several canons. Although I have tried to do everything correctly, it's possible that I may have made a mistake somewhere. I also wanted to write the correct index (Iv) for each exercise, but...

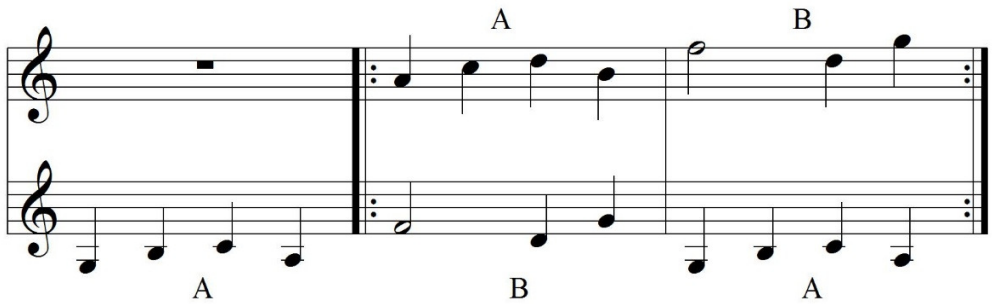
17.



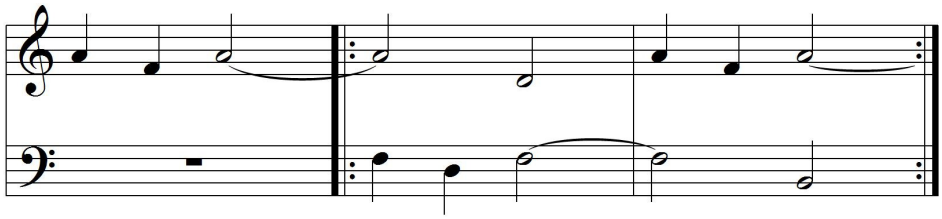
18.



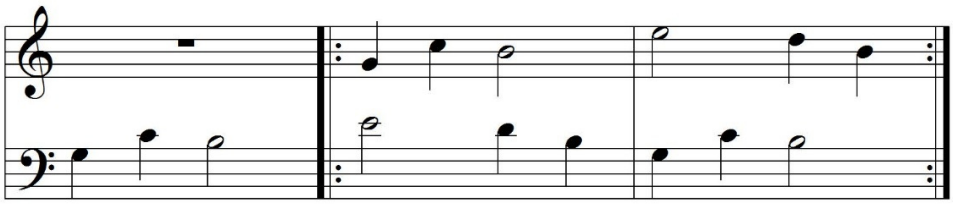
19.



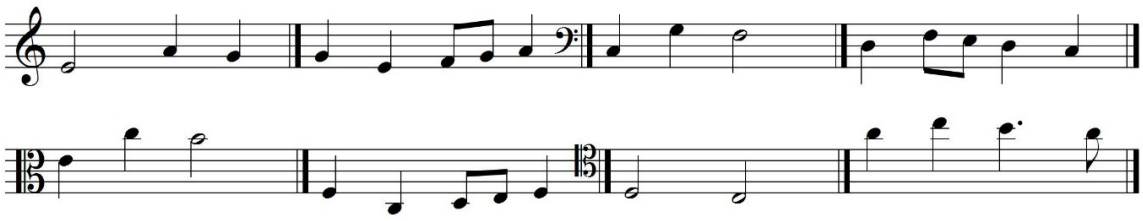
20.



21.



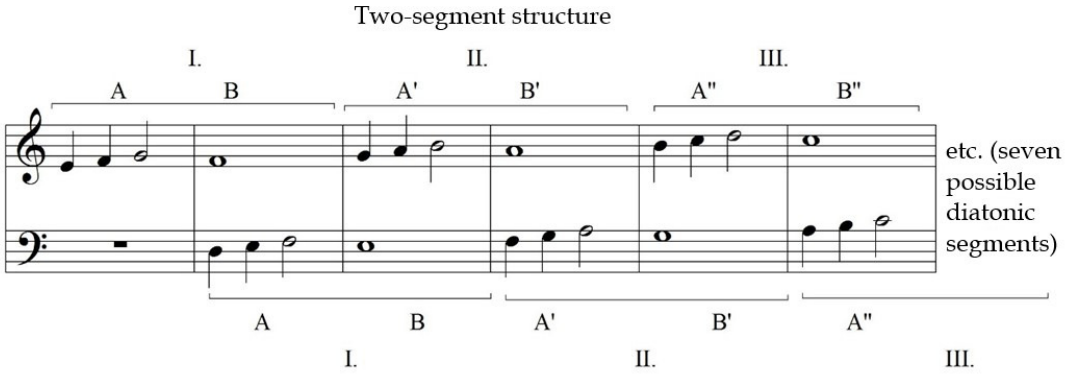
22. Propostae for solving



Two-Segmented Canonic Sequence

On the basis of the already explained formula Iv=2n Taneyev demonstrates us a modification of it, which concerns the canonical sequence. This is a variant of the canon that is combined with a sequence:

1.



This canonical sequence goes a third upwards. It starts from E in the first measure - the beginning of the first segment. The second segment starts from G in m. 3. Then it starts from B at the beginning of segment III, measure 6. On the same principle is also the lower voice - segment I starts on the second measure from D. Then from F in m. 4, from A in m. 6 and so on.

If I am given: 1. a one-measure proposta, 2. an index, 3. an instruction whether the sequence ascends or descends, and 4. at what interval I can use a Taneyev's formula and calculate at what interval my imitation should be in the second voice.

2.

Iv = 2n± m

Iv is the invertible counterpoint index. n is the goal we are looking for - the interval of imitation from the first note of Segment A (I) to the first note of Segment A in the other voice. ± is the action we have to take depending on whether the sequence is ascending or descending and in which voice. m is the interval at which the sequence ascends or descends.

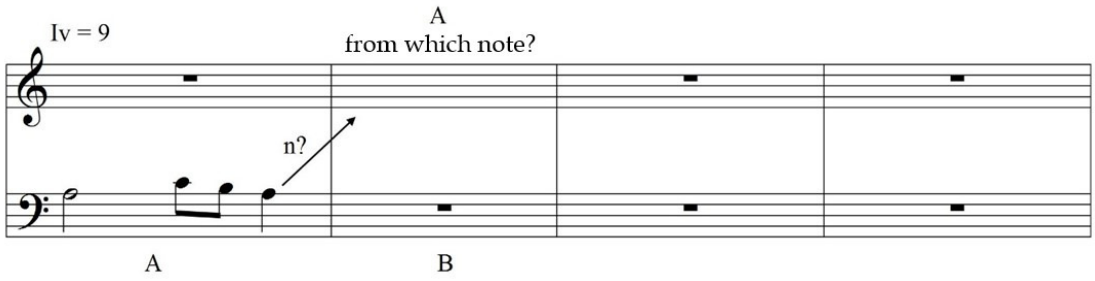
In addition to this we should also use the following table to find out whether it is a plus or a minus:

3.

Voice	Ascending	Descending
Upper	+	-
Lower	-	+

For example: my proposta is in the lower voice (one segment = one measure):

4.



Iv = 9.
The lower voice (which is first in this exercise) should go a second downwards in each segment.

I arrange the formula: Iv = 2n, and then I look at the table of example 3.
The lower voice is descending, so I need +
At the end I write m. A second in the Taneyev's system is 1:

5.

$$9 = 2n + 1$$

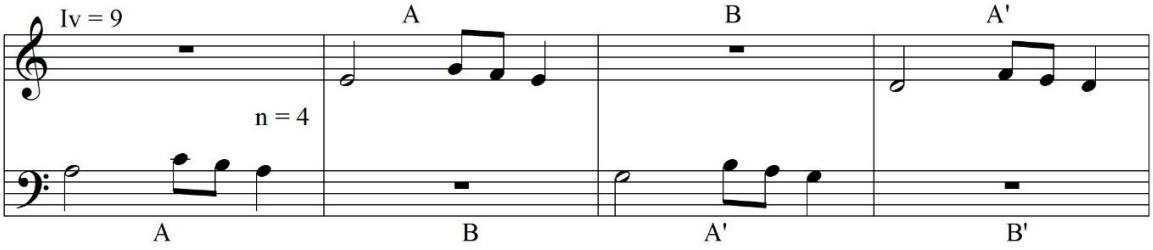
Then I solve the formula. Because one must move to the other side of the equality, the sign is changed:

6.

$$9 - 1 = 2n$$

So I have to divide 8 by two and I get n = 4 (fifth).
My next step will be to write the imitation in the upper voice:

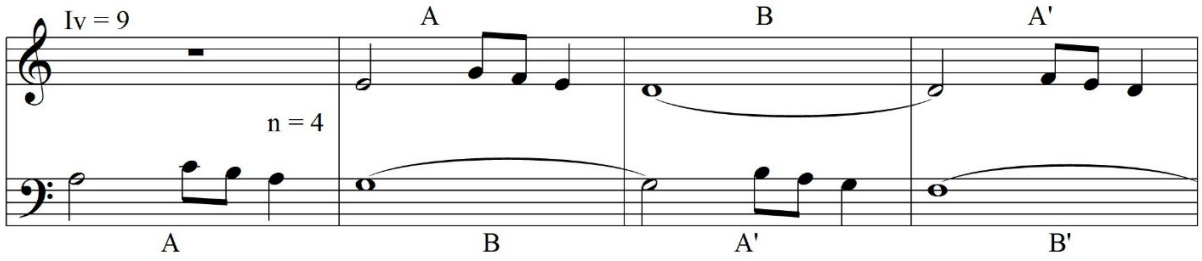
7.



As it can be seen, I have already written A' in the lower voice in m. 3 a second downwards. I have done the same with Segment A' in m. 4 in the upper voice.

Now all I have to do is write segment B in the lower voice, measure 2, and add it to m. 4, and then transpose it to the upper voice in m. 3.

8.



In this solution, there is no similar motion between the voices, i.e. from measure 3 onwards, no invertible-contrapuntal problems will arise.

Second example solution

Proposta in the upper voice Iv = 11. Sequence: a third upwards:

9.

11 = 2n + 2

Component	Value	Explanation	Remarks
Iv	11	Invertible counterpoint at the twelfth	The sixth is prohibited (except transient one)
2n	?	Interval of imitation from segm. A to segm. A in the other voice	We don't know n yet, that's our goal
+		Direction of the sequence movement	Because the upper voice is going upwards
2	Third	Interval of the sequence movement	

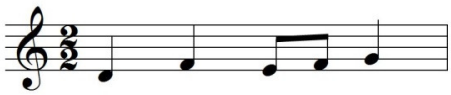
10.

(Iv = 2n±m)
11 = 2n + 2
11 – 2 = 2n
9 = 2n

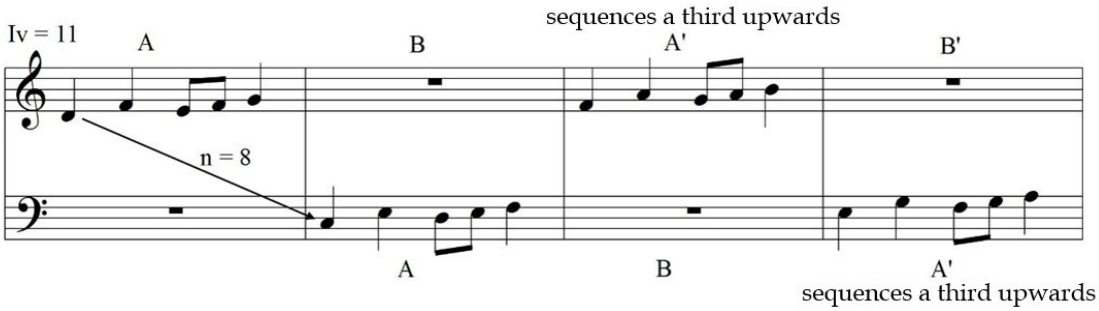
As I mentioned in the previous chapter, we can't divide an odd number by two. We add one more octave (7) to Iv in our equation:

11.
9 + 7 = 2n
16 = 2n
n = 8 – ninth

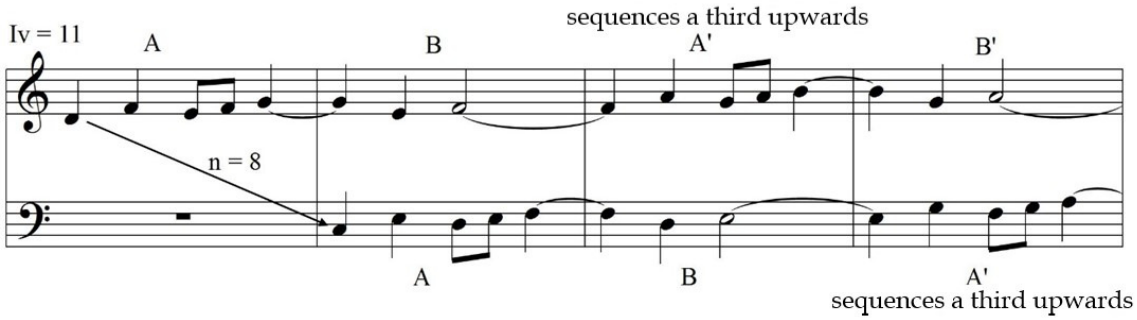
12. Proposta:



13. First stage - placing all segments A.

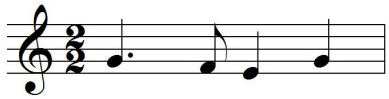


14. Second and final stage: solving the exercise in the mode of invertible counterpoint at the twelfth - placing segments B.



Third example solution

15. Conditions

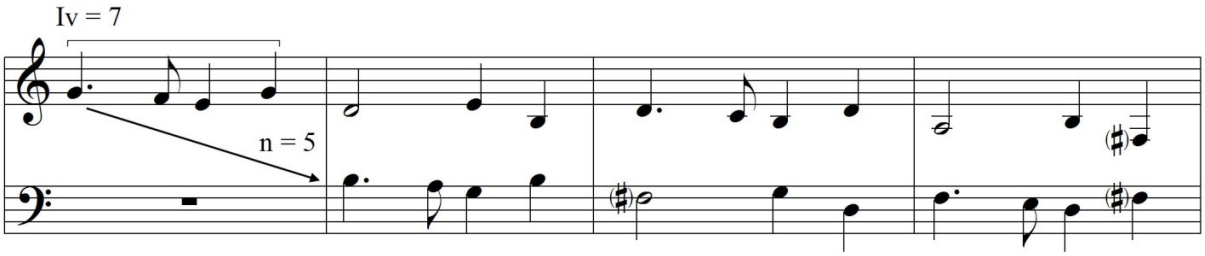
Proposta	Iv	Upper voice - downwards	Sequence at the fourth
	7	According to the table - minus -	3

16. Formula

(Iv = 2n ± m)

7 = 2n– 3
7 + 3 = 2n
10 = 2n
n = 5 – sixth

17. Completed solution, no fifths (except transient ones)



Tritones may occur - these are corrected with fictas.
The larger the sequence interval, the more difficult the exercise becomes.

An advice to the teacher - the fewer notes or leaps there are in the proposta, the easier the exercise. I recommend that it should be solved first and then given to the students/pupils - sometimes these exercises are very difficult if they have a more complex proposta.

Even the slightest error in the equation - an incorrect digit or plus/minus - will make the exercise impossible to solve.

Three-Segmented Canon

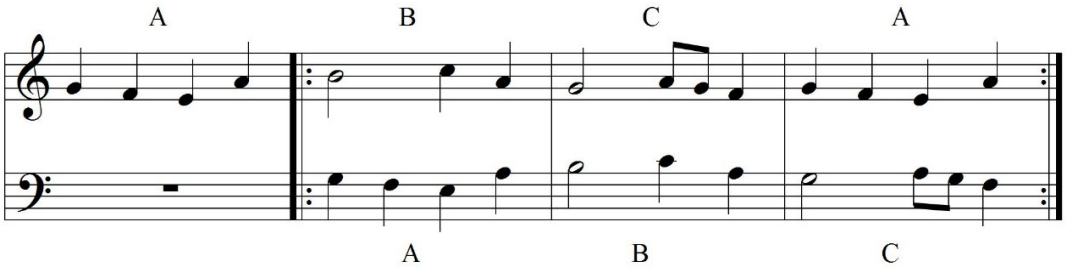
This type of canon doesn't use invertible counterpoint. The segments are three:

1.

A		B	C	A
		A	B	C

The voices swap their places, but there are no matching segments: there is A-B (reading from bottom to the top) but no B-A, there is B-C but no C-B, there is C-A but no A-C.

2. Here is an example of such a canon:



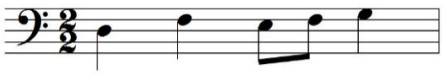
There are no mathematical formulas. The imitation interval can be any [1] – this example is a canon at the octave.

Taneyev instructed us to solve such a canon with projection - as I showed in the chapter on introduction to the canon.

Writing

The proposta is one measure:

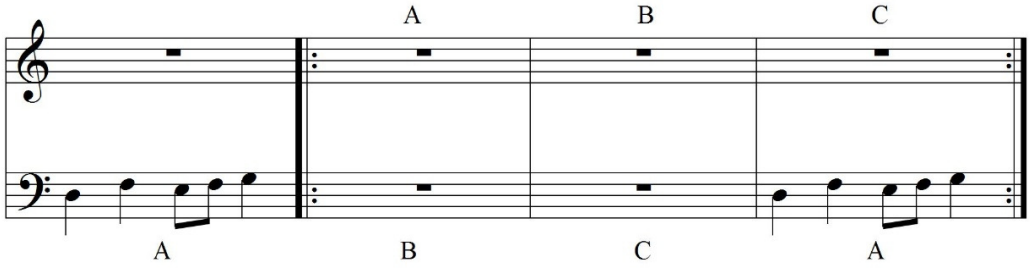
3.



First step: Since this is a lower voice, I place the proposta in the first and last measure of my construction.:

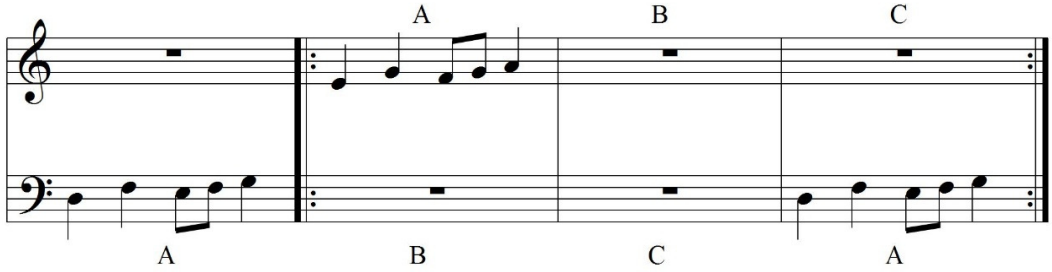
[1]If the interval of imitation is small - for example, a second, the two voices may in a sense interfere with each other or cross each other. Although this is not a problem, it can make the solution a little more difficult.There are other "more crooked" intervals of imitation, as I show a little further on.

4.



Second step: I am imitating segment A in the second measure in the upper voice at a random interval:

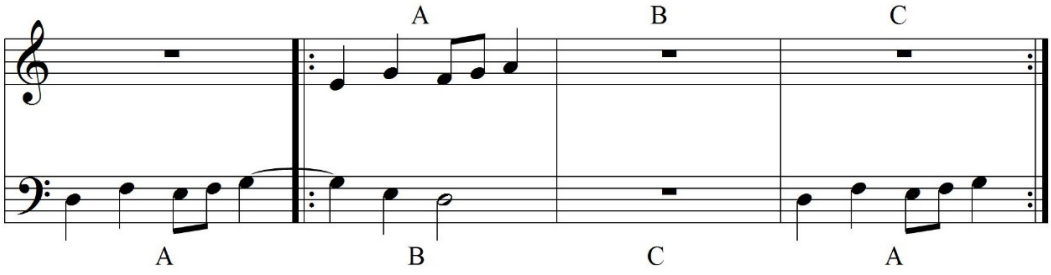
5.



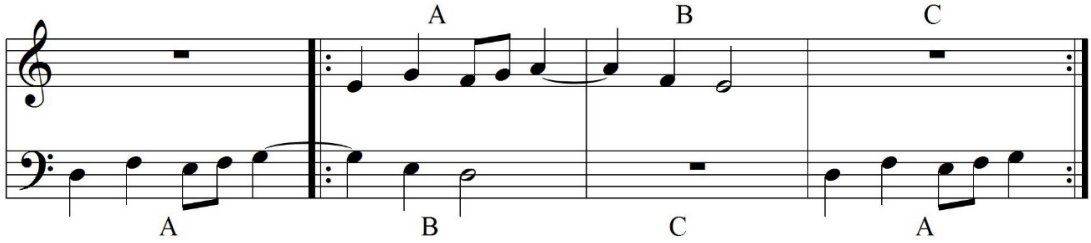
The interval of imitation is a ninth.

Third step - I am counterpointing segment A in the lower voice in step 2 - this will be my segment B:

6.



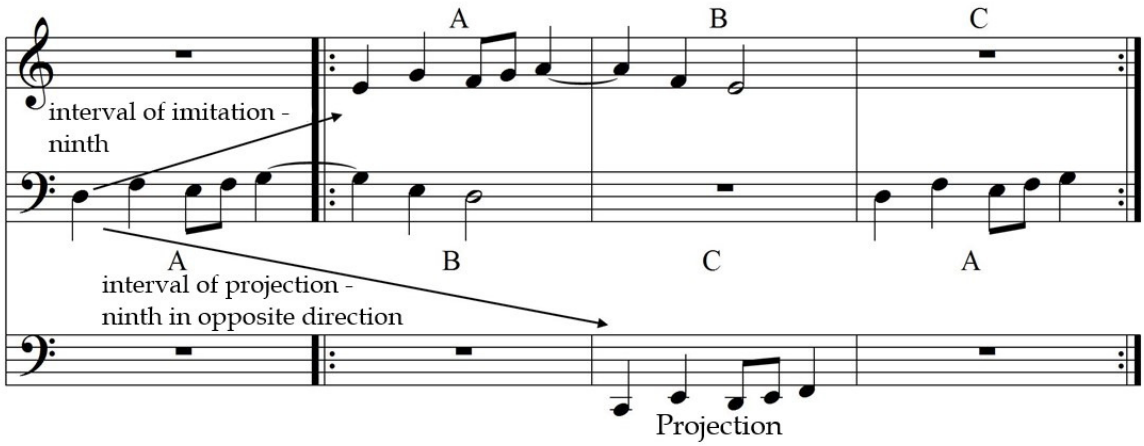
7. Fourth step - I am transcribing Segment B in the upper voice in m. 3 at the corresponding interval:



Now it's time for segment C in the lower voice, measure 3.

The projection should be written here. The first voice is the lower one. The projection should therefore be written below the lower voice. The interval of the projection should be the interval of imitation, i.e. ninth. This should be done in the opposite direction - descending, below the lower voice:

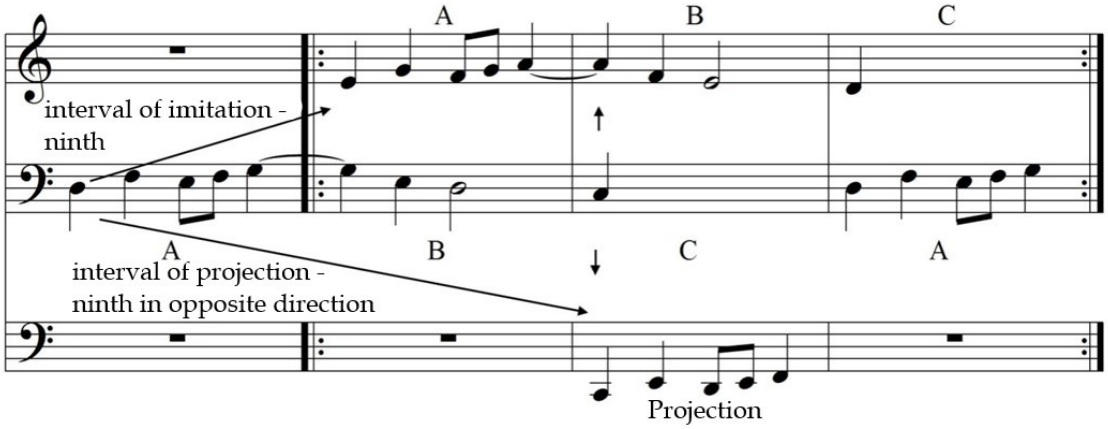
8. Fifth step - placing a projection



Now comes the most difficult part. I have to write Segment C in the lower voice, m. 3. In order to do this, I need to choose a note that forms consonances with the upper voice in m. 3 (Segment) B and *at the same time* it needs to form consonances with the projection.

This is the Sixth step.

9.



The first note of segment C is C (the only other option is E) because it forms a consonance with both the A in the upper voice (m. 3, Segment B) and the projection - also m. 3, first note - C.

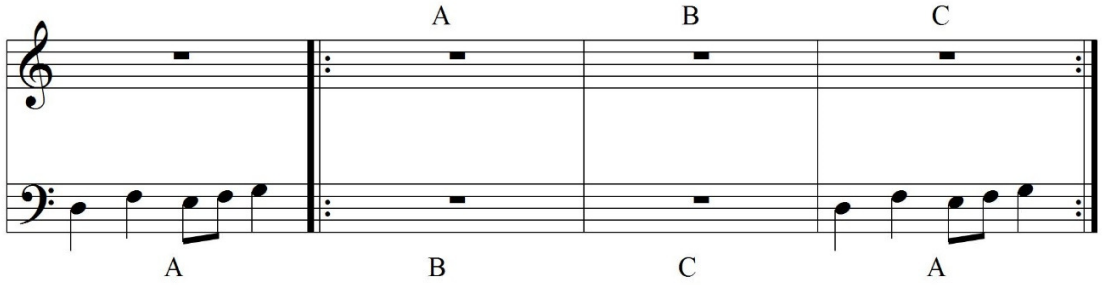
I transpose it into the upper voice in segment C at the ninth and I see that it works out.

Hereafter the exercise has no solution - it is impossible. On the second quarter note of segment C the only possible note is B (possibly B flat). I can't write this note. The exercise stops here[2].

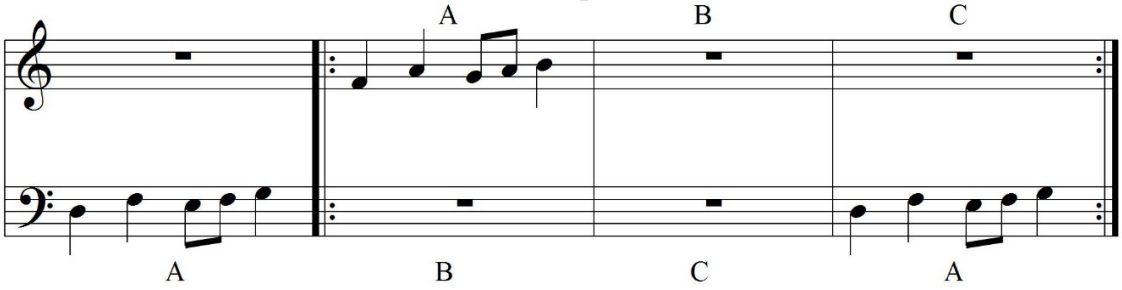
This example demonstrates a sad but true fact - there might not be a solution. In that case, I have to go back. I can keep trying different solution variants until I am satisfied that the problem can be written either 1. with a modified proposta, or 2. with a modified interval of imitation.

In the following series of examples, I show a solution of the same proposta with an interval of imitation tenth:

10. Step 1:

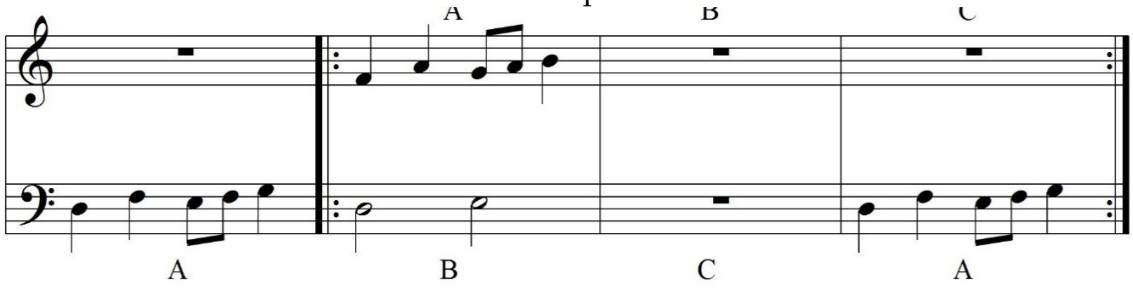


11. Step 2:

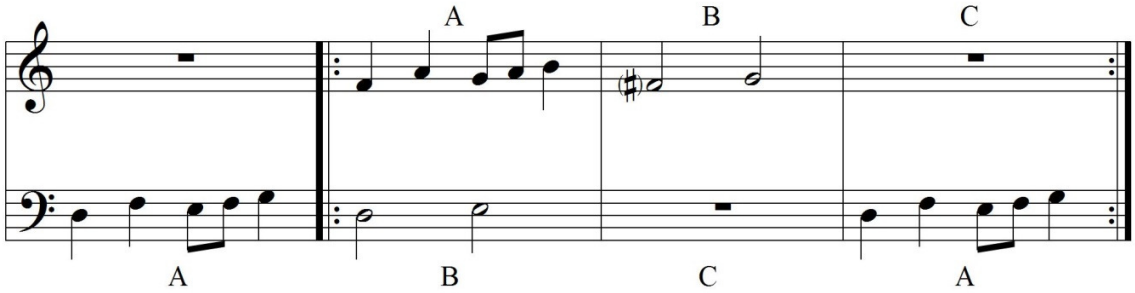


[2] The main reason for the impossibility to solve it is the interval of imitation. The note is divided into only two consonants - two fifths. All other combinations of two intervals yield one consonance and one dissonance. The direct result of this is that when resolving a projection at the ninth, the possible combinations are very few.

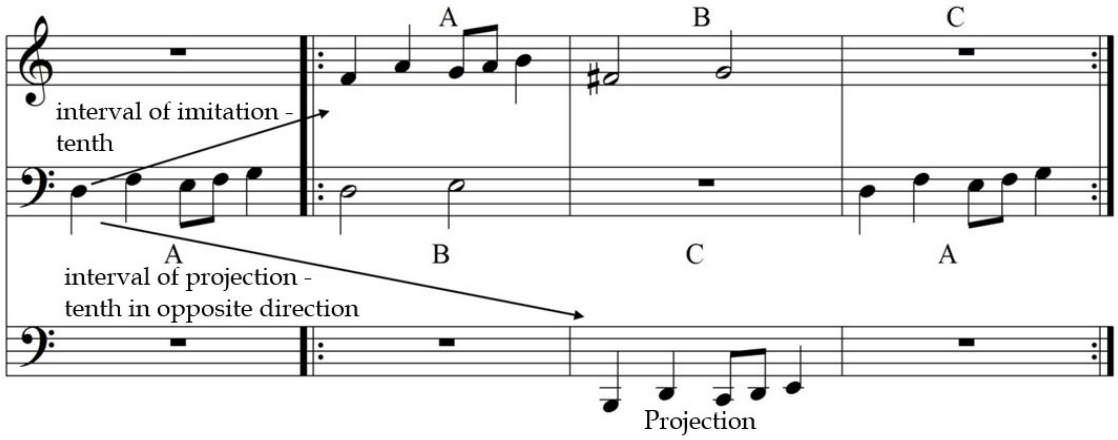
12. Step 3:



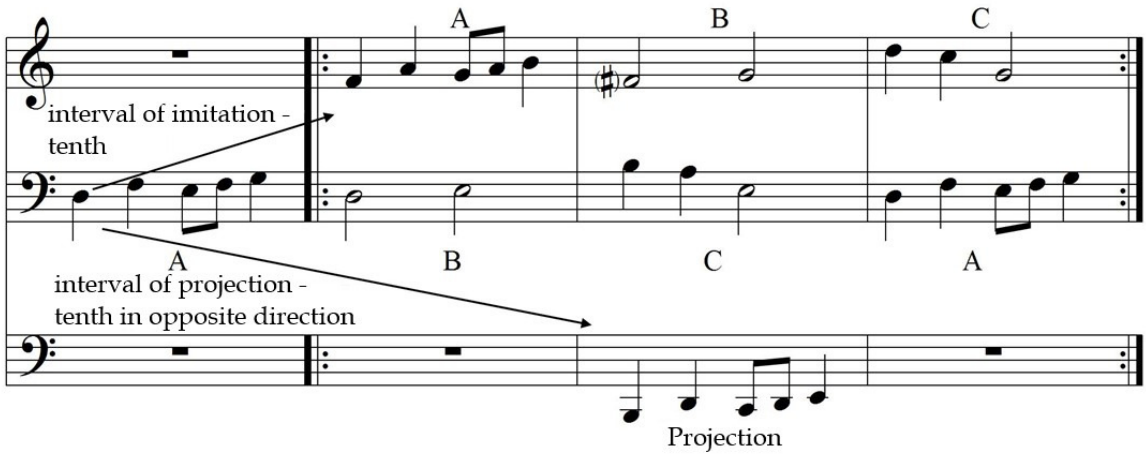
13. Step 4:



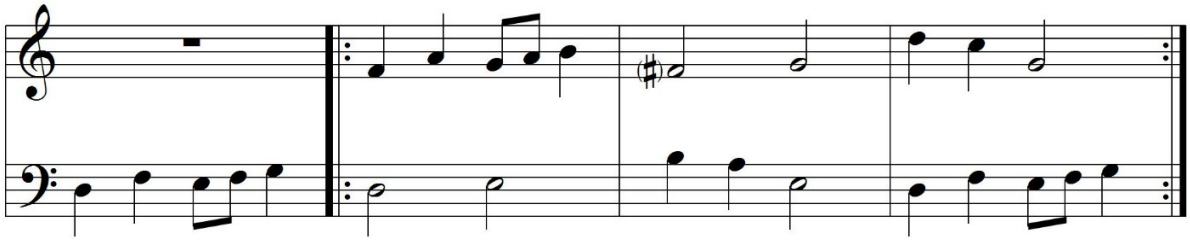
14. Step 5:



15. Step 6. Here the solution proves to be possible.



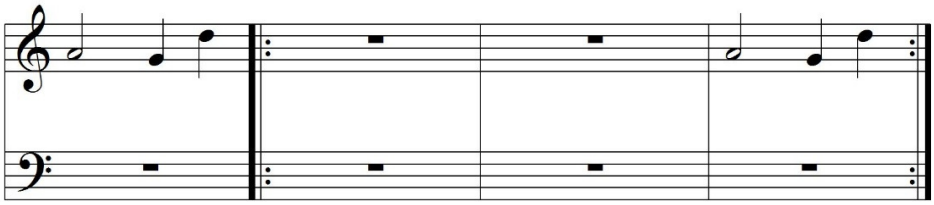
16. I erase the projection, the letters, the markings:



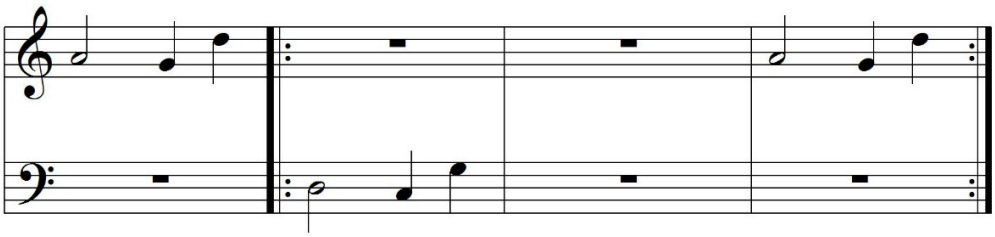
If we understand how to solve the projection, we don't need anything else. But the selected notes in Segment C must be consonant with Segment B and with the projection, and there should be no parallel or hidden intervals, nor leaps from or to dissonance anywhere. This also applies between the measures.

One more solution

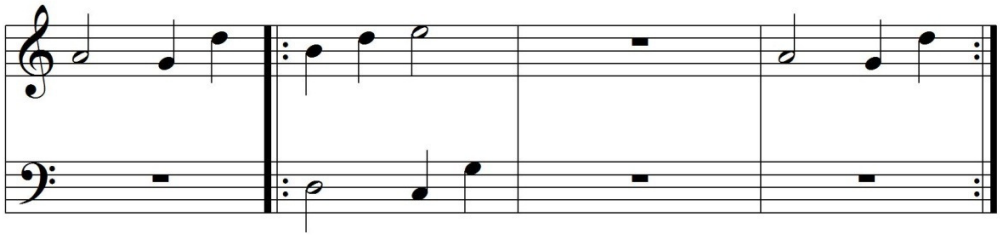
17. Step 1



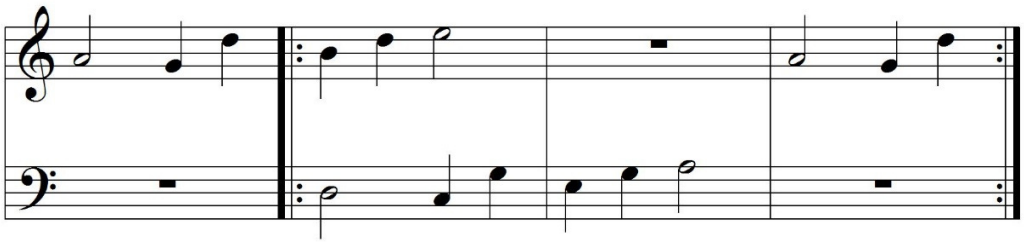
18. Step 2. Imitation at the twelfth



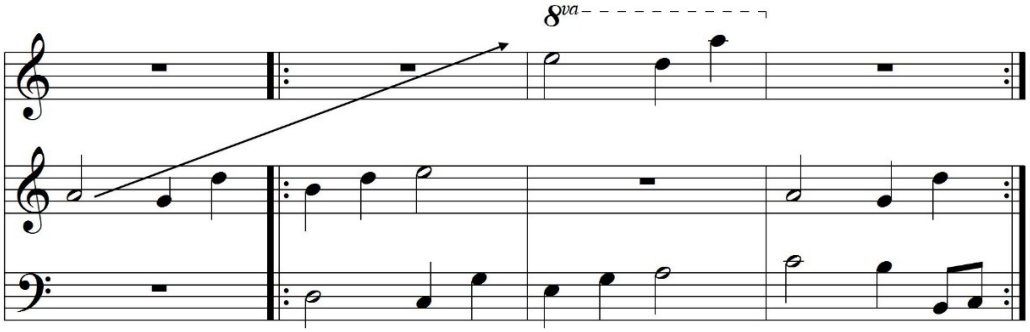
19. Step 3



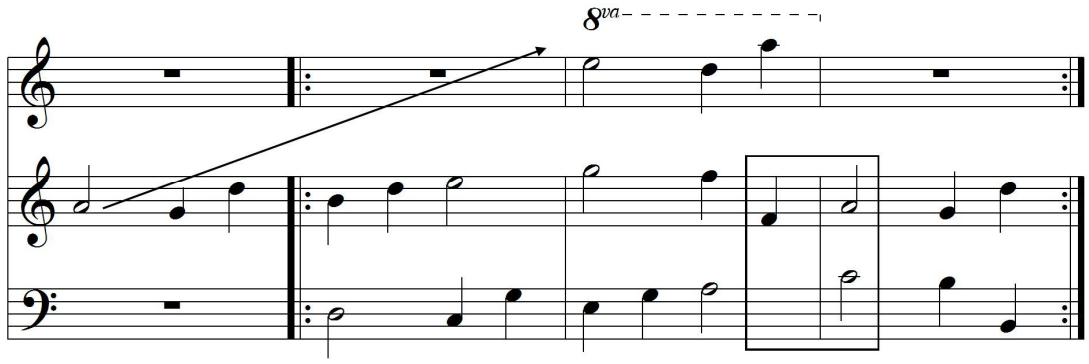
20. Step 4



21. Step 5. Projection at the twelfth from above



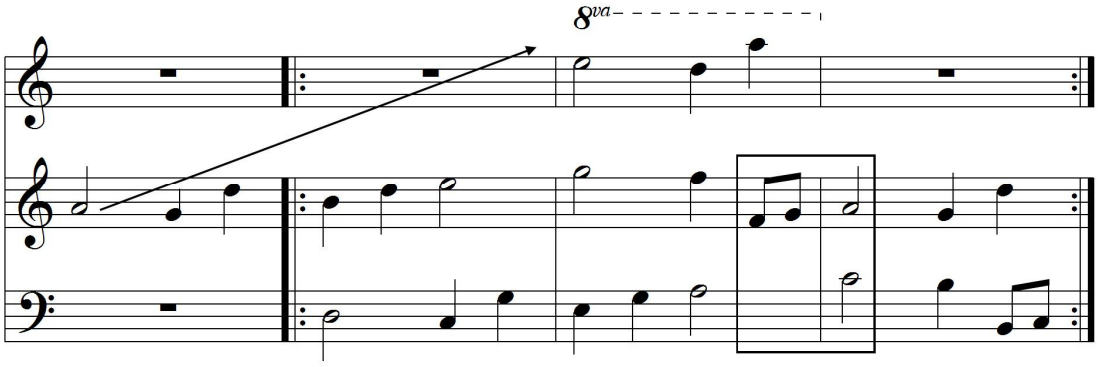
22. Step 6. Construction of Segment C



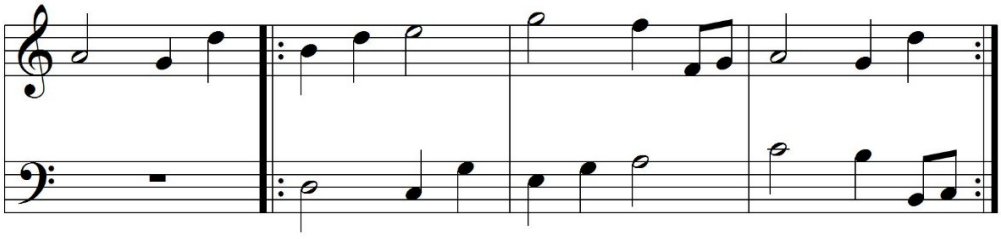
Everything seems fine with one exception. In the rectangle there is a noticed error - both voices are leaping simultaneously in one direction.

To correct this, I use a “blurring” technique - I replace the quarter note F with two eighth notes:

23.



24. Done. It took me ten minutes.



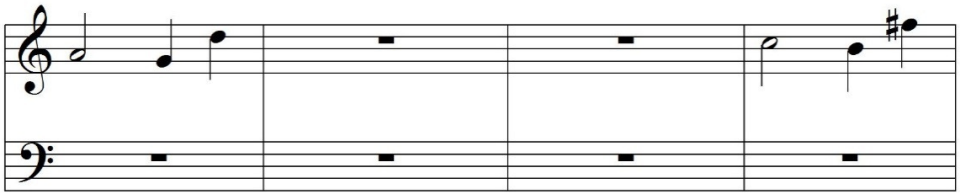
Three-Segmented Canonic Sequence

This type of canon is written in much the same way as the three-segment one.

Modified Step 1:

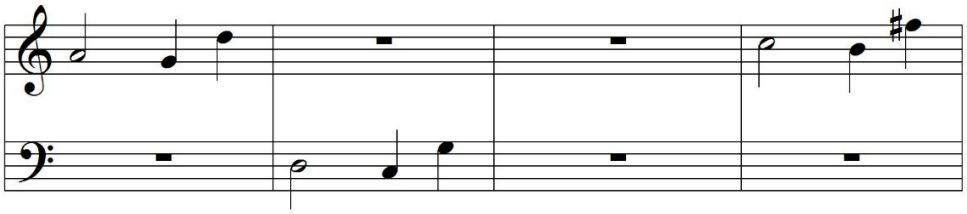
After I write the proposta in the first measure, I don't transcribe it literally in the last one. If I write it a second upwards, for example, it means that this canon is not infinite, but sequential. There are no repeat signs here either:

1. Step 1

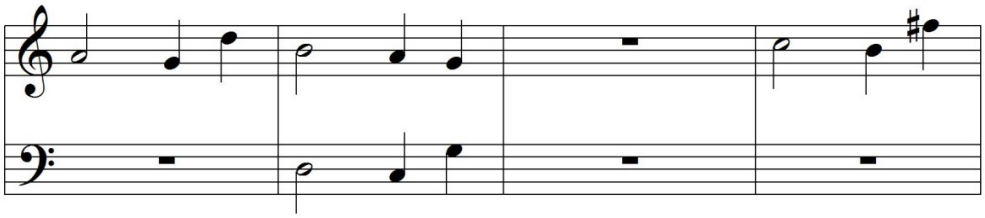


Segment A' (m. 4) is not the same as Segment A in m. 1. Thus it is clear that this is a construction that will sequence a third upwards.

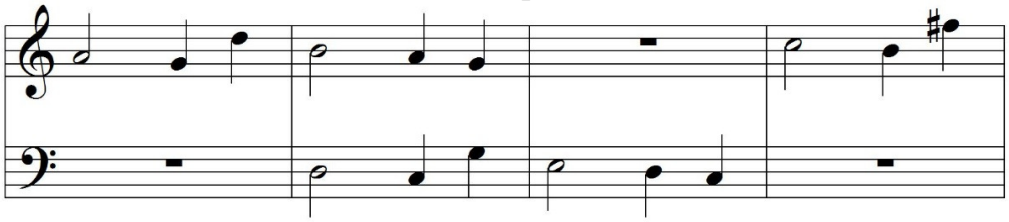
2. Step 2



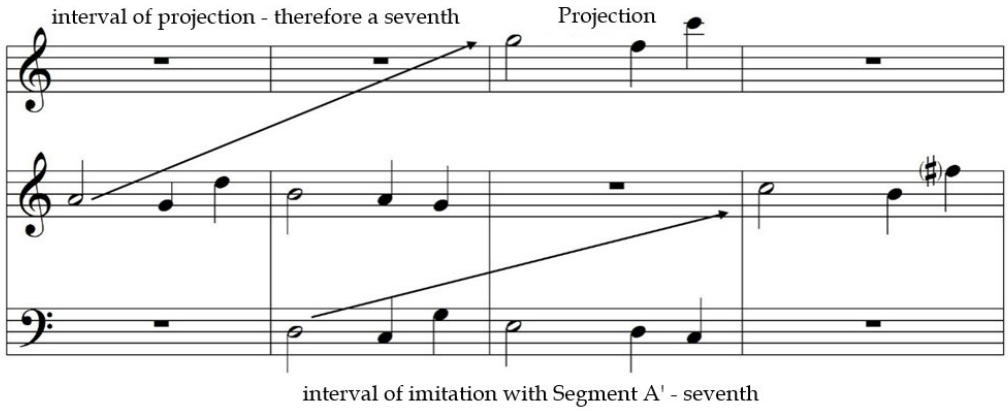
3. Step 3



4. Step 4



5. Modified Step 5:

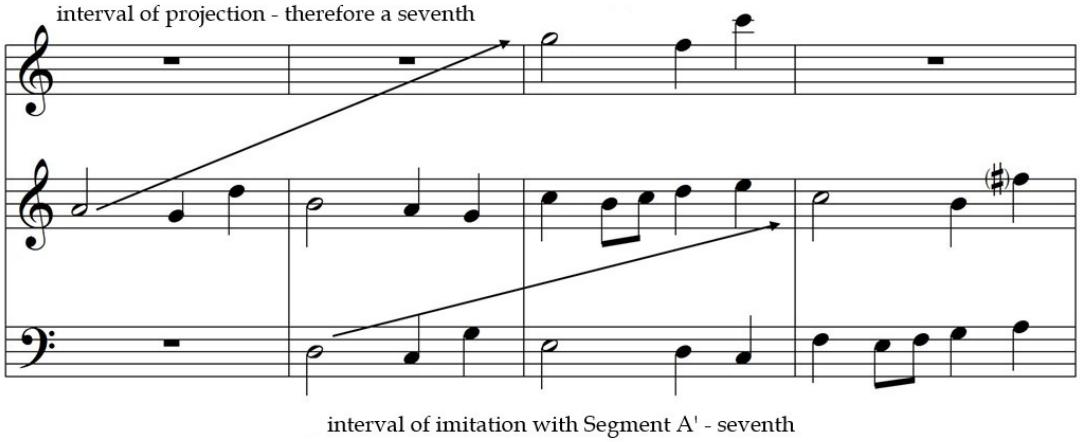


The projection interval is not derived from the distance between Segment A and Segment A in the other voice.

Instead, the interval is derived between Segment A' in the upper voice (middle staff, measure 4) and Segment A in the other (in this case lower) voice - measure 2.

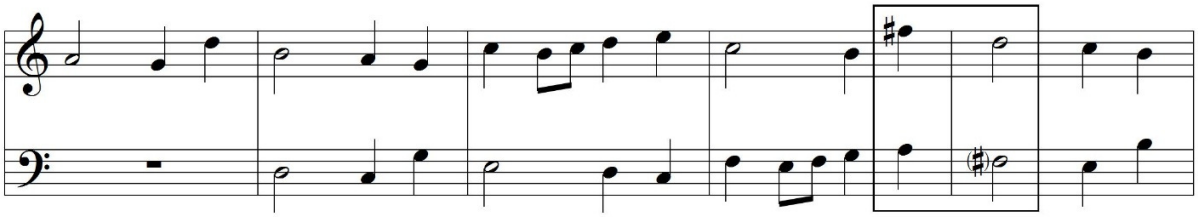
In this particular case, that distance is a seventh. The projection is therefore built a seventh upwards - in the opposite direction, as viewed from m. 1, upper voice.

6. Step 6



The exercise is *de facto* solved. I need to check for errors, and I am especially watching out for errors between the measures. Here I find that there is one similar problem:

7.

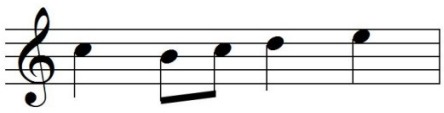
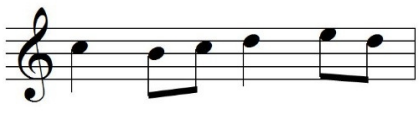


A leap of both voices in one direction.

In order to solve this problem, I go back and correct my Step 6 a bit. I am using the "blurring" technique that I showed at the end of the last chapter.

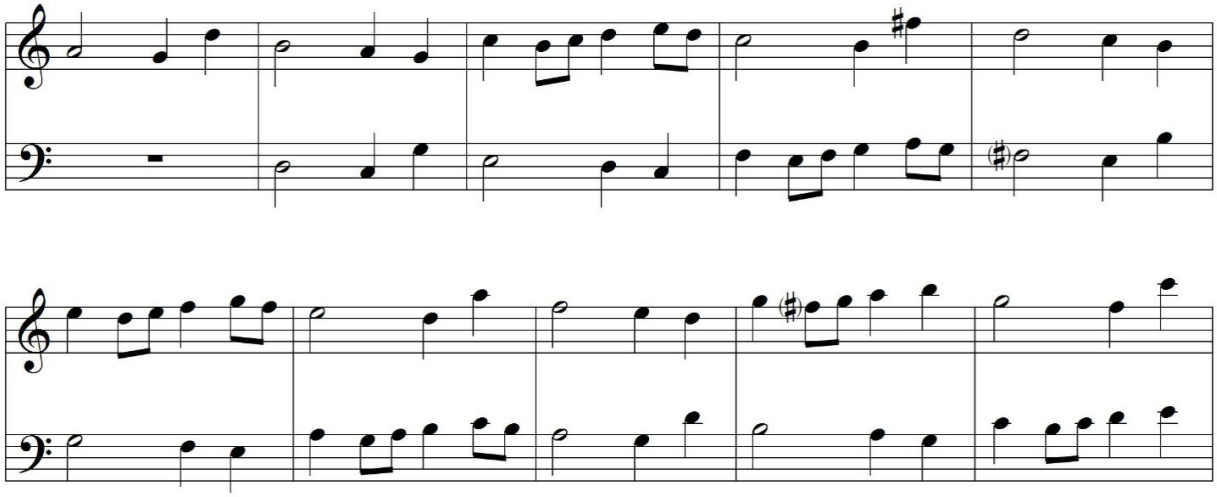
Modifying segment C:

8.

Original Segment C	Modified Segment C
	

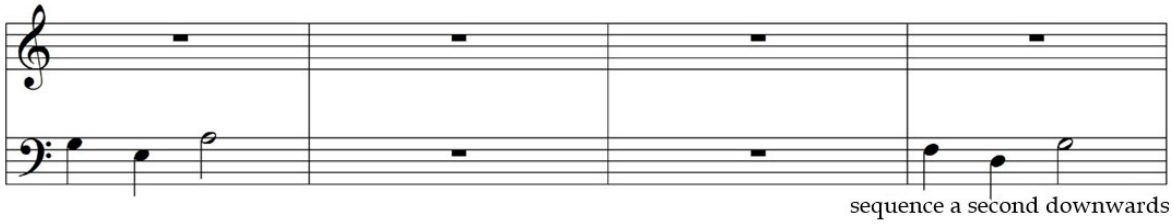
Here I show ten measures of the solved exercise:

9.

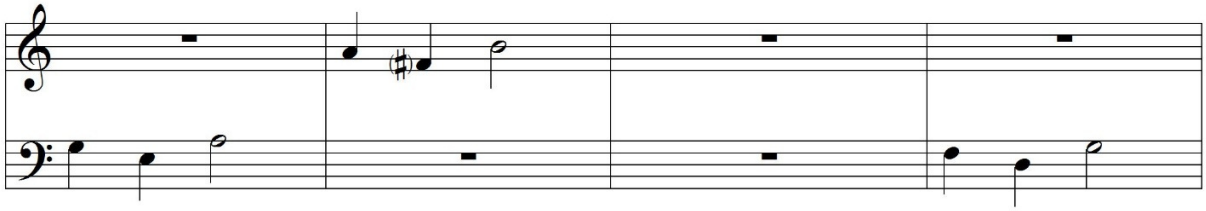


There is not always a solution. Below I am showing two more examples.

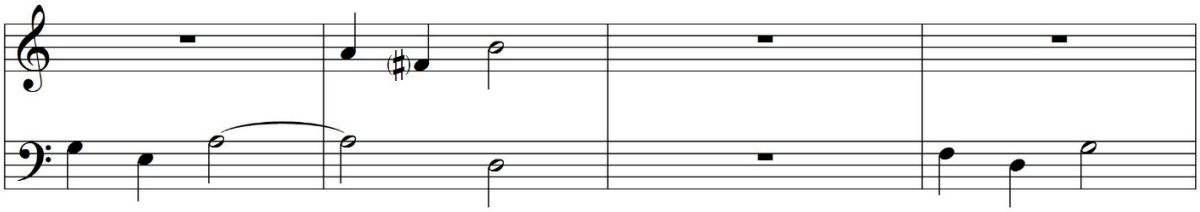
10. Step 1



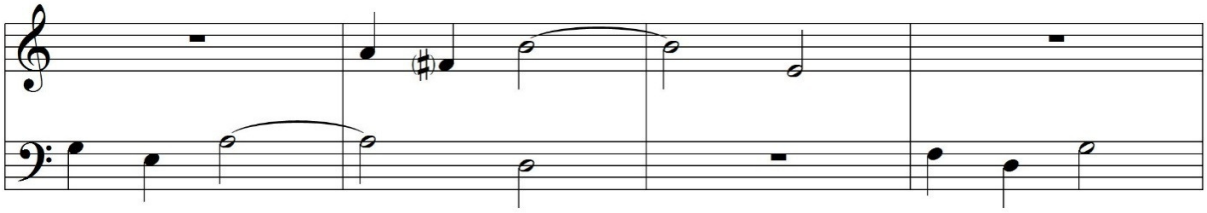
11. Step 2



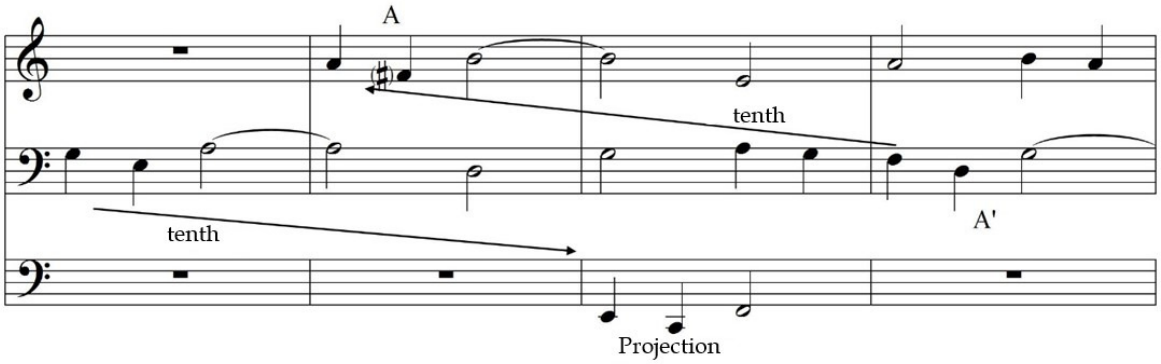
12. Step 3



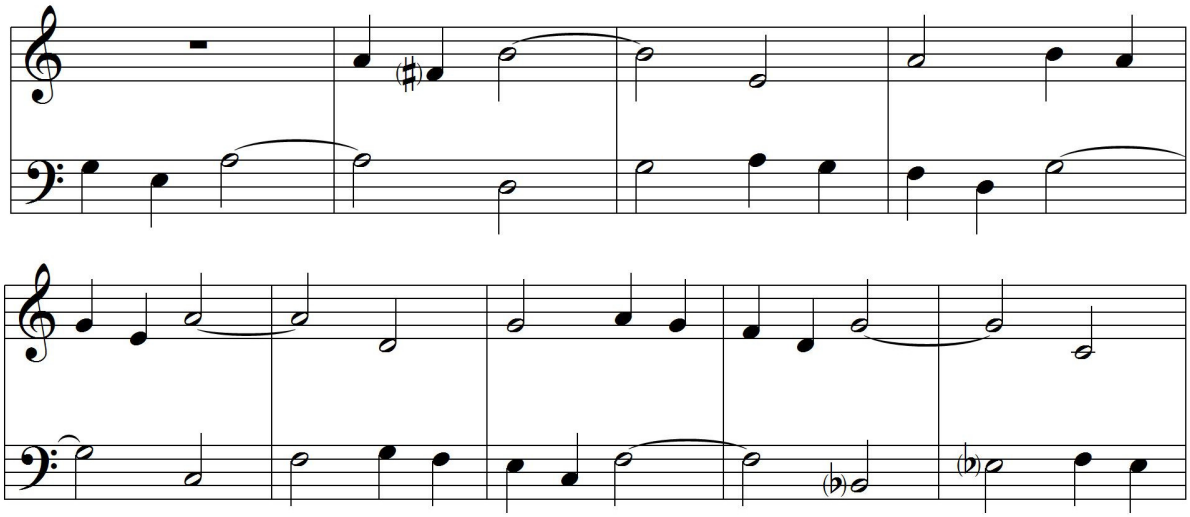
13. Step 4



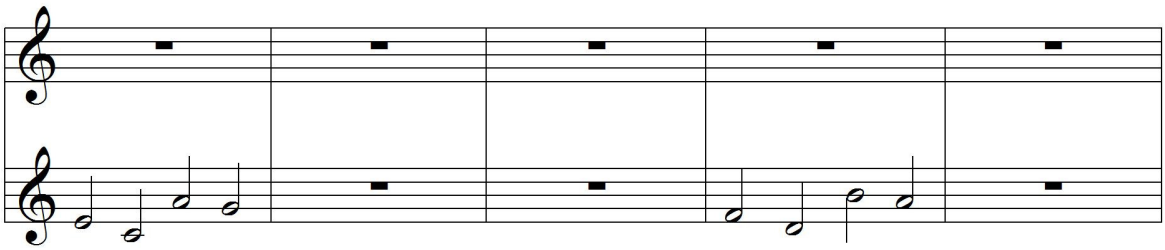
14. Step 5



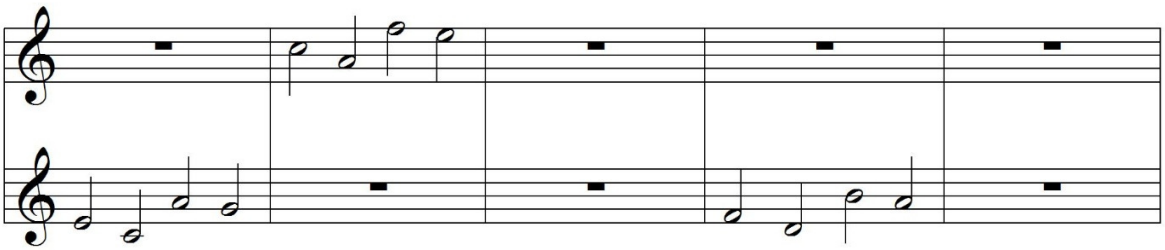
15. Result



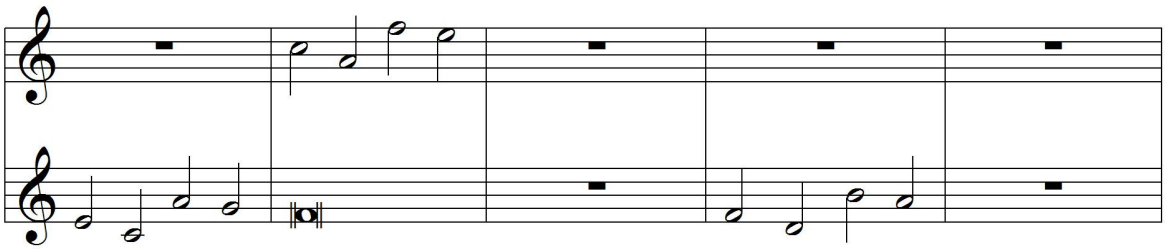
16. One more example



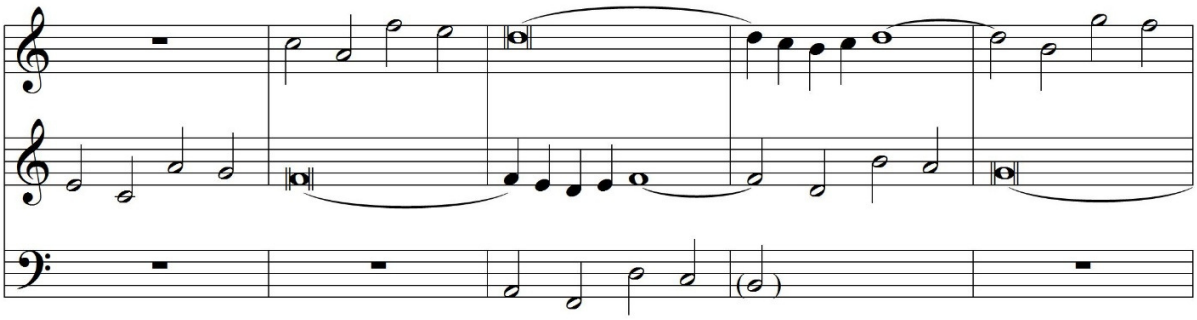
17.



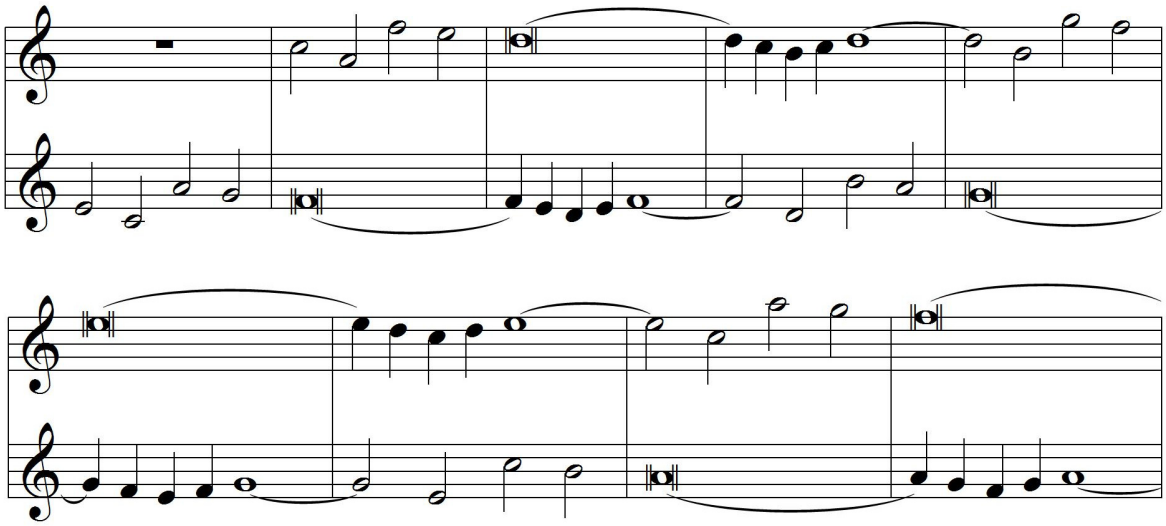
18.



19.



20. Excerpt from the result



In principle, the exercises are somewhat easier to solve if the projection interval is an octave, tenth or twelfth (in theory also fourteenth - seventh over an octave).

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